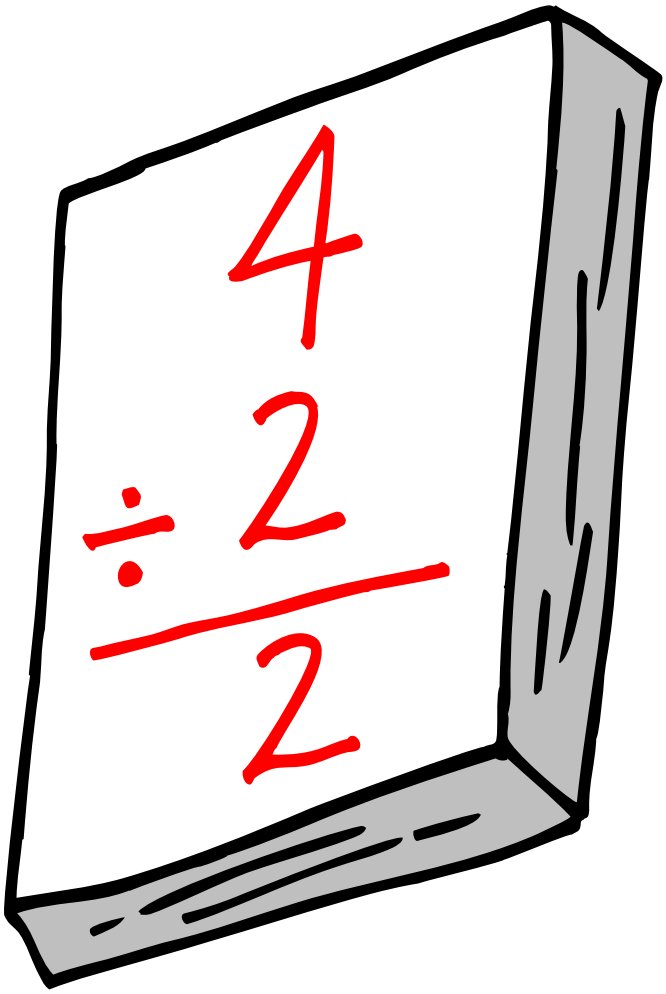
UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

Invitational A • 2017



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1. **Evaluate:** $5.4 \div (\frac{3}{2})^{-1} - (1)^{-2} \times 3! + 4.5$

   (A) -3.9 **(B)** $\frac{1}{6}$ **(C)** $\frac{3}{5}$ **(D)** 2.1 **(E)** 6.6

2. **If $\frac{3}{8}$ of A is 87.5% more than B, then A is what percent of B?**

   (A) 5% **(B)** 20% **(C)** 50% **(D)** $233\frac{1}{3}\%$ **(E)** 500%

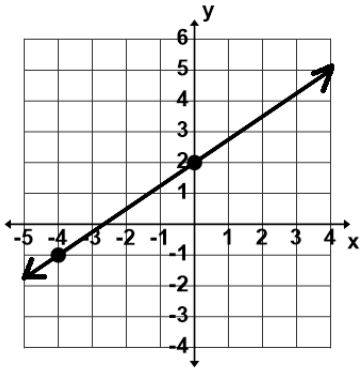
3. **One billion two and three-fourth million five hundred six is added to six million fifty-four thousand three hundred twenty-one. How many digits in the sum are twos?**

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

4. **Kookie Baykur baked some cookies. She took 20% of them to her grandmother. Then she ate 4 for lunch. After lunch she sold $\frac{1}{2}$ of what was left at her school's bake sale. She had 6 left to share with her parents for after supper. How many cookies did she bake originally?**

   (A) 18 **(B)** 20 **(C)** 24 **(D)** 30 **(E)** 36

5. **A line parallel to the line shown containing the point (6, 3) contains which of the following points?**



   (A) $(-3, -4)$ **(B)** $(-6, -6)$ **(C)** $(0, -2)$ **(D)** (9, 7) **(E)** (10, 6)

6. **Let p and q be the roots of $2x^2 + 3x - 5 = 0$. Find $p^3 + 3p^2q + 3pq^2 + q^3$.**

   (A) 15.625 **(B)** 6.5 **(C)** 3.625 **(D)** -1.125 **(E)** -3.375

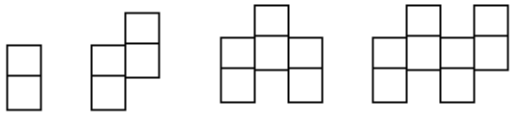
7. **Phil D. Belly budgets \$53.00 per week for lunch. He spends \$7.00 each day that he goes to McDee's Grill and \$9.00 each day that he goes to Queen's Burger. How much more does he spend at McDee's Grill than at Queen's Burger during a 7 day week?**

   (A) \$1.00 **(B)** \$15.00 **(C)** \$17.00 **(D)** \$33.00 **(E)** \$35.00

8. **$\angle A$ and $\angle B$ are supplementary. If m $\angle A = 3x + 4$ and m $\angle B = 2x + 1$, the measure of the larger angle is:**

   (A) 55° **(B)** 61° **(C)** 109° **(D)** 112° **(E)** 115°

9. **The four shapes below are made up of 1 cm squares. If the pattern continues, find the perimeter of the shape consisting of 16 squares.**



