T M S C A H I G H S C H O O L

M A T H E M A T I C S

T E S T # 9 ©

J A N U A R Y 2 8 , 2 0 1 7

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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2016-2017 TMSCA Mathematics Test Nine

1. **Evaluate $6+\pi^0\times11-18-40\div(2\times8)$. (nearest tenth)**

   (A) 20.1 **(B)** −161.0 **(C)** −3.5 **(D)** 56.5 **(E)** −37.8

2. **How many proper fractions in lowest terms have a denominator of 42?**

   (A) 12 **(B)** 10 **(C)** 7 **(D)** 16 **(E)** 9

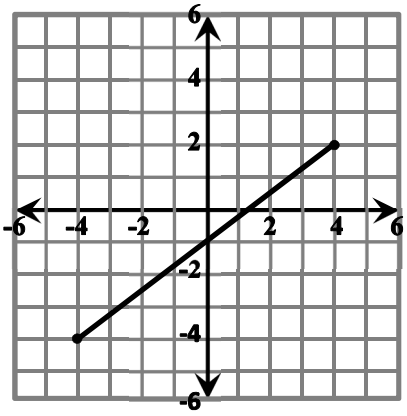
3. **A water company charges $3.81 per hundred cubic feet for the first 1000 cubic feet used, then $3.99 per hundred cubic feet for the next 1000 cubic feet. How much should a homeowner expect to pay if he uses 10,000 gallons of water in a billing period?**

   (A) $38.10 **(B)** $13.44 **(C)** $42.81 **(D)** $51.54 **(E)** $55.08

4. **Simplify $\dfrac{(n+2)!}{n!}\div\dfrac{1}{n}\times\dfrac{(n-1)!}{(n+1)!}$.**

   (A) $\dfrac{n+2}{n^2}$ **(B)** $\dfrac{n+1}{n^2}$ **(C)** $n+2$ **(D)** $n+1$ **(E)** n^2+2n

5. **Which of the following is an equation of the perpendicular bisector of the line segment shown:**



   (A) $4x-3y=3$ **(B)** $4x+3y=-1$ **(C)** $4x-3y=-1$

   (D) $4x-3y=3$ **(E)** $4x+3y=-3$

6. **Solve for y: $2xy+3x-5=y-x$.**

   (A) $\dfrac{x+5}{2x+4}$ **(B)** $\dfrac{x-5}{2x+4}$ **(C)** $\dfrac{x-5}{2x+2}$ **(D)** $\dfrac{5-4x}{2x-1}$ **(E)** $\dfrac{5}{x+4}$

7. **If $15x^2-2x-8=(5x-4)(ax+b)$, then $a+b=$ ____.**

   (A) 3 **(B)** −3 **(C)** −5 **(D)** −2 **(E)** 5

8. **A solid begins as a cube. If the length is increased by 20%, the width is doubled and the height is decreased by 25%, what is the percent increase in the total volume?**

   (A) 10% **(B)** 80% **(C)** 30% **(D)** 60% **(E)** 40%

9. **Two soccer teams, A and B, play a series of 3 games. The probability that team A will win any given game is 0.5, while the probability that team B wins any given game is 0.4. What is the probability that the series will end in a tie?**

   (A) $\dfrac{3}{25}$ **(B)** $\dfrac{1}{1000}$ **(C)** $\dfrac{121}{1000}$ **(D)** $\dfrac{31}{1000}$ **(E)** $\dfrac{3}{100}$

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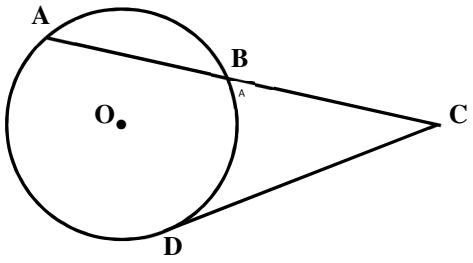
10. If $h(x)=x+3$, $g(x)=x^2$ and $f(x)=0.5x$, then $f\left(g\left(h(-4)\right)\right)=$?

