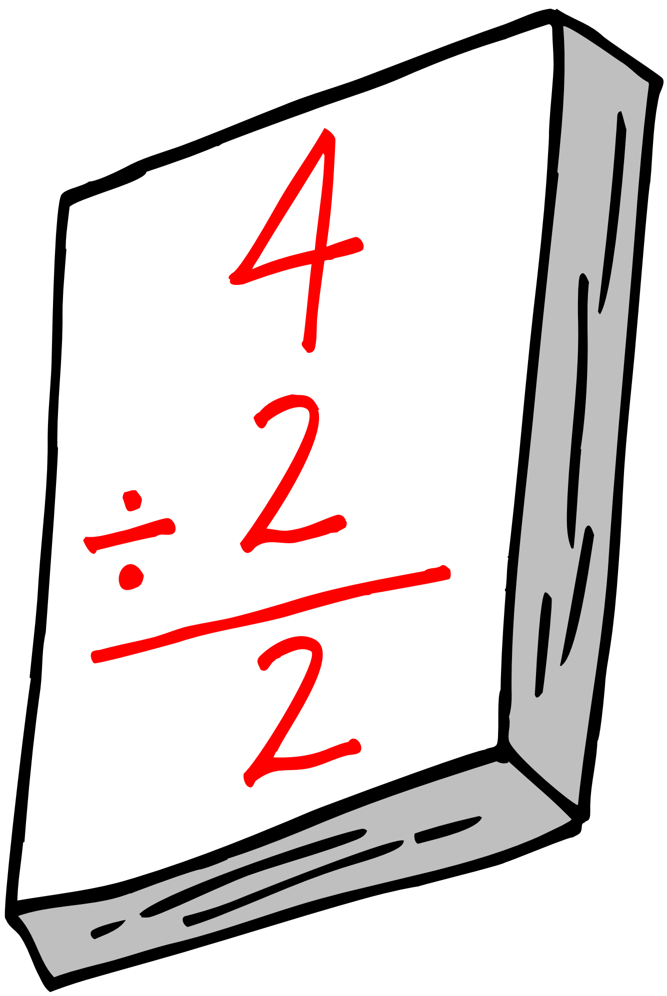
UNIVERSITY INTERSCHOLASTIC LEAGUE

# Mathematics

## State • 2017




DO NOT TURN THIS PAGE UNTIL

YOU ARE INSTRUCTED TO DO SO!

1. **Evaluate: $(1 - 2) + 3^4 - 5 \div 5 \times (4^3 + 2) - 1$**

**(A) $-81\frac{1}{66}$ (B) $-56$ (C) 1 (D) 13 (E) 989**

2. **Find the sum of the multiples of 8 that are greater than 20 and less than 200.**

**(A) 2,352 (B) 2,376 (C) 2,396 (D) 2,576 (E) 2,600**

3. **Three million two hundred one thousand four hundred twenty-two is subtracted from one billion, two million, three hundred fifty-seven thousand eleven. What is the sum of the digits in the difference?**

**(A) 34 (B) 42 (C) 50 (D) 60 (E) 62**

4. **On a map legend, 2.5 inches represents 500 miles. Big Sur, California is 1 foot 3 inches from Surfside, Florida on the map. What is the distance from Big Sur to Surfside?**

**(A) 3,250 miles (B) 3,750 miles (C) 2,600 miles (D) 2,750 miles (E) 3,000 miles**

5. **If $24x^2 + ax - 15 = (bx - 5)(4x + c)$ then $a + b + c$ = _____.**

**(A) $-1$ (B) 1 (C) 5 (D) 7 (E) 11**

6. **Let p and q be the roots of $6x^2 - 3x - 1 = 0$. Find $p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4$.**

**(A) 0.0625 (B) 1.23456... (C) 8. (D) 6.25 (E) 0.375**

7. **The equation of a line through point P (2, 4) and perpendicular to the line $3x - 5y = 1$ is $ax + by = c$. Find $a + b + c$.**

**(A) $-20$ (B) $-14$ (C) 6 (D) 10 (E) 30**

8. **What is the sum of the digits in the tens place and the units place of $7^{(91)}$ ?**

**(A) 1 (B) 3 (C) 7 (D) 9 (E) 13**

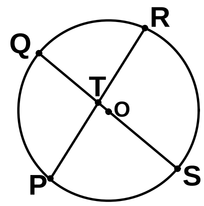
9. **Simplify:** $2\sqrt[3]{8w^5} \div \sqrt[4]{16w^8}$