(A) 26 (B) 30 (C) 32 (D) 34 (E) 40

10. **M. T. Tank has a rectangular based water tank that is empty. The length of the tank is twice the width and the height is half of the width. How many gallons of water will he need to fill the tank if the height is 4 feet? (nearest gallon)**

(A) 3,830 gal (B) 3,456 gal (C) 3,192 gal (D) 2,608 gal (E) 2,095 gal

11. **The point (3 , — 4) is rotated 450° clockwise about the origin. The coordinates of the point after the rotation is _____.**

(A) (— 3, 4) (B) (4, — 3) (C) (— 3, — 4) (D) (3, 4) (E) (— 4, — 3)

12. **If $\frac{5x-2}{3x+1} + \frac{Ax-B}{x+4} = \frac{11x^2+5x-13}{3x^2+13x+4}$, where A and B are constants, then A × B equals:**

(A) — 3 (B) — 1 (C) 3 (D) 7 (E) 10

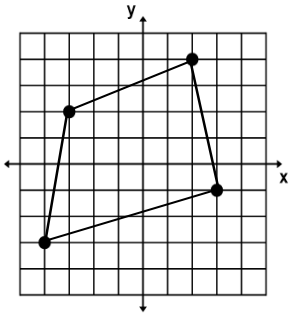
13. **The roots of the equation $2x^3 - x^2 - 5x - 2 = 0$ are —1, 2, and R. Find R.**

(A) 2.5 (B) 1 (C) — 0.5 (D) — 1 (E) — 1.5

14. **Let A = $\begin{bmatrix} 1 & 6 \\ -9 & -7 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & -1 \\ 0 & -7 \end{bmatrix}$. Find $|A - B|$.**

(A) — 15 (B) — 3 (C) 33 (D) 61 (E) 63

15. **Find the area of the quadrilateral. The coordinates of the vertices are integers.**



(A) 31.5 sq. units (B) 30 sq. units (C) 28.5 sq. units (D) 28 sq. units (E) 26 sq. units

16. **Determine the frequency of $f(x) = 3 + 5\sin[4\pi(x - 2)]$.**

(A) $\frac{1}{2}$ (B) 2 (C) 3 (D) 4 (E) 5

17. A plane is 120 miles north and 85 miles east of an airport. What bearing should the plane take to fly directly to the airport?

(A) 65° (B) 55° (C) 45° (D) 35° (E) 25°

18. Given the arithmetic sequence 15, a, b, 37, c, ..., find $a + b + c$.

(A) $96\frac{1}{3}$ (B) 114 (C) $148\frac{2}{3}$ (D) $81\frac{2}{3}$ (E) $73\frac{1}{3}$

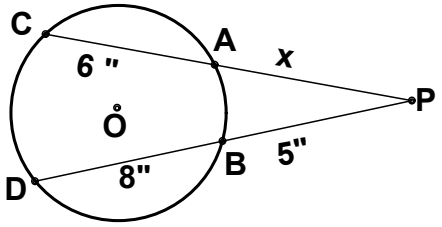
19. Find the remainder when $x^3 + 2x^2 - 3x + 4$ is divided by $x + 1$.

(A) 10 (B) 8 (C) 7 (D) 5 (E) 4

20. Find the eccentricity of the ellipse, $16x^2 + 100y^2 = 1600$. (nearest hundredth)

(A) 0.87 (B) 0.90 (C) 0.92 (D) 0.95 (E) 0.98

21. Given the circle with center O shown. Find x. (nearest tenth).



(A) 3.8 (B) 5.6 (C) 6.3 (D) 6.7 (E) 9.6

22. What is the sum of the digits in the tens place and the units place of $7^{(65)}$?

(A) 1 (B) 3 (C) 7 (D) 9 (E) 13

23. The function $f(x) = x^2$ is concave up on which of the following open intervals?
I. (0, 5) II. (— 5, 5) III. (— 5, 0)

(A) I only (B) II only (C) III only (D) I, II & III (E) none of them

24. The graph of $g(x) = (x^3 + 3x^2 + 3x + 1) \div (x^2 - 1)$ has vertical asymptote(s) at:

(A) $x = 1$ (B) $x = -1$ (C) $x = 1$ and -1 (D) $x = 0$ (E) g(x) has no vertical asymptotes

25. Let $f''(x) = 18x + 4$, $f'(-1) = 6$, and $f(1) = 6$. Find $f(-2)$.

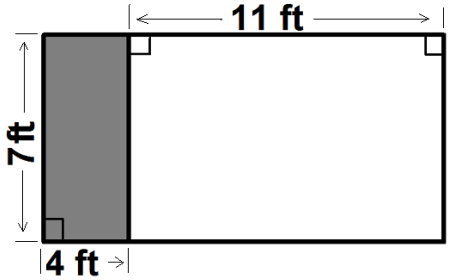
(A) — 34 (B) — 32 (C) — 18 (D) — 12 (E) 12

26. Suppose A, B, and C are positive integers such that $\frac{32}{5} = A + \frac{1}{B + \frac{1}{C+1}}$.

The value of $3A + 2B + 5C$ equals:

(A) 9 (B) $9\frac{2}{5}$ (C) $13\frac{1}{2}$ (D) 27 (E) 37

27. Spud Pharmer's son, Tater, buried his daddy's shovel in their rectangular garden. What is the probability that it was buried in the shaded section shown? (nearest whole percent)



(A) 7% (B) 20% (C) 25% (D) 27% (E) 36%

28. Lyn Koln flipped a penny four times and recorded the results. What are the odds of three or more consecutive heads occurring?