(A) –0.5 (B) 0.5 (C) 0.25 (D) –0.25 (E) 7

11. An operation "Φ" is defined by $a\Phi b=a^2+b^2-ab$. What is the value of $(-2\Phi 1)\Phi(0)$?

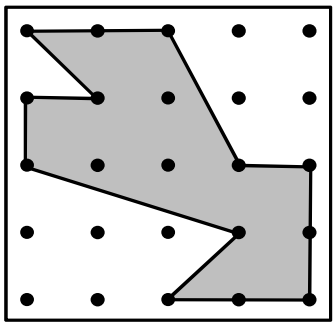
(A) 42 (B) 9 (C) 6 (D) 49 (E) 12

12. Given the circle with center O shown, find CD if AB = 9' and BC = 11'. (nearest inch)



(A) 17 (B) 13 (C) 12 (D) 10 (E) 15

13. A rubber band was stretched on the geoboard to form this 10-sided figure. What is the area?



(A) 8.5 units^2 (B) 8 units^2 (C) 9 units^2

(D) 9.5 units^2 (E) 10.5 units^2

14. If $\sqrt{x^5\left(\sqrt{x^2\left(\sqrt[3]{x}\right)}\right)}=\sqrt[n]{x^k}$, where *k* and *n* are relatively prime and $x>0$, then $k=$?

(A) 35 (B) 12 (C) 37 (D) 6 (E) 33

15. $\sec^2(\theta)\csc^2(\theta)=$

(A) $\sec^2(\theta)+\csc^2(\theta)$ (B) $\csc(2\theta)$ (C) $2\sec(\theta)$ (D) $2\csc(2\theta)$ (E) $2\sec(\theta)\csc(\theta)$

16. $3+10+29+66+127+\ldots+1333+1730=$?

(A) 3,171 (B) 6,084 (C) 17,448 (D) 6,108 (E) 6,500

17. Let the "1" at the top of Pascal's triangle be row 0. Determine the third number in row 22.

(A) 253 (B) 210 (C) 242 (D) 239 (E) 231

18. How many 3-digit numbers exist such that the sum of their digits equals 11?

(A) 63 (B) 61 (C) 59 (D) 58 (E) 60

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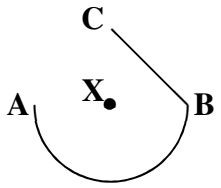
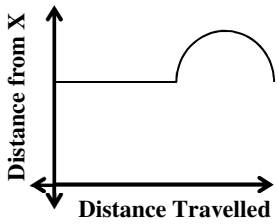
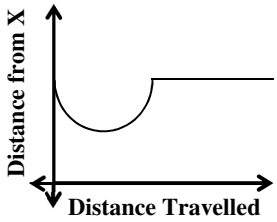
19. A boat travelled from points A to B along a semi-circular path centered on island X. Then it travelled along a straight path from B to C. Which of the graphs below could represent the boat's distance from Island X as it moves along this course?



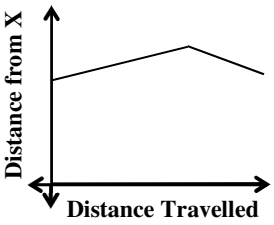
(A)



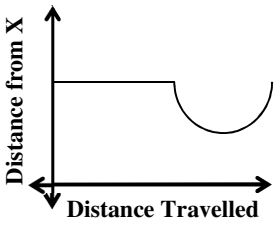
(B)



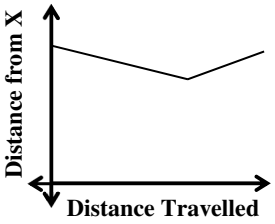
(C)



(D)



(E)



20. If $\log(ab)=12$ and $\log\left(\frac{a}{b}\right)=\frac{1}{4}$, what is the value of $\log(a)$?

(A) 2.875 **(B)** 2.5 **(C)** 4.5 **(D)** 6.125 **(E)** 5.5

21. Which of the following is a reference angle for $\frac{22\pi}{3}$?

(A) $\frac{\pi}{6}$ **(B)** $\frac{\pi}{3}$ **(C)** $\frac{\pi}{2}$ **(D)** $\frac{\pi}{5}$ **(E)** $\frac{\pi}{4}$

22. Given the Fibonacci characteristic sequence $a,9,b,12,c,27,\ldots$, find $a+b+c$.

(A) 12 **(B)** 9 **(C)** 6 **(D)** 3 **(E)** 8

23. The fraction 0.656565... in base 7 can be written as which of the following fractions in base 7?

(A) $\frac{13}{14}$ **(B)** $\frac{14}{33}$ **(C)** $\frac{32}{33}$ **(D)** $\frac{65}{66}$ **(E)** $\frac{31}{33}$