**(A) $\frac{1}{2w}$ (B) $\frac{2}{\sqrt[3]{w}}$ (C) 2w (D) $\frac{1}{w}$ (E) $\sqrt[3]{w}$**

10. **The sum of the measures of the interior angles of a single face of a regular convex dodecahedron is:**

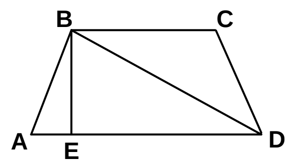
**(A) 180° (B) 360° (C) 540° (D) 720° (E) 900°**

11. **Given the circle with center O shown. Find $m\overset{\frown}{PQ}$ if $m\angle QTR = 75°$ and $m\overset{\frown}{RS} = 110°$.**




**(A) 110°　(B) 105°　(C) 95°　(D) 75°　(E) 70°**

12. **ABCD is an isosceles trapezoid with altitude BE = 20 cm and diagonal BD = 25 cm. What is the area of ABCD? (nearest cm)**




**(A) 187.5 $cm^2$　(B) 250 $cm^2$　(C) 300 $cm^2$　(D) 375 $cm^2$　(E) not enough information given**

13. **The point $(-5, 12)$ is rotated 630° clockwise about the origin. The coordinates of the point after the rotation is _____.**

**(A) $(-5, -13)$　(B) $(5, 12)$　(C) $(5, -12)$　(D) $(5, 13)$　(E) $(-5, -12)$**

14. **If $\frac{Ax-8}{3x-7} + \frac{2x-B}{4x+1} = \frac{34x^2-42x-1}{12x^2-25x-7}$, where A and B are constants, then $(A+B)(A-B)$ equals:**

**(A) 64　(B) 36　(C) 42　(D) 56　(E) 48**

15. **Let $f(x) = 2 - x$, $g(x) = x + 2$, $h(x) = 1 - 2x$, and $f\Big(g\big(h(x)\big)\Big) = 3$. Find x.**

**(A) 5　(B) 4　(C) 3　(D) 2　(E) 1**

16. **If 7 QTs equal 5 MTs and 3 MTs equal 2 ETs, then how many ETs does it take to make 4 QTs?**

**(A) $2\frac{1}{10}$　(B) $\frac{10}{21}$　(C) $1\frac{19}{21}$　(D) $1\frac{2}{19}$　(E) $3\frac{11}{15}$**

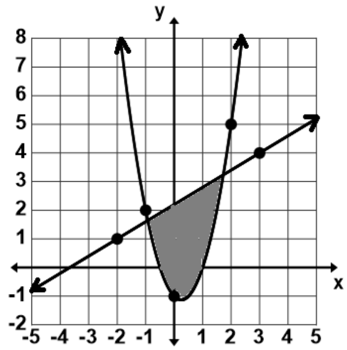
17. **The roots of the equation $x^3 + bx^2 + cx + d = 0$ are $3 + i$, $3 - i$, and 2. Find $b + c + d$.**

**(A) $-6$　(B) 10　(C) $-50$　D) $-10$　(E) 4**

18. **Which of the following is an identity for $(\tan^2\theta) \div (\tan^2\theta + 1)$?**

**(A) $1 + \cot^2\theta$　(B) $\sin^2\theta + 1$　(C) $\cot^2\theta$　(D) $\sin^2\theta$　(E) $\sec^2\theta$**

19. Which of the following system of inequalities would be best represented by the shaded region?




(A) $3x - 5y \geq 11$
$y \leq 2x^2 - x - 1$

(B) $3x - 5y \leq -11$
$y \geq 2x^2 - x - 1$

(C) $3x - 5y \leq 11$
$y \leq 2x^2 - x + 1$

(D) $3x - 5y \geq -11$
$y \geq 2x^2 - x + 1$

(E) $3x - 5y \geq -11$
$y \geq 2x^2 - x - 1$

20. Which of the following expressions is not equal to 1?

(A) $\cot(\theta)\sin(\theta)\sec(\theta)$ (B) $\tan(\theta)\csc(\theta)\cos(\theta)$ (C) $\cos^2(\theta) + \sin^2(\theta)$
(D) $\sec^2(\theta) - \tan^2(\theta)$ (E) $\cot^2(\theta) - \sec^2$

21. Given the geometric sequence a, b, 45, c, d, 1215, ... find $a + b + c + d$.

(A) 410 (B) 560 (C) 605 (D) 1,260 (E) 1,812

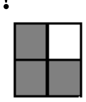
22. Let $x^5 + 4x^4 - 3x^2 + x - 6 = 0$. According to Descartes' Rule of Signs, how many possible negative roots are there?

