

TMSCA HIGH SCHOOL MATHEMATICS

T E S T # 3 ©

NOVEMBER 6, 2021

GENERAL DIRECTIONS

1. About this test:

A. You will be given 40 minutes to take this test.

B. There are 60 problems on this test.

2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading on Scantrons and Chatsworth cards.

3. If you are using a Chatsworth or Scantron card, please follow the specific instructions given at your particular meet.

4. You may write anywhere on the test itself. You must write only answers on the answer sheet.

5. You may use additional scratch paper provided by the contest director.

6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.

7. Calculators <u>MAY NOT</u> be used on this test.

8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.

9. In case of ties, percent accuracy will be used as a tie breaker.

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(D) **-29.625**

(C) **–**1

1. Evaluate: $\left[1 \times 2! - 3 \cdot 4\left(\frac{5!}{6}\right)\right] \div 8$

(B) –28

(A) **-29.75**

(E) 28

2. Simplify:
$$(a^{2}b^{-2})^{3} \times \frac{b^{3}}{a^{4}} + (a^{6}b^{2})^{-2}$$

(A) $\frac{1}{a^{10}b^{7}}$ (B) $a^{14}b$ (C) $a^{15}b^{2}$ (D) $\frac{a^{12}}{b}$ (E) $\frac{a^{22}}{b^{7}}$
3. 1230A897B + 3 has a remainder of 2. The greatest value of A+B is:
(A) 9 (B) 11 (C) 13 (D) 15 (E) 17
4. The x-intercept of the line parallel to \overrightarrow{AB} and passing through point P is (x,0). The value of x is:
(A) -6.5 (B) -5.5 (C) -4.5 (D) -6 (E) -5 (E

8. A triangle with side lengths 8",12",13" is considered to be which of the following:

(A)cute	(O)btuse	(R)ight		
(S)calene	(E)quilateral	(I)sosceles		
(A) A & S	(B) O & S	(C) A & I	(D) O & I	(E) R & S
9. Jeremy is measur with an angle of o	ring the height of a tr elevation of 40° from	ee in his front yard. the tip of the shadov	At 9 AM, the tree cas w to the top of the tre	sts a shadow 20 feet long e. How tall is the tree?
(A) 24.7 ft	(B) 22.3 ft	(C) 17.9 ft	(D) 16.8 ft	(E) 23.8 ft
10. Which of the fol	lowing is a tetrahedr	al number?		
(A) 6	(B) 10	(C) 21	(D) 36	(E) 60
11. What is the area	a of the shaded region	n given each grid squ	are is 1 cm ² .	
(A) 11.5 cm ²	(B) 12 cm ²	(C) 13 cm ²	E	в
(D) 12.5	5 cm ² (I	E) 13.5 cm²		C

12. Given the geometric sequence: 27, a, b, -125, ... The sixth partial sum is:

(A) $\frac{7448}{9}$ (B) $-\frac{3125}{9}$ (C) $-\frac{1862}{9}$ (D) $\frac{421}{3}$ (E) $\frac{6125}{3}$

13. The set of positive, even numbers is closed under how many of the following operations?

I. Addition	II. Subtraction	III. Multiplication	on IV. Divisio	Dn
(A) 1	(B) 2	(C) 3	(D) all of them	(E) none of them

14. Evaluate:
$$\sin\left(\frac{\pi}{6}\right) \times \cos\left(\frac{\pi}{6}\right) \times \sec\left(\frac{5\pi}{6}\right) \div \csc\left(\frac{5\pi}{4}\right) \times \tan\left(\frac{5\pi}{3}\right) \div \cot\left(\frac{11\pi}{6}\right)$$

(A) $\frac{\sqrt{2}}{4}$ (B) $-\frac{\sqrt{2}}{4}$ (C) $\frac{\sqrt{2}}{8}$ (D) $\frac{\sqrt{6}}{8}$ (E) $-\frac{\sqrt{6}}{8}$

- 15. A game is played with a die, a coin and a standard deck of cards. The player wins if two of the following events occur or if all three of the following events occur: the die lands on an even number, the coin lands on tails, and/or a Jack is drawn from the deck. What is the probability of a player winning the game?
 - (A) 2% (B) 22% (C) 25% (D) 29% (E) 33%