24. The polynomial x^2-4x+3 is a factor of $x^3+(a-4)x^2+(3-4a)x+3$. Find the value of a.

(A) -3 **(B)** 0 **(C)** -1 **(D)** 1 **(E)** 3

25. Let $f(x)=\frac{3x-7}{4x+5}$, where $x\neq -1.25$. Find $f^{-1}(2)$.

(A) -0.08 **(B)** -3.4 **(C)** -1.4 **(D)** 0.6 **(E)** 1.5

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26. Find the volume of the solid generated by revolving the region completely enclosed by $f(x) = 2x^2$ and $g(x) = 3 - x$ around the line $y = -1$. (nearest tenth)

(A) 78.3 (B) 98.2 (C) 3.9 (D) 102.7 (E) 88.4

27. A triangle has vertices $A(9,9)$, $B(3,2)$, and $C(9,4)$. Find the area of triangle ABC.

(A) 30 (B) 18 (C) 20 (D) 15 (E) 12

28. Old's General Store carries 8 flavors of lollipops. They decide to package them in bundles of five. How many distinct bundles are possible?

(A) 1287 (B) 462 (C) 792 (D) 1039 (E) 330

29. BHS math team consists of 7 number sense players, 11 math test players and 8 calculator players. In how many ways can their coach pick 4 of each to compete at a math meet?

(A) 435 (B) 23,135 (C) 11,620 (D) 646,800 (E) 808,500

30. Out of a total freshman class of 348, three hundred thirty are taking math classes, two hundred eighty-eight are taking science classes and thirteen students are not taking either. How many students are taking both?

(A) 288 (B) 285 (C) 283 (D) 217 (E) 281

31. Solve $3\sin^2\theta = \cos^2\theta$, for $0^\circ \le \theta \le 180^\circ$.

(A) 30°, 120° (B) 60°, 120° (C) 60°, 150° (D) 0°, 180° (E) 30°, 150°

32. What is the slope of the normal to $2x^2 + 3y^2 + xy = 9$ at the point $(2,-1)$.

(A) $\frac{4}{7}$ (B) $\frac{7}{4}$ (C) $\frac{4}{5}$ (D) $-\frac{5}{4}$ (E) $-\frac{4}{7}$

33. Find the mean of the median and mode of: 3.75, 3.5, 4.25, 2.75, 3.5, 5.25, 4, and 5.

(A) 3.9375 (B) 3.75 (C) 3.875 (D) 3.6875 (E) 4

34. Angle A is complementary to angle B and supplementary to angle C. If $m\angle B = 4x - 1$ and $m\angle C = 12x + 1$, find $m\angle A$.

(A) 43° (B) 52° (C) 46° (D) 38° (E) 55°

35. Find the lateral surface area of a right square pyramid if the diagonal of the base and the height of the pyramid both have lengths of 14 cm. (nearest square centimeter)

(A) 85 cm^2 (B) 170 cm^2 (C) 294 cm^2 (D) 339 cm^2 (E) 240 cm^2

36. If $2x - y = 3$, $x - y = -3$ and $ax - 3y = 3$, then $a =$?

(A) –2 (B) 3 (C) 1 (D) 5 (E) –4

37. On Karen's way to work, she averaged only 10 mph due to construction. On her way home, she averaged 60 mph. The total driving time to and from work was 35 minutes. How long did it take Karen to drive home?

(A) 10 min (B) 5 min (C) 6 min (D) 4 min (E) 7 min

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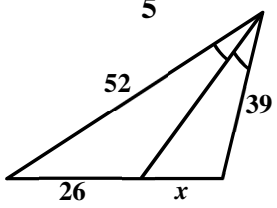
38. $1+\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}+...+\frac{1}{55}=$ _____.