(A) $\frac{3}{16}$ (B) $\frac{1}{7}$ (C) $\frac{1}{8}$ (D) $\frac{5}{11}$ (E) $\frac{3}{13}$

29. Which of the following mathematicians are associated with for working with prime numbers?
I. Erastosthenes of Cyrene II. Sophie Germain III. Marin Mersenne

(A) I only (B) I & II (C) I & III (D) I, II & III (E) none of them

30. The number 13 is a member of which of the following sets of numbers?
(A)bundant (E)vil (L)ucas (P)rimeval

(A) L & P only (B) P only (C) E & L only (D) none of them (E) all of them

31. If 2 Babs equal 3 Bibs and 5 Bibs equal 7 Bobs, then how many Babs does it take to make 3 Bobs?

(A) $3\frac{1}{3}$ (B) $2\frac{5}{7}$ (C) $1\frac{3}{7}$ (D) $1\frac{1}{2}$ (E) $\frac{7}{10}$

32. Let U (universal set) = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, P = {2, 3, 5, 7, 9}, and Q = {2, 1, 3, 4, 7}. Let $R = (P \cap Q)^C$. Set R contains how many distinct elements?

(A) 3 (B) 5 (C) 7 (D) 10 (E) none

33. Soh Yung is 7 years older than her sister Tu Yung. In 3 years Soh will be twice as old as Tu. How old will Tu be in 5 years ?

(A) 4 (B) 9 (C) 11 (D) 14 (E) 16

34. Seven students in Miss Work's math class had test scores of 75, 83, 85, 92, 95, 98, and 100. Three of her students haven't take the test yet. What will the remaining three students have to average so that the entire class average is 88?

(A) 84 (B) 85 (C) 86 (D) 87 (E) 88

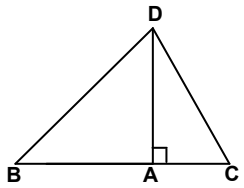
35. Rusty Yatt sailed his boat to Junk Yard Bay and back home. The trip took 12 hours going and 9 hours coming back. His average speed coming back was 20 kph. What was his average speed going?

(A) 8 kph (B) 11 kph (C) 15 kph (D) 21 kph (E) 28 kph

36. Dee Deeler has a standard deck of cards consisting of 4 Aces, 12 face cards, and 36 number cards. No Joker is allowed. Dee wants to see how many 5 card hands he can create such that each hand has 1 Ace, 2 face cards, and 2 number cards. How many such hands can he make?

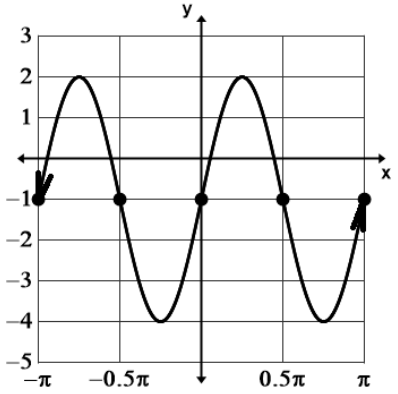
(A) 166,320 (B) 260 (C) 8,640 (D) 1,728 (E) 665,280

37. Find the perimeter of $\triangle BCD$ if AD = 3", $m\angle ADB = 45°$, and $m\angle ACD = 60°$. (nearest tenth)



(A) 7.6" (B) 8.2" (C) 10.2" (D) 12.4" (E) not enough information given

38. The equation y = _____ will produce this graph.



(A) $3\cos(2x + \pi)$ (B) $3\sin(2x) - 1$ (C) $3\cos(2x + \pi) - 1$
(D) $3\sin(2x - 1)$ (E) $2 - 3\sin(x)$

39. The function $f(x) = 3x^2 - 4x - 4$ crosses the x-axis at two points. Find the distance between the two points.

(A) $3\frac{1}{2}$ units (B) $1\frac{1}{3}$ units (C) 4 units (D) $2\frac{2}{3}$ units (E) $1\frac{1}{2}$ units

40. The point of concurrency of the angle bisectors of a triangle is called the:

(A) incenter (B) centroid (C) orthocenter (D) circumcenter (E) line of Euler

41. If $a_1 = -1$, $a_2 = -2$, $a_3 = 3$, and $a_n = (a_{n-1})(a_{n-3}) - (a_{n-2})$, where $n > 3$ then $a_6 = ?$

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

42. Determine the range of $(x) = 3 + 5\sin[4\pi(x - 2)]$.

(A) $[-5, 5]$ (B) $[-5, 8]$ (C) $[-2, 8]$ (D) $[-2, 5]$ (E) $[-8, 8]$

43. Find the area (in square units) of the region bounded by $y = -x^2$ and $y = -4$.

(A) 16 (B) $11\frac{1}{3}$ (C) $10\frac{2}{3}$ (D) 8 (E) $5\frac{1}{3}$

44. $\frac{1+4+9+16+...+64+81}{1+3+6+10+...+36+45} =$ _____.

(A) $1\frac{4}{5}$ (B) $1\frac{9}{11}$ (C) $1\frac{64}{81}$ (D) $1\frac{74}{101}$ (E) $1\frac{8}{11}$

45. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. If $f_n = 121{,}393$ then n is:

(A) 20 (B) 22 (C) 24 (D) 26 (E) 28

46. Willie Pikette is going to randomly pick two different numbers from the set { 2, 1, 3, 4, 7, 11}. What is the probability that the sum of the two numbers he picks will be a prime number?