16. The function $f(x) = 4x^5 + 6x^2 - 12$ has a point of inflection at (x, y). The value of x + y is:

(A) -13 (B) -11 (C) -10 (D) 1 (E) -5

17. This Greek philosopher/mathematician was one of the first known to deal with the idea of infinity.

(A) Archimedes (B) Aryabhata (C) Leonardo Bigollo (D) Claudius Ptolemy (E) Zeno of Elea

18. The equilateral triangular prism shown is being used as a trough to give livestock water. Each side length of the base is 15 inches and the overall length of the trough is 20 feet. How many gallons of water can the trough hold when it is 90% full?

(A) 4.5 gal (B) 364 gal (C) 91.1 gal

(D) 30.4 gal (E) 33.7 gal



19. The two roots of $f(x) = -4x^2 + 6x + 12$ are p and q. Find the value of $2p^3 + 6p^2q + 6pq^2 + 2q^3$.

(A) $\frac{27}{16}$ (B) $\frac{3}{2}$ (C) $\frac{27}{8}$ (D) $\frac{27}{4}$ (E) $\frac{27}{2}$

20. Aiden has six colored crayons. He wants to draw four concentric circles without repeating colors. How many ways can he color the circles such that each combination is unique?

(A) 48 (B) 24 (C) 120 (D) 144 (E) 360

- 21. The function $f(x) = 3x^3 7x^2 2$ has a minimum value on the interval [1,5] that is equal to the maximum value of $h(x) = -(9x 14)^2 + C$. The value of C is:
 - (A) -8.1 (B) -7.6 (C) -6 (D) 1.56 (E) 198
- 22. Determine the area of the shaded region of the cubic function. All indicated points have integer values.
 - (A) $\frac{64}{3}$ (B) $\frac{63}{4}$ (C) $\frac{32}{3}$

(D)
$$\frac{128}{3}$$
 (E) 8



- 23. Byron has won 8 of 17 head to head math competitions this year. There are still 10 events left. What is the minimum number of competitions Byron must win in order to have a winning percentage of at least 55%?
 - (A) 5 (B) 6 (C) 7 (D) 8 (E) 9
- 24. All three lines in the diagram are coplanar and m || t. Which of the following is true?



(A) I only(B) II only(C) I & II only(D) I & III only(E) II & III only

- 25. What is the greatest evil number less than 30?
 - (A) 24 (B) 25 (C) 27 (D) 28 (E) 29
- 26. Given the function $f(x) = 2x^5 6x^4 + x^3 + 4x^2 + x + 12$, there is a possibility of how many negative real zeros?
 - (A) 1 (B) 2 or 0 (C) 3 or 1 (D) 4, 2 or 0 (E) 5, 3 or 1

(A) $2\sin(x-1)-2$ (B) $3\sin(x-1)-2$ (C) $-3\sin(x-1)-2$ (D) $3\cos(x-\frac{5}{2})+1$ (E) $-3\sin(x+2)-3$



28. Given $\sqrt[3]{x\sqrt{x\sqrt[5]{x^2}}} = \sqrt[n]{x^k}$ where n and k are relatively prime, then n - k = ?

- (A) -3 (B) -2 (C) -1 (D) 1 (E) 13
- 29. $132_5 \times 3_5 = __{10}$ (A) 44 (B) 79 (C) 126 (D) 1001 (E) 396
- **30.** It has been said that "a monkey could guess better" when a student does not make higher than a 25% on a multiple choice test that has four answer options. What is the probability on an eight question multiple choice test with four answer options a guesser can get at least 3 questions correct?
 - (A) 63% (B) 32% (C) 11% (D) 2% (E) 21%

31. Let $h(x) = \frac{2-x}{x+6}, x \neq -6$, then $h^{-1}(x) =$ (A) $\frac{2-6x}{x+1}$ (B) $\frac{x+6}{2-x}$ (C) $\frac{6x-2}{x+1}$ (D) $\frac{x-2}{-x-6}$ (E) $\frac{x+6}{x-2}$ 32. If $\frac{1}{x} = \frac{3}{y+5}$, then y =