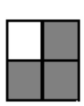
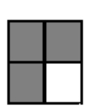
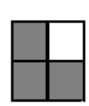
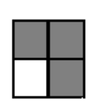
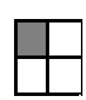
(A) 2 or 0 (B) 3 or 1 (C) 0 (D) 4, 2, or 0 (E) 5, 3, or 1

23. Find $m + n$ if $\begin{bmatrix} -1 & 2 \\ m & 3 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ n \end{bmatrix} = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$

(A) $2\frac{3}{8}$ (B) $-4\frac{1}{2}$ (C) $\frac{1}{2}$ (D) $-7\frac{1}{2}$ (E) $1\frac{7}{8}$

24. The figure shown is rotated 180° counter clockwise. Then it is reflected over its horizontal axis. Then it is rotated 90° clockwise. Finally, it is reflected over its negative diagonal. Which of the following figures is the result of these four transformations?



(A)  (B)  (C)  (D)  (E) 

25. **Find $a + b + c + d$ given the Fibonacci characteristic sequence: $a, b, -1, 1, c, d, 1, ...$**

(A) 6 (B) $-3$ (C) 1 (D) $-1$ (E) 0

26. **The graph of $g(x) = (x - 2) \div (2x^2 + 2x - 5)$ has how many asymptotes?**

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

27. **Let $f(x) = x^3 - 2x^2 - 15x + 2$. Find the sum of the x-values of the critical points of the function.**

(A) $4\frac{2}{3}$ (B) 3 (C) 2 (D) $1\frac{2}{3}$ (E) $1\frac{1}{3}$

28. **Evaluate: $\int_{-2a}^{2a} (9 - 5x)\ dx$**

(A) 20a (B) 18 (C) $4a(9 - 5a)$ (D) 36a (E) does not exist

29. **The sequence $3, p, q, r, \frac{1}{2}$ is a harmonic progression. Find the value of $p + q + r$. (nearest tenth)**

(A) 2.8 (B) 3.3 (C) 5.8 (D) 6.3 (E) 6.5

30. **William Penn is putting together a 5-pack of colored pens. He has red pens, blue pens, black pens, and green pens. How many different 5-packs of pens can he make?**

(A) 70 (B) 625 (C) 96 (D) 2,880 (E) 56

31. $(422_7 - 124_7) \times 5_7 =$ ____________________ $_7$

(A) 2054 (B) 1520 (C) 1325 (D) 1655 (E) 2155

32. **An operation " $\odot$ " is defined by: $a \odot b = b^a + a^b$. What is the value of $(-1 \odot 3)(2 \odot -2)$?**

(A) $-\frac{8}{51}$ (B) $-2\frac{5}{6}$ (C) $-3\frac{7}{12}$ (D) $-5\frac{2}{3}$ (E) $-4\frac{11}{12}$

33. **Let $f_1 = 4, f_2 = 9, f_3 = 13, f_4 = 22, ...$ be the terms of a Fibonacci characteristic sequence. Find $f_{15}$.**

(A) 4,378 (B) 2,706 (C) 4,305 (D) 3,542 (E) 4,325

34. **Given the sequence, $\frac{11}{(1\times1+1)} - \frac{11}{(2\times2-1)} + \frac{11}{(3\times3+1)} - \frac{11}{(5\times5-1)} + \frac{11}{(8\times8+1)} - ...$, find the digit in the ten-thousandths place.**

(A) 2 (B) 5 (C) 6 (D) 7 (E) 9

35. **If $y^3 = 2 - 11i$, $y^2 = 3 - 4i$ and $y = a + bi$ then $a + b$ equals:**

(A) $-2$ (B) $-1$ (C) 0 (D) 1 (E) 3

**36. Lotta Hare bought a ribbon that was 3 yards long. She cut it into four smaller ribbons such that the ratio of the lengths was 1:3:6:10. How much shorter was the shortest piece than the longest piece? (nearest inch)**

**(A) 1 ft 4 in (B) 1 ft 5 in (C) 1 ft 6 in (D) 1 ft 7 in (E) 1 ft 8 in**

**37. Otto Mobeal went to the Retread Discount Tire store to get some new tires. He got 20% off of the regular price of a set of 4 tires. It cost an extra $5.50 per tire for mounting and balancing and a disposal fee of $2.50 per tire. What was his final cost before taxes for a set of 4 tires if the regular price was $74.95 per tire?**

**(A) $247.84 (B) $265.44 (C) 271.84 (D) $299.80 (E) $331.80**

**38. David, Phyllis, and Jin scored a team total of 542 points at the TTU math camp. David scored thirty-two points less than three-fourths of the points Phyllis' scored. Jin scored two points more than half the sum of the points scored by David and Phyllis. What was the teams highest score?**