(A) 20% (B) $26\frac{2}{3}\%$ (C) $33\frac{1}{3}\%$ (D) 40% (E) $53\frac{1}{3}\%$

47. The function *f* is defined by $f(x) = 2 + \ln(x + 3)$. The inverse function of *f* is $f^{-1}(x) = ?$

(A) $(2 + \ln(x + 3))^{-1}$ (B) $\ln(x - 2)$ (C) $e^{(x+2)} - 3$
(D) $e^{(x-2)} - 3$ (E) $-(2 + \ln(x + 3))^{-1}$

48. Let $f(x) = \frac{x^3 - 3x^2}{x^2 - 1}$ and s(x) be the slant asymptote of f. Find the value of s(4).

(A) $1\frac{1}{15}$ (B) -1 (C) $\frac{15}{16}$ (D) 1 (E) 7

49. Alice, Bob, Charlie, Dan, and Edith sit randomly in a row of five chairs. What is the probability that Alice and Edith sit next to each other? (nearest percent)

(A) 3% (B) 7% (C) 20% (D) 35% (E) 40%

50. Find the slope of the line tangent to the curve $y = x^2 - 3x + 5$ at (3, 5).

(A) 2 (B) 3 (C) 5 (D) 6 (E) 10

51. If the three numbers 78, 169, and 246 are each divided by the number D, each of their quotients will have the same remainder R. Find R.

(A) 7 (B) 5 (C) 3 (D) 2 (E) 1

52. Let $f(x) = x^3 + 2x^2 - 4x$. Find the sum of the x-values of the critical points of the function.

(A) 2 (B) 1 (C) $-\frac{2}{3}$ (D) $-1\frac{1}{3}$ (E) $-2\frac{2}{3}$

53. Let $g(x) = x^2 + 2x + 1$. Find k if $g(k + 1) - g(k) = 7$.

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

54. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. How many digits are in f_{21}?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

55. $14_5 + 32_5 \times 23_5 =$ _____ $_5$

(A) 1410 (B) 1300 (C) 1113 (D) 2314 (E) 2323

56. If $15x^2 + cx - 12 = (5x + a)(bx - 4)$ then $a + b + c =$ _____.

(A) -5 (B) -2 (C) 3 (D) 6 (E) 17

57. Let $f(x) = 5x - 2$, $g(x) = x + 4$, $h(x) = 3x + 1$, and $g\Big(f\big(h(x)\big)\Big) = 10$. Find x.

(A) $-\frac{13}{15}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{5}$ (D) $\frac{7}{9}$ (E) $1\frac{2}{15}$

58. How many 3-digit numbers can be made using the digits 2, 1, 3, 4, and 7?

(A) 64 (B) 60 (C) 32 (D) 30 (E) 15

59. The sequence 6, p, q, 1.5 is a harmonic sequence. Find the value of p + q.

(A) $\frac{5}{6}$ (B) $1\frac{1}{5}$ (C) $3\frac{3}{4}$ (D) 5 (E) 7.5

60. A right triangle, $\triangle ABC$, with leg lengths 15" and 20" and the right angle at vertex B is congruent to right triangle, $\triangle BDE$, with the right angle at vertex D. Point C lies on segment BD and points A and E are on the same side of segment BD. Find the distance between points A and E. (nearest eighth of an inch).

(A) $20\frac{5}{8}$" (B) $20\frac{1}{4}$" (C) $19\frac{7}{8}$" (D) $18\frac{3}{8}$" (E) $17\frac{1}{2}$"

University Interscholastic League
Mathematics Contest

WRITE ALL ANSWERS WITH
CAPITAL LETTERS

	Score	Initials
Final	_____	_____
2nd	_____	_____
1st	_____	_____

Contestant # _____ **Conference** _____

1. _____	21. _____	41. _____
2. _____	22. _____	42. _____
3. _____	23. _____	43. _____
4. _____	24. _____	44. _____
5. _____	25. _____	45. _____
6. _____	26. _____	46. _____
7. _____	27. _____	47. _____
8. _____	28. _____	48. _____
9. _____	29. _____	49. _____
10. _____	30. _____	50. _____
11. _____	31. _____	51. _____
12. _____	32. _____	52. _____
13. _____	33. _____	53. _____
14. _____	34. _____	54. _____
15. _____	35. _____	55. _____
16. _____	36. _____	56. _____
17. _____	37. _____	57. _____
18. _____	38. _____	58. _____
19. _____	39. _____	59. _____
20. _____	40. _____	60. _____