- (A) 3(x-2) (B) 3x+5 (C) 5-3x (D) 3(x-2)+1 (E) 3(x-2)-1
- 33. Nitish is looking for an ancient relic in the forest. He has to hike deep in the woods to even have a chance of finding it. From his vehicle, he hikes 12 miles on a bearing of N54°W. Then, he hikes another 10 miles on a bearing of S70°W. How far is he from his vehicle?
 - (A) 6.7 mi (B) 10.5 mi (C) 15.6 mi (D) 17.6 mi (E) 19.4 mi

- 34. The region bounded by the x-axis and the function $f(x) = -x^2 + 6$ is rotated about the x-axis. What is the volume of the solid formed?
 - (A) 295 units³ (B) 62 units³ (C) 312 units³ (D) 148 units³ (E) 256 units³

35. A train is travelling 30 miles per hour and passes a bird flying in the same direction at 15 miles per hour. What is the relative speed with which the train passes the bird?

- (A) $22\frac{in}{sec}$ (B) $44\frac{in}{sec}$ (C) $264\frac{in}{sec}$ (D) $528\frac{in}{sec}$ (E) $123\frac{in}{sec}$
- 36. Seven students are running a race. There are three boys and four girls. In how many ways can first, second and third place be assigned such that two girls and one boy place in the top three?
 - (A) 35 (B) 108 (C) 210 (D) 216 (E) 54

37. A, B, and C are integers in the equation $\frac{37}{16} = A + \frac{1}{B + \frac{1}{C+1}}$. What is the value of A - B + C?

- (A) -3 (B) -1 (C) 0 (D) 1 (E) 3
- 38. The first few terms in a Fibonacci characteristic sequence are 2, M, N, P, 19, 31, 50, The value of M + N ÷ P is:

39. A new operator is defined such that: $a \odot b = a^b - b^a$. Evaluate $(-2 \odot -1) \odot (3 \odot -2)$.

(A) -26.9 (B) -2.8 (C) 2.8 (D) 13.4 (E) not possible

40. Ronald Reagan High School currently has 124 students enrolled in science, math and English classes. There are 34 students just enrolled in a math class and 63 enrolled in all three classes. The rest of the students are taking either science and math or science and English. No students are just taking English. How many students are taking a science class?

(A) 23 (B) 27 (C) 36 (D) 90 (E) 97

41. Albert is trying to estimate the area of the region under the curve on the interval (0,8) and has defined the following characteristics: f'(x) < 0 and f''(x) < 0 on the interval (-3,8). If he decides to use Right Riemann Sums to approximate the area, which of the following must be true?

I. The function	must be quadratic on	this interval	
II. The approxi	mation will be an unde	erestimate	
III. The approx	imation will be an ove	restimate	
(B) II only	(C) III only	(D) I & III only	(E) II & III only
			_

42. In circle A, determine the length of BC given GD = 5 in, AD = 4 in and $EF \perp CD$.

(A) I only



(A) 1.4 in (B) 3.6 in (C) 4.8 in (D) 6.4 in (E) 8 in

- 43. Consider the function $h(x) = \frac{6x^2 7x + 12}{5x^2 x 6}$. The function has a horizontal asymptote at $y = \frac{a}{b}$ and vertical asymptotes at x = c and x = d where c < d and a & b are relatively prime. The value of a + b c + d is:
 - (A) -0.2 (B) 3.8 (C) 5.8 (D) 9.6 (E) 13.2

44. Let $a_0 = 5$, $a_1 = 6$, $a_n = 2(a_{n-1}) + 3(a_{n-2})$, $n \ge 2$. What is the value of $a_3 + a_4$?

- (A) 27 (B) 93 (C) 297 (D) 426 (E) 945
- 45. The equation $-2^{-x-4} = -x^2 + 6x + 4$ has two solutions. What is the absolute value difference of the two solutions?
 - (A) 5.99 (B) 6.89 (C) 7.22 (D) 7.48 (E) 8.49