(A) $1\frac{109}{132}$ **(B)** $1\frac{2}{3}$ **(C)** $1\frac{313}{396}$ **(D)** $1\frac{9}{11}$ **(E)** $1\frac{4}{5}$

39. Find the value of x on the illustration shown.

(A) 13 **(B)** 18 **(C)** 19.5 **(D)** 21.5 **(E)** 26



40. The point of concurrency of the medians of a triangle is called the_____.

(A) Incenter **(B)** Centroid **(C)** Orthocenter **(D)** Circumcenter **(E)** Euler Line

41. Which of the following is not a solution to $|5x+3|>7x-5$?

(A) -2 **(B)** $-\frac{9}{8}$ **(C)** $\frac{3}{2}$ **(D)** 4 **(E)** 1

42. The point (x,y) is a point of inflection on $f(x)=\frac{\cos x}{1+\sin x}$. Find the value of x.

(A) $-\pi$ **(B)** π **(C)** $-\frac{\pi}{2}$ **(D)** 0 **(E)** $\frac{\pi}{2}$

43. If $\frac{a}{x+5}+\frac{b}{x+7}=\frac{-x-1}{x^2+12x+35}$, then $a-b=$?

(A) -5 **(B)** 1 **(C)** -3 **(D)** -1 **(E)** 5

44. What is the constant term in the expansion of $\left(2x-\frac{1}{x}\right)^6$?

(A) -160 **(B)** 20 **(C)** -8 **(D)** 8 **(E)** 160

45. Simplify: $\left(-2-3\sqrt{-10}\right)\left(4\sqrt{-8}\right)$.

(A) $112\sqrt{10}$ **(B)** $48\sqrt{5}-16\sqrt{2}\,i$ **(C)** $80\sqrt{10}$ **(D)** $80\sqrt{3}$ **(E)** $48\sqrt{5}+16\sqrt{2}\,i$

46. Let the vector $u=(1,4)$ and $v=(5,-3)$. Find the obtuse angle between the two vectors. (nearest degree)

(A) 107° **(B)** 104° **(C)** 111° **(D)** 121° **(E)** 113°

47. A lock's combination consists of 3 digits. The first digit is a power of 3, the second is a perfect number and the third is a factor of 12. How many unique combinations fit this criterion?

(A) 9 **(B)** 8 **(C)** 12 **(D)** 15 **(E)** 24

48. Four letters are chosen at random from the word PERPENDICULAR. What are the odds that the selection will not contain any vowels?

(A) 126:715 **(B)** 14:129 **(C)** 14:143 **(D)** 126:589 **(E)** 7:71

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49. A chord of a circle has a length of 24 cm and is 5 cm from the center of the circle. The area of the circle is _____cm^2. (nearest whole square centimeter)

(A) 75 (B) 452 (C) 79 (D) 531 (E) 82

50. A waiter received a 15% tip from table one in the amount of \$9.00 and a 25% tip from table two in the amount of \$2.50. What was the positive difference in the bills from table one and two not including the tip?

(A) \$8.45 (B) \$60.00 (C) \$50.00 (D) \$5.83 (E) \$7.23

51. The reaction time of human beings are normally distributed with a mean of 0.76 seconds and a standard deviation of 0.06 seconds. What is the probability that the reaction time of a person chosen at random will be greater than 0.67 seconds? (nearest hundredth)

(A) 0.07 (B) 0.43 (C) 1.00 (D) 0.93 (E) 0.57

52. Let $a = \log x$, $b = \log y$ and $c = \log z$. Write $\log\left(\frac{x^2\sqrt{y^3}}{z^3}\right)$ in terms of a, b and c.

(A) $\frac{3ab}{2} - 3c$ (B) $\frac{a^2\sqrt{b^3}}{c^3}$ (C) $\frac{-3ab}{2c}$ (D) $a^2 + \sqrt{b^3} - c^3$ (E) $2a + \frac{3}{2}b - 3c$

53. $A93B5_{13} - A6C5_{13} =$ _____$_{13}$.

(A) 9B9CO (B) 9B8B0 (C) 948C (D) 948B (E) 9B8C0

54. Point P(2,3) undergoes several transformations to point Q. First it is reflected across the line $y = x$. Then it is reflected across the y-axis. Then it is translated 3 units horizontally in the positive direction. What are the coordinates of Q?

(A) $(6,-2)$ (B) $(0,2)$ (C) $(0,-2)$ (D) $(3,-2)$ (E) $(3,2)$

55. The sum of the first n terms of a sequence is given by $S_n = 3n^2 - n$, where $n \in \mathbb{Z}^+$. Find the eighth term of the sequence.

(A) 30 (B) 44 (C) 184 (D) 38 (E) 46

56. Find the units digit of 2017^{2017}.

(A) 0 (B) 1 (C) 3 (D) 7 (E) 9

57. Simplify to the nearest ten-thousandth place: $1 + (1.5) + \frac{(1.5)^2}{2!} + \frac{(1.5)^3}{3!} + \frac{(1.5)^4}{4!} + \ldots$.