**(A) 212 (B) 214 (C) 224 (D) 242 (E) 254**

**39. Poly Gawn drew a rectangle with the length being 6 inches longer than the width. He drew a second rectangle with the length being half the original rectangle's length and the width being 2 inches shorter than the original rectangle's width. The perimeter of the second rectangle is 18 inches less than the perimeter of the original one. Find the perimeter of the original rectangle.**

**(A) 66" (B) 44" (C) 32" (D) 26" (E) 22 "**

**40. The *HAIR express* travels 50% faster than the *TERTAL coupe*. Both start from point A at the same time and reach point B 75 km away from point A at the same time. On the way, the *HAIR* stopped to rest for 12 minutes 30 seconds. Find the speed of the *TERTAL*.**

**(A) 150 kmph (B) 120 kmph (C) 100 kmph (D) 80 kmph (E) 75 kmph**

**41. Using the following array, determine the value of the last term of row 24.**

| | | | | | |
|---|---|---|---|---|---|
| 1 | | | | | (row 1) |
| 2 | 3 | | | | (row 2) |
| 4 | 5 | 6 | | | (row 3) |
| 7 | 8 | 9 | 10 | | (row 4) |
| 11 | 12 | 13 | 14 | 15 | (row 5) |
| ... | | | | | ( ... ) |

**(A) 293 (B) 296 (C) 300 (D) 305 (E) 311**

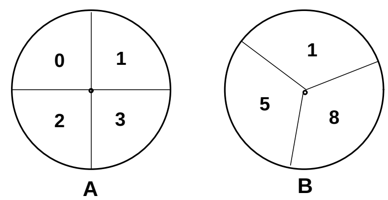
**42. Willie Fall was skiing at Snow Bowl, Arizona last winter. He was at the top of Sunset Peak. He measured the angle of depression to the bottom of the run to be 14°. He read that the actual length of the run is 2675 feet. What is the change in altitude to the bottom of the run? (nearest foot)**

**(A) 2,596 ft (B) 187 ft (C) 647 ft (D) 2,599 ft (E) 667 ft**

43. A rectangular prism water tank has a base width of 3 feet and a base length of 6 feet. The tank is being filled at a constant rate of 5 gallons per second. What is the rate of change of the height of the water in the tank? (nearest hundredth)

(A) 0.45 in/sec (B) 1.25 in/sec (C) 3.6 in/sec (D) 4.55 in/sec (E) 5.3 in/sec

44. Spinner A is divided into four equal sectors and spinner B into three equal sectors. Willie Whenn spins each spinner once. If the product of the two numbers is prime then Willie gets that number of points If the product in not prime then Willie loses that number of points. What is the mathematical expectation of spinning the spinners many times?




(A) $-4\frac{5}{6}$ pts (B) $-5\frac{1}{6}$ pts (C) $-5\frac{1}{3}$ pts (D) $-5\frac{2}{3}$ pts (E) $-6\frac{1}{6}$ pts

45. Poly Gawn wrote down the coordinates of a non-regular convex quadrilateral. She used a special technique to find the area of the quadrilateral called "Area the Easy Way". The technique is mostly associated with which of the following mathematicians?

(A) Archimedes (B) Descartes (C) Diophantus (D) Erastosthenes (E) Theano

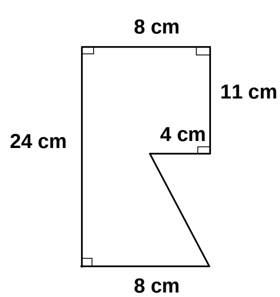
46. $\left\{(x\,,\,y)\middle|\ x,y\ \in \{\text{Integers}\}, -5 \le x \le 5, \text{ and } -3 \le y \le 7\right\}$ is the solution set of $4x - 3y = 2$. How many such ordered pairs exist?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 7

47. Saul Dewould cut 4 dowel rods. The lengths of the rods are 3 feet, 4 feet, 5 feet and 6 feet. How many different acute triangles can he make using three rods at a time?