46. The graph of $4x^2 - 3y^2 - 6x + 6y - 48 = 0$ has asymptotes with a slope of ±m. What is the value of m?

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{2}{\sqrt{3}}$ (C) 4 (D) $2\sqrt{3}$ (E) $\frac{\sqrt{3}}{6}$

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- 47. A monkey gets on a Ferris wheel seat when the seat is at the minimum height. The distance from the monkey's chair to the chair 180° on the opposite side is 50 feet. What is the total distance travelled by the monkey if the Ferris wheel completes 2 rotations per minute and the monkey rides for 7.5 minutes before getting bored and jumping off?
 - (A) 814 yards (B) 628 yards (C) 393 yards (D) 785 yards (E) 1,570 yards

48. Given $\triangle ABC \sim \triangle DEF$ and AB = 6x + 2, DF = 2x + 5, AC = 4x, DE = 24. What is the length of the side opposite angle C?

(A) 5 (B) 15 (C) 20 (D) 32 (E) 38

49. A ball is being dropped off of a 258 foot building. Assuming there is no terminal velocity or wind resistance, how long does it take for the ball to hit the ground? (nearest half second)

(A) 3.5 sec (B) 4 sec (C) 4.5 sec (D) 5 sec (E) 5.5 sec

50. Let $(6+20i)(3+2i) \div (5-i) = a + bi$, then a - b = ?

(A) -20 (B) -6 (C) 6 (D) 20 (E) 21

51. Jester's Pizza Company is competing with King's Pizza Company for business. Jester's Pizza Company printed flyers offering 3 sizes of crust, 4 types of cheese and 8 unique toppings. King's Pizza Company offers 4 sizes of crust, 5 types of cheese and 9 unique toppings. Assuming each pizza ordered has one size of crust, one type of cheese and no doubling of toppings, how many more pizza combinations does King's Pizza Company offer?

- (A) 84 (B) 3,072 (C) 7,168 (D) 8,124 (E) 10,240
- 52. Sarah decides to order from Jester's Pizza because Jester's is offering 10% off when ordering ten extra large, one topping pizzas that are pepperoni, ham or sausage. In how many ways can Sarah order the ten pizzas to get the discount?
 - (A) 36 (B) 45 (C) 120 (D) 66 (E) 72
- 53. Based on the graphed function, f(x), how many of the statements are true?



- 54. A large cube is formed from 1331 identical cubic blocks. How many cubes can be seen from the outside if the large cube formed is completely solid?
 - (A) 222 (B) 482 (C) 596 (D) 602 (E) 726

55. Victor wants to take his family to see *The Monkey's Paw*. It costs 2 adults and 5 children \$62.00 before tax and 4 adults and 2 children \$60 before tax. What is the final cost of Victor taking two other adults and three children if the tax rate is 9%?

(A) \$50.14 (B) \$62.13 (C) \$66.49 (D) \$61.07 (E) \$64.29

 $56. \int e^x x^2 dx = ?$

(A)
$$e^{x}\left(\frac{x^{3}}{3} + \frac{x}{2} + \frac{1}{2}\right) + C$$
 (B) $\frac{e^{x}x^{3}}{3} + C$ (C) $e^{x}\left(x^{2} + 2x + 2\right) + C$
(D) $e^{x}\left(x^{2} - 2x + 2\right) + C$ (E) $e^{x}\left(-x^{2} + 2x - 2\right) + C$

57. Let $g(x) = 2\sin(3x - \pi) - 4$. What is the value of the phase shift plus the maximum of the function?

(A) -3.05 (B) -0.95 (C) 1.14 (D) 2.05 (E) 3.05

58. Use the following table to approximate a left Riemann sum for $\int f(x) dx$ using four subintervals of equal length.

		X	2	3	4	6	8	9	10			
		f (x)	3	6	8	9	8	7	5			
(A) 23	(B) 28		(C) 56	5				(D)	60	(E)	76

59. The inner bullseye of a dartboard has a diameter of one half inch and a playable diameter of seventeen and three quarters inches. What are the odds that a single dart thrown without attempting to aim lands in the inner bullseye, given the dart hits in the playable area?