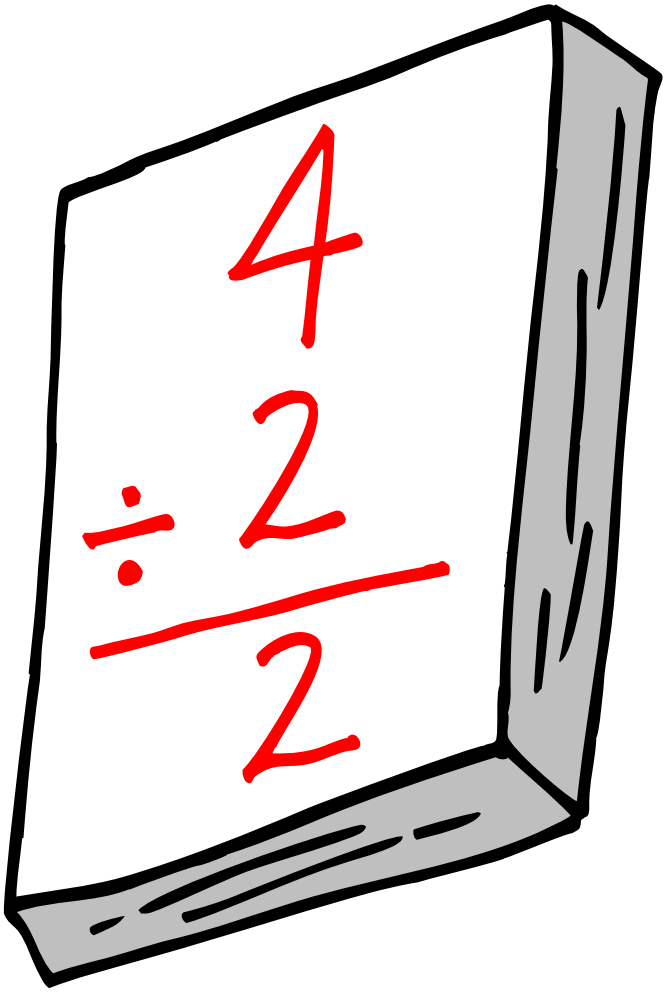
University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitation A ● 2017
Answer Key

1. E	**21. B**	**41. A**
2. E	**22. C**	**42. C**
3. B	**23. D**	**43. C**
4. B	**24. A**	**44. E**
5. B	**25. C**	**45. D**
6. E	**26. D**	**46. D**
7. C	**27. D**	**47. D**
8. C	**28. E**	**48. D**
9. D	**29. D**	**49. B**
10. A	**30. B**	**50. B**
11. E	**31. C**	**51. E**
12. E	**32. C**	**52. D**
13. C	**33. B**	**53. E**
14. E	**34. A**	**54. C**
15. B	**35. C**	**55. A**
16. B	**36. A**	**56. A**
17. D	**37. D**	**57. C**
18. A	**38. B**	**58. B**
19. B	**39. D**	**59. D**
20. C	**40. A**	**60. A**

UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational B • 2017



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1. **Evaluate:** $2 \div (10 - 20) + 17 \times 3 - 11 \times 20 \div (1 + 7)$

   (A) -48.5 **(B)** -23.3 **(C)** -0.4 **(D)** 22.9 **(E)** 98.5

2. **Rose Thorn's flower shop is having a spring plant sale. The regular price of a yellow rose bush is $6.95 and a red rose bush regularly sell for $8.50. Yellow roses are on sale for 20% off and red roses are on sale for 10% off. What would it cost Rose to buy 4 yellow rose bushes and 2 red rose bushes on sale before tax?**

   (A) $39.71 **(B)** $38.62 **(C)** $38.08 **(D)** $37.54 **(E)** $31.36

3. **If** $P = \{p, l, u, s\}$, $M = \{m, i, n, u, s\}$, $T = \{t, i, m, e, s\}$ **and** $O = \{o, p, e, r, a, t, i, o, n\}$ **then** $(P \cup M \cup T) \cap O$ **contains how many distinct elements?**

   (A) 1 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 8

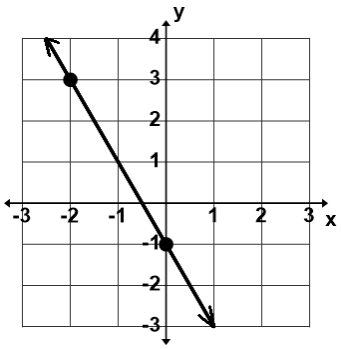
4. **Three-fourths is the same part of two-fifths as one-half is of _____.**

   (A) $3\frac{3}{4}$ **(B)** $3\frac{2}{5}$ **(C)** $2\frac{3}{10}$ **(D)** $\frac{15}{16}$ **(E)** $\frac{3}{20}$

5. **Which of the following multiples of 6 is the average of two consecutive prime numbers?**

   (A) 24 **(B)** 36 **(C)** 42 **(D)** 54 **(E)** 66

6. **Which of the following linear equations has the graph of a line perpendicular to the line shown and containing the point** $(-4, -1)$**?**



   (A) $2x + y = -9$ **(B)** $x - 2y = -7$ **(C)** $2x + y = -6$ **(D)** $x - y = -2$ **(E)** $x - 2y = -2$

7. **The set of positive composite numbers {4, 6, 8, 9, 10, 12, ...} is closed under how many of these operations:** $+$ **addition** $-$ **subtraction** $\times$ **multiplication** $\div$ **division**

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** none of them

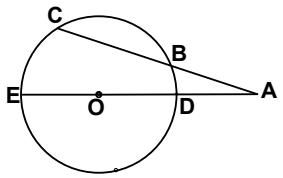
8. **Seymore Wirk can paint a picket fence in 8 hours. His brother Les Wirk can paint the same fence in 6 hours. How long would it take the Wirk brothers to paint the fence if they worked together? (nearest minute)**

   (A) 3 hrs 26 min **(B)** 3 hrs 30 min **(C)** 3 hrs 37 min **(D)** 3 hrs 43 min **(E)** 3 hrs 52 min