(A) 4 (B) 3 (C) 2 (D) 1 (E) 0

48. Find the area of the hexagon shown.




(A) 140 in$^2$ (B) 152 in$^2$ (C) 166 in$^2$ (D) 175 in$^2$ (E) not enough data

49. $\angle$PQR is an acute angle. Point A lies on segment PQ and point B lies on segment QR. AQ = 12", BQ = 10", and AB = 7". Find BR if segment AR bisects $\angle$PAB. (nearest inch)

(A) 15" (B) 14" (C) 11" (D) 9" (E) 5"

50. Let $\|V_1\| = 5$, $\|V_2\| = 12$, where the direction angles of $V_1$ and $V_2$ are 50° and 120°, respectively. Find the direction angle of $\|V_1 + V_2\|$. (nearest degree)

(A) 101° (B) 110° (C) 50° (D) 119° (E) 70°

51. Let f(x) = $\left(2x - 5 - \frac{3}{x}\right) \div \left(5x - 13 - \frac{6}{x}\right)$. The domain of f(x) is $\{x : x \neq a, b, c$, where x is a rational number$\}$. Find $a + b + c$.

(A) 5.1 (B) 4.3 (C) 3.4 (D) 2.6 (E) 2.4

52. If $(\log_k x)(\log_5 k) = 2.5$, find x. (nearest tenth)

(A) 97.7 (B) 2.0 (C) 76.8 (D) 1.7 (E) 55.9

53. Find the digit in the thousandth place of the series $\frac{7^0}{0!} - \frac{7^2}{2!} + \frac{7^4}{4!} - \frac{7^6}{6!} + \frac{7^8}{8!} - \ldots$ .

(A) 0 (B) 3 (C) 5 (D) 7 (E) 9

54. The function $f(x) = x^4 - 3x^3 + 3x^2 + 1$ is concave down on which of the intervals?

I. $\left(\frac{3}{5}, \frac{7}{8}\right)$ II. $\left(\frac{1}{9}, \frac{5}{6}\right)$ III. $\left(\frac{3}{4}, 1\frac{1}{2}\right)$ IV. $\left(\frac{2}{5}, 1\frac{1}{10}\right)$

(A) I only (B) II only (C) II & III (D) I & IV (E) I, II, & III

55. Roland Bones tossed a fair 6-sided die 5 times. What is the probability that he rolled at least one 4? (nearest whole percent)

(A) 33% (B) 60% (C) 67% (D) 74% (E) 87%

56. Let $T_n$ be the nth triangular number, $S_n$ be the nth square number, and $P_n$ be the nth pentagonal number. Then $T_n + S_{(n+1)}$ has the same value as:

(A) $P_{(n+1)}$ (B) $P_{(n+2)}$ (C) $T_{(2n+1)}$ (D) $S_{(2n+2)}$ (E) $P_{(2n)}$

57. Mark DeCard labels blank cards with the numbers from the set { 1, 3, 6, 10, 15, 21} with one number per card. He selects two cards at random. What are the odds that the absolute value difference is an odd number?

(A) $\frac{2}{3}$ (B) $\frac{8}{7}$ (C) $\frac{2}{5}$ (D) $\frac{2}{1}$ (E) $\frac{8}{15}$

**58. The number 2017 is a member of which of the following sets of numbers?**

**I. Evil    II. Unhappy    III. Polite**

**(A) I, II, & III    (B) I & II only    (C) II only    (D) II & III only    (E) III only**

**59. Let $f_0 = 0, f_1 = 1, f_2 = 1, f_3 = 2, f_4 = 3$, ... be the terms of the Fibonacci sequence. If $f_{(k)} = 832{,}040$ then $f_{(k-3)} = ?$**

**(A) 277,347    (B) 121,393    (C) 196,418    (D) 1,346,269    (E) 317,811**

**60. A lock's combination consists of three positive digits. The first digit is a Fibonacci number, the second digit is a composite number, and the last digit is a triangular number. How many unique combinations fit this criteria?**

**(A) 48    (B) 54    (C) 60    (D) 66    (E) 72**

**University Interscholastic League**
**MATHEMATICS CONTEST**
**HS ● State ● 2017**
**Answer Key**

| | | |
|---|---|---|
| 1. D | 21. B | 41. C |
| 2. B | 22. A | 42. C |
| 3. D | 23. A | 43. A |
| 4. E | 24. D | 44. C |
| 5. D | 25. E | 45. B |
| 6. A | 26. D | 46. B |
| 7. E | 27. E | 47. C |
| 8. C | 28. D | 48. C |
| 9. B | 29. A | 49. B |
| 10. C | 30. E | 50. A |
| 11. A | 31. A | 51. D |
| 12. C | 32. B | 52. E |
| 13. E | 33. E | 53. B |
| 14. E | 34. D | 54. A |
| 15. D | 35. D | 55. B |
| 16. C | 36. A | 56. A |
| 17. A | 37. C | 57. B |
| 18. D | 38. C | 58. D |
| 19. E | 39. B | 59. C |
| 20. E | 40. B | 60. C |