(A) 4:5,037 (B) 2:69 (C) 4:5,041 (D) 2:71 (E) 4:3,705

60. How many of the following series are convergent?

I. $\sum_{n=2}^{\infty} \left(\frac{2}{3}\right)^n$ II. $\sum_{n=3}^{\infty} \frac{1}{n^2 - n - 2}$ III. $\sum_{n=1}^{80} \left(\frac{3}{2}\right)^{n+1}$ IV. $\sum_{n=0}^{\infty} \frac{(-1)^{n-3} \sqrt{n}}{n+2}$ (A) 1 (B) 2 (C) 3 (D) none of them (E) all of them

1.	Α	21. B	41. B
2.	В	22. A	42. C
3.	Ε	23. C	43. E
4.	В	24. D	44. C
5.	С	25. E	45. C
6.	Α	26. C	46. B
7.	В	27. B	47. D
8.	Α	28. E	48. D
9.	D	29. C	49. B
10.	В	30. B	50. A
11.	D	31. A	51. C
12.	C	32. D	52. D
13.	В	33. E	53. B
14.	Α	34. A	54. D
15.	D	35. C	55. B
16.	В	36. B	56. D
17.	Ε	37. E	57. B
18.	C	38. B	58. C
19.	D	39. A	59. A
20.	Ε	40. D	60. E

2.
$$= a^{6}b^{-6}a^{-4}b^{3}a^{12}b^{4}$$

$$= a^{14}b$$
3.
$$30 + A + B = 2 \mod 3$$
4.
$$y = 3 + \frac{2}{3}(x+1)$$
5.
$$\frac{5(88.6) + T_{6}}{6} = 89.5$$
6.
$$let x = -1$$
8.
$$8^{2} + 12^{2} > 13^{2}$$
9.
$$h = 20 \tan 40^{\circ}$$
10.
$${}_{5}C_{3}$$
11.
$$\frac{1}{2} | (2 \cdot 0 + 1 \cdot 1 + 5(-1) + 4(-3) + 5 \cdot 0) - (0 \cdot 1 + 2 \cdot 5 + 1 \cdot 4 + (-1) \cdot 5 + (-3) \cdot 0) |$$
27
$$r^{3} = -125$$
12.
$$8_{6} = 27 \left(\frac{1 - (-5/3)^{6}}{1 - (-5/3)} \right)$$
13. Addition, Multiplication

14.
$$\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)\left(\frac{-2}{\sqrt{3}}\right)\left(\frac{-\sqrt{2}}{2}\right)\left(-\sqrt{3}\right)\left(\frac{-1}{\sqrt{3}}\right)$$

Respectively, consider WWW + WLW + LWW + WWL 15. $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{13}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{13}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{$

16.
$$\begin{aligned} f''(x) &= 80x^3 + 12 \\ POI &= (-0.53, -10.48) \end{aligned} \qquad V &= Bh \\ 18. V &= \left(\frac{\sqrt{3}(15/12)^2}{4}\right)(20)(0.9)(7.481) \end{aligned}$$

$$p = -1.14$$
19. $q = 2.64$

$$20._{6}P_{4}$$
21. min @ $\left(\frac{14}{9}, -7.646\right)$
22. $\int_{-1}^{3} (x+1)(x-3)^{2} dx$

$$2(p+q)^{3}$$

23.
$$\frac{8+x}{27} > 0.55$$
 24. I & III true 26. $\frac{f(-x) = -2x^5 - 6x^4 - x^3 + 4x^2 - x + 12}{3 \text{ sign changes}}$