9. If $\frac{x+5}{x-4} + \frac{x-4}{x+5}$ is written as the mixed number $A\frac{B}{C}$ then B = ?

(A) 9 (B) 16 (C) 20 (D) 25 (E) 81

10. Given the circle with segment AE containing center O as shown. Find OE if AC = 12 cm, BC = 6 cm, and AD = 4 cm.



(A) 4 cm (B) 7 cm (C) 8 cm (D) 14 cm (E) 15 cm

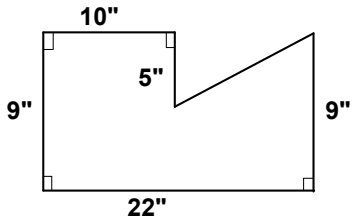
11. The sum of the measures of the interior angles of a single face of a regular octahedron is:

(A) 180° (B) 360° (C) 540° (D) 720° (E) 900°

12. The orthocenter of which of the following triangles lies outside the triangle?

(A) equilateral (B) acute isosceles (C) right scalene (D) acute scalene (E) obtuse scalene

13. Find the perimeter of the hexagon shown.



(A) 69 " (B) 68 " (C) 55 " (D) 52 " (E) not enough data

14. If $\frac{Ax+B}{2x-5} - \frac{3x+2}{5x-3} = \frac{14x^2+4x+7}{10x^2-31x+15}$, where A and B are constants, then A + B equals:7

(A) 3 (B) 2 (C) 5 (D) −1 (E) −3

15. Find the range of the function $y = 5 - 3|x+2|$ given that the domain is restricted to $\{x \in \text{Reals} \mid -5 \le x \le 1\}$.

(A) {all Reals} (B) $\{y \mid y \in \{\text{Reals}\}, -4 \le y \le 5\}$ (C) $\{y : y \le 5\}$
(D) $\{y : y > 5\}$ (E) $\{y \mid y \in \{\text{Reals}\}, -1 \le y \le 5\}$

16. Willis Quik flew his plane to Sumplace. The speed with a tailwind was 183 km/h. The speed on the return trip was 141 km/h going into the wind. Find the speed of the wind.

(A) 35 km/h (B) 28 km/h (C) 21 km/h (D) 14 km/h (E) 7 km/h

17. Find the value of $5A + 3B + 2C$, where A, B, and C are greater than zero and

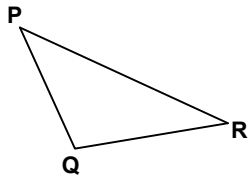
$$\frac{23}{5} = A + \left(\frac{1}{B + \left(\frac{1}{C+1}\right)}\right).$$

(A) 26 (B) 6.5 (C) 23.5 (D) 5.5 (E) 24

18. Determine the amplitude of $f(x) = 4 - 3\sin(2x + 1)$.

(A) 1 (B) 2 (C) 3 (D) π (E) 4

19. Find the height of $\triangle PQR$ from point P if $m\angle PQR = 110^o$, PQ = 20 cm, and QR = 15 cm. (nearest tenth)



(A) 19.3 cm (B) 18.8 cm (C) 17.5 cm (D) 14.1 cm (E) 13.2 cm

20. Sir Vayer places a stake in the ground. He walks 50 yards on a course heading 75^o west of south. Then he turns and walks 75 yards on a course heading 100^o east of north. How far will he have to walk to get back to his stake? (nearest yard)

(A) 36 yds (B) 59 yds (C) 30 yds (D) 38 yds (E) 45 yds

21. If $\log_3(x + 24) - \log_3(x + 2) = 2$ then x = ?

(A) $\frac{3}{4}$ (B) $\frac{7}{8}$ (C) $1\frac{1}{3}$ (D) $\frac{2}{3}$ (E) $1\frac{1}{7}$

22. Given the geometric sequence $-0.5, a, b, 0.0625, c, \ldots$, find $a + b + c$.

(A) $\frac{1}{2}$ (B) $\frac{13}{32}$ (C) $\frac{11}{32}$ (D) $\frac{1}{8}$ (E) $\frac{3}{32}$

23. Find mn if $\begin{bmatrix} -1 & m \\ n & 3 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 4 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \end{bmatrix}$

(A) -8 (B) -3 (C) -2 (D) $-1\frac{2}{3}$ (E) -1

24. Find k when $g(x) = 2x^2 + kx + 1$ divided by $h(x) = x - 2$ has a remainder of 2.

(A) -3.5 (B) -3 (C) 2 (D) 3.5 (E) 6

25. The numbers greater than 1 are arranged in the array below. In which column will 2017 fall.

(A)	(B)	(C)	(D)	(E)
	2	3	4	5
9	8	7	6	
	10	11	12	13
17	16	15	14	
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

26. Which of the following sequences are divergent?

I. $\left\{\frac{\ln(n)}{n^2}\right\}$ **II.** $\left\{\ln\left(\frac{1}{n-1}\right)\right\}$ **III.** $\left\{\frac{n^2+2}{2n^2-1}\right\}$

(A) I & III (B) II & III (C) III only (D) II only (E) none of them

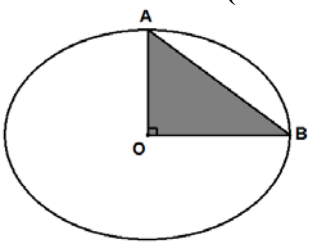
27. Evaluate: $\int_{1-a}^{a+1} (6x-2)\,dx$

(A) 8a (B) 10a + 2 (C) 12a (D) 16a + 10 (E) does not exist

28. Willie Pickette randomly chooses exactly one letter from each of the sets, {Q,U,I,C,K} and {P,I,C,K}. What is the probability of choosing two vowels?