28.
$$x^{\frac{1}{3}\left(\frac{1}{2}(2/5+1)\right)+1} = x^{17/30}$$
 29. $(5^2 + 3 \cdot 5 + 2)(3)$ 30. $1 - \text{binomcdf}\left(8, \frac{1}{4}, 2\right)$

$$\begin{aligned} x &= \frac{2 - y}{y + 6} \\ 31. & x(y + 6) = 2 - y \\ xy + y = 2 - 6x \\ b^{-1}(x) &= \frac{2 - 6x}{x + 1} \end{aligned}$$

$$32. & y = 3x - 5 \\ y = 3(x - 2) + 1 \end{aligned}$$

$$33. & d^{2} = 10^{2} + 12^{2} - 2(10)(12)\cos(70^{\circ} + 54^{\circ}) \\ y = 3(x - 2) + 1 \end{aligned}$$

$$34. & \pi \int_{-10}^{\sqrt{6}} (6 - x^{2})^{2} dx$$

$$35. & 15\left(\frac{22}{15}\right)(12)$$

$$36. & (_{3}C_{1})(_{1}C_{2})(_{3}P_{3})$$

$$37. & B = 3 \\ 5 = C + 1 \\ C = 4 \\ 2 - 3 + 4 \end{aligned}$$

$$\begin{aligned} M + N - P = 0 \\ M - N = -2 \\ 38. & N + P = 19 \\ M = 5, N = 7, P = 12 \\ 5 + 7/12 \end{aligned}$$

$$39. & \left(-\frac{3}{2}\right) \odot \left(\frac{73}{9}\right)$$

$$40. & 124 - 34 \qquad 41. \text{ II only} \end{aligned}$$

$$\frac{5}{4} = \frac{8}{x + 5} \\ 42. & \frac{BG}{BG} = x = 1.4 \\ 6.4^{2} + (BC)^{2} = 8^{2} \end{aligned}$$

$$43. & x = -1, x = \frac{6}{5} \\ 44. & a_{3} = 2(72) + 3(6) = 72 \\ 6 + 5 - (-1) + \frac{6}{5} \\ a_{4} = 2(72) + 3(27) = 225 \end{aligned}$$

$$4\left(x^{2} - \frac{3}{2}x\right) - 3(y^{2} - 2y) = 48 \\ 4(x^{-3})^{2} - 3(y^{-1})^{2} = 54 \\ 45. & x = -0.62, 6.61 \end{aligned}$$

$$46. & \frac{(x - 3)^{2}}{54/4} - \frac{(y - 1)^{2}}{54/3} = 1 \\ m = \pm \sqrt{\frac{54/3}{54/4}} = 1 \\ m = \pm \sqrt{\frac{54/3}{54/4}} = 1 \\ m = \pm \sqrt{\frac{54/3}{54/4}} \end{aligned}$$

51.
$$(4)(5)\left(\sum_{n=0}^{9} {}_{9}C_{n}\right) - (3)(4)\left(\sum_{n=0}^{8} {}_{8}C_{n}\right)$$

52. ${}_{12}C_{10}$
53. I & IV
2A + 5C = 62
54. $2(11)^{2} + 2(9)(11) + 2(9)^{2}$
55. $4A + 2C = 60$
1.09 $(3 \cdot 11 + 3 \cdot 8)$

56.

Alternating Signs	u	dv
+	\mathbf{x}^2	e ^x
-	2x	e ^x
+	2	e ^x
-	0	e ^x

57. phase shift = $\frac{\pi}{3}$	58. $2(3+8+9+8)$
max imum = -2	

59.
$$\frac{\pi(1/4)^2}{\pi((71/8)^2 - (1/4)^2)}$$

true

PLACE LABEL IN THIS RECTANGLE

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FILL IN RECTANGLE TO THE RIGHT

Name
Grade ID Number
School
City
Classification
FINAL SCORE

HIGH SCHOOL MATHEMATICS ANSWER SHEET CAPITAL PRINTED LETTERS REQUIRED

1	21	41
2	22	42
3	23	43
4	24	44
5	25	45
6	26	46
7	27	47
8	28	48
9	29	49
10	30	50
11	31	51
12	32	52
13	33	53
14	34	54
15	35	55
16	36	56
17	37	57
18	38	58
19	39	59
20	40	60
******	******	******
1 st Grader	2 nd Grader	3 rd Grader
# Correct	# Correct	# Correct
# Incorrect	# Incorrect	# Incorrect
6 x # Correct =	6 x # Correct =	6 x # Correct =
2 x # Incorrect =	2 x # Incorrect =	2 x # Incorrect =
Score =	Score =	Score =