(A) $22\frac{2}{9}\%$ **(B) 10% (C) 65% (D)** $33\frac{1}{3}\%$ **(E) 40%**

29. N. D. Shaid throws a dart that hits in the circle with center O and having a diameter of 12". What are the odds the dart hits in the shaded area? (nearest whole percent)



(A) 16% (B) 18% (C) 19% (D) 22% (E) 25%

30. The odds of the Ruff Ryders baseball team winning a game is $\frac{5}{11}$**. How many games can they expect to lose if there are 120 games in the season?**

(A) 54 (B) 66 (C) 75 (D) 82 (E) 85

31. Which of the following mathematicians first introduced the term "polytope" and had a good grasp of the concept of four dimensional geometry?

(A) Ada Byron (B) Benoit Mandelbrot (C) Agnesi (D) Hypatia (E) Alicia Stott

32. A happy number that is also perfect is:

(A) 6 **(B) 11** **(C) 28** **(D) 44** **(E) 100**

33. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. How many digits are in f_{32}?

(A) 4 **(B) 5** **(C) 6** **(D) 7** **(E) 8**

34. Willie Sawette had a log that was 12 feet long. He cut it into three smaller logs such that the ratio of the lengths was 2:3:5. How much longer was the longest piece than the shortest piece? (nearest inch)

(A) 1 ft 2 in **(B) 2 ft 5 in** **(C) 2 yds** **(D) 1 yd 2 ft** **(E) 1 yd 7 in**

35. There were 148 students competing at last year's state math contest. The number of young men competing was eight more than four times the number of the young ladies competing. How many young ladies competed?

(A) 18 **(B) 20** **(C) 24** **(D) 28** **(E) 36**

36. The ratio of the lateral area of a right cylinder to its total surface area is 5:8. Find the radius of the cylinder's base if it is 8 cm shorter than the height of the cylinder.

(A) $2\sqrt{3}$ cm **(B) $2\sqrt{10}$ cm** **(C) 6 cm** **(D) 12 cm** **(E) 20 cm**

37. If the probability that a student studies for an exam is 85%, and the probability that a student who studies passes the test is 90%, then the probability that a student both studies and passes the test is:

(A) 5% **(B) 76.5%** **(C) 80%** **(D) 87.5%** **(E) 95%**

38. Lotta Dough puts 3 one-dollar bills, 4 five-dollar bills, 2 ten-dollar bills, and 1 twenty-dollar bill in a box. She selects two bills randomly without replacement, what is the probability that the sum of the two bills selected is \$15.00 or more?

(A) 15% **(B) 20%** **(C) 40%** **(D) $44\frac{4}{9}\%$** **(E) $55\frac{5}{9}\%$**

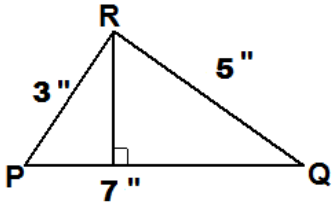
39. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. Find f_{23}.

(A) 17,711 **(B) 28,657** **(C) 23,765** **(D) 24,476** **(E) 46,368**

40. Find the least common multiple of 44, 60, and 76.

(A) 180 **(B) 3,135** **(C) 12,540** **(D) 50,160** **(E) 200,640**

41. Find the length of the altitude from point R to segment PQ. (nearest hundredth)



(A) 2.14" (B) 1.98" (C) 1.86" (D) 4.2" (E) not enough information given

42. If $18x^2 - 15x + 2 = (ax - 1)(bx - 2)$ then $a - b =$ _____, where a and b are integers.

(A) 2 (B) 3 (C) 5 (D) 6 (E) 9

43. Find the area of the circle, $x^2 + y^2 - 4x - 6y + 8 = 0$. (nearest tenth)

(A) 9.4 units^2 (B) 14.4 units^2 (C) 15.7 units^2 (D) 20.4 units^2 (E) 25.1 units^2

44. The equation of a parabola with its vertex at (3, 3) and its focus at (3, 2.5) is y = ?

(A) $-0.5x^2 + 3x - 15$ (B) $-0.5x^2 - 3x - 1.5$ (C) $-0.5x^2 + 3x - 1.5$
(D) $-0.5x^2 - 3x + 7.5$ (E) $-0.5x^2 + 3x + 1.5$

45. $1 + 3 + 6 + 10 + 15 + ... + 55 + 66 =$ _____.

(A) 286 (B) 298 (C) 306 (D) 368 (E) 398

46. $(110101_2 - 1001_2) \times 11_2 =$ _____ $_2$

(A) 1101100_2 (B) 10000100_2 (C) 1010101_2 (D) 1101110_2 (E) 110011_2

47. The function $f(x) = 5x^3 + 30x^2 + x + 1$ is concave down on which of the following open intervals?

(A) (1.1, 2.2) (B) (− 2.2, − 1.1) (C) (− 1.5, 1.5) (D) (− 4, 0) (E) (− 3.5, − 2.5)

48. $\{(x, y) \mid x,y \in \{\text{Integers}\}, -3 \le x \le 7, \text{ and } -1 \le y \le 9\}$ is the solution set of $2x - 5 = 3y$. How many such ordered pairs exist?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

49. Point M (− 1, 2) is the midpoint of the line segment with endpoints P (x , 1) and R (1, y). Find PR. (nearest tenth unit)

(A) 2.4 (B) 2.8 (C) 3.1 (D) 4.5 (E) 4.8

50. If $a_1 = -2$, $a_2 = 0$, $a_3 = 2$, and $a_n = (a_{n-3})^{(a_{n-2})} - (a_{n-1})$, where $n > 3$ then $a_7 = ?$

(A) $1\frac{1}{2}$ (B) 1 (C) 0 (D) $-\frac{1}{2}$ (E) -1

51. Let $\|V_1\| = 6$, $\|V_2\| = 8$, where the direction angles of V_1 and V_2 are 60° and 80°, respectively. Find the direction angle of $\|V_1 + V_2\|$. (nearest degree)

(A) 11° **(B)** 14° **(C)** 20° **(D)** 24° **(E)** 71°

52. Let $f(x) = ax - 5$ and $g(x) = bx + 3$, where a and b are positive integers. Find $a + b$ if $f(g(x)) = g(f(x))$.

(A) 5 **(B)** 4 **(C)** 3 **(D)** 2 **(E)** 1

53. The fraction $\frac{22}{30}$ in base 7 can be written as which of the following decimals in base 7?

(A) $0.4333..._7$ **(B)** $0.5222..._7$ **(C)** $0.6111..._7$ **(D)** $0.444..._7$ **(E)** $0.2555..._7$

54. Let $\frac{dy}{dx} = 4x^3 - 2x + 1$, and $y = 3$ when $x = 0$. Find y when $x = 3$.

(A) 103 **(B)** 4 **(C)** 66 **(D)** 96 **(E)** 78

55. Given the function $f(x) = 2x^2 + 1$, find the slope of the secant line between $x = 2$ and $x = 5$.

(A) 12 **(B)** 14 **(C)** 20 **(D)** 28 **(E)** 60

56. In the decimal number 2x3y4z, the letters x, y, and z represent digits where all six digits are distinct. If the number is divisible by 30 then $x + y + z$ could be:

(A) 12 **(B)** 13 **(C)** 14 **(D)** 16 **(E)** 18

57. How many 4-digit numbers can be created from the set of positive digits where the digits are not repeated?

(A) 6,561 **(B)** 6,480 **(C)** 5,040 **(D)** 4,536 **(E)** 3,024

58. The sum of the coefficients of the 3rd term in the expansion of $(x + 1)^3$, the 3rd term of $(x + 1)^4$, and the 3rd term of $(x + 1)^5$ is:

(A) 12 **(B)** 14 **(C)** 17 **(D)** 19 **(E)** 36

59. Let $f(x) = 4x$, $g(x) = 3 - x$, $h(x) = x + 2$, and $h\big(f(\,g(x)\,)\big) = 5$. Find x.

(A) $2\frac{1}{4}$ **(B)** $1\frac{3}{4}$ **(C)** 0 **(D)** -1 **(E)** -6

60. Points A,B, C, and D are coplanar. Point A lies on segment DC and point B exists such that DB = DC, BA = BC, and $m\angle ADB = \frac{\pi}{6}$. Find the ratio of DB to BC. (nearest tenth)

(A) 3.9 **(B)** 0.5 **(C)** 1.0 **(D)** 0.9 **(E)** 1.9

University Interscholastic League
Mathematics Contest

WRITE ALL ANSWERS WITH
CAPITAL LETTERS

	Score	Initials
Final	_____	_____
2nd	_____	_____
1st	_____	_____

Contestant # _____ **Conference** _____

1. _____	21. _____	41. _____
2. _____	22. _____	42. _____
3. _____	23. _____	43. _____
4. _____	24. _____	44. _____
5. _____	25. _____	45. _____
6. _____	26. _____	46. _____
7. _____	27. _____	47. _____
8. _____	28. _____	48. _____
9. _____	29. _____	49. _____
10. _____	30. _____	50. _____
11. _____	31. _____	51. _____
12. _____	32. _____	52. _____
13. _____	33. _____	53. _____
14. _____	34. _____	54. _____
15. _____	35. _____	55. _____
16. _____	36. _____	56. _____
17. _____	37. _____	57. _____
18. _____	38. _____	58. _____
19. _____	39. _____	59. _____
20. _____	40. _____	60. _____

University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitation B ● 2017
Answer Key

1. B	**21. A**	**41. C**
2. D	**22. E**	**42. B**
3. C	**23. A**	**43. C**
4. A	**24. A**	**44. C**
5. C	**25. A**	**45. A**
6. E	**26. D**	**46. B**
7. A	**27. A**	**47. E**
8. A	**28. B**	**48. C**
9. E	**29. C**	**49. D**
10. B	**30. D**	**50. D**
11. A	**31. E**	**51. E**
12. E	**32. C**	**52. D**
13. B	**33. D**	**53. B**
14. C	**34. E**	**54. E**
15. B	**35. D**	**55. B**
16. C	**36. D**	**56. A**
17. E	**37. B**	**57. E**
18. C	**38. C**	**58. D**
19. B	**39. B**	**59. A**
20. A	**40. C**	**60. E**