(A) 4.4817 (B) 4.4055 (C) 3.0707 (D) 4.9975 (E) 3.2619

58. How many solutions are there to $6x + 8y = 218$ such that $x, y \in \mathbb{Z}^+$

(A) 8 (B) 7 (C) 10 (D) 9 (E) 11

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59. Two hikers, May and Nan, part and set off on paths at a 103° to each other. If may hikes at a rate of 2.5 mph and Nan hikes at a rate of 4.25 mph, how far are the two apart after 2.5 hours? (nearest tenth of a mile)

(A) 5.4 mi **(B) 12.3 mi** **(C) 6.8 mi** **(D) 13.5 mi** **(E) 10.8 mi**

60. If $\int_{-1}^{4} f(x)\,dx = 17$, what is the value of $\int_{-1}^{4} \left[2f(x)+3\right]dx$?

(A) 37 **(B) 46** **(C) 49** **(D) 32** **(E) 30**

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Test Nine Answer Key

1. C	21. B	41. D
2. A	22. A	42. E
3. D	23. D	43. E
4. C	24. D	44. A
5. E	25. B	45. B
6. D	26. B	46. A
7. E	27. D	47. D
8. B	28. C	48. B
9. C	29. E	49. D
10. B	30. C	50. C
11. D	31. E	51. D
12. E	32. E	52. E
13. A	33. D	53. A
14. C	34. C	54. B
15. A	35. C	55. B
16. D	36. D	56. D
17. E	37. B	57. A
18. B	38. D	58. D
19. D	39. C	59. D
20. D	40. B	60. C

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Test Nine Select Solutions

8. $(1.2)(0.75)(2)=1.8$, **for an 80% increase in volume.**

12. $11(20)=x^2$ **for** $x\approx 14.8$.

13. $A=\dfrac{2I+P}{2}-1=\dfrac{6+13}{2}-1=8.5$

16. The terms of the sequence follow the pattern k^3+2 **, so**
$\sum_{k=1}^{12}\left(k^3+2\right)=\left[\dfrac{12(13)}{2}\right]^2+2(12)=6108$

17. The third number in the nth row of Pascal's triangle is the (n-1)th triangular number for $\dfrac{21(22)}{2}=231$

20. $\log(ab)+\log\left(\dfrac{a}{b}\right)=\log\left(ab\bullet\dfrac{a}{b}\right)=12+\dfrac{1}{4}$, **so** $\log\left(a^2\right)=\dfrac{49}{4}$,
$2\log(a)=\dfrac{49}{4}$ **for** $\log(a)=6.125$.

24. The roots of the quadratic factor are 1 and 3. Evaluate the cubic at either for $1+a-4+3-4a+3=0$ **and** $a=1$.

26. $\int_{-1.5}^{1}\left[\pi(3-x+1)^2-\pi\left(2x^2+1\right)^2\right]dx\approx 98.2$

28. ${}_{8+5-1}C_5=792$

38. The denominators are all triangular numbers, with 55 being the 10th triangular number. So evaluate
$\sum_{n=1}^{10}\left(\dfrac{2}{k(k+2)}\right)=1\dfrac{9}{11}$

39. Let the angle the angle bisector makes with the base of the greater triangle is *y*, and the two congruent angles be *z*. Set up the law of sines for both smaller triangles:
$\dfrac{\sin y}{52}=\dfrac{\sin z}{26}$ **and** $\dfrac{\sin y}{39}=\dfrac{\sin(180-z)}{x}$, **then since sines of supplementary angles are equal, solve** $\dfrac{52}{26}=\dfrac{39}{x}$ **for** $x=19.5$

43. $a(x+7)+b(x+5)=-1-x$. **Let** $x=-7$ **and solve** $-2b=6$ **for** $b=-3$, **then let** $x=-5$ **for** $2a=4$ **and** $a=2$ **and** $a-b=2+3=5$

44. The constant term is ${}_6C_3(2x)^3\left(-\dfrac{1}{x}\right)^3=-160$

47. The digits that are powers of 3 are 1, 3 and 9. The digit that is a perfect number is 6, and 1, 2, 3, 4, and 6 are all factors of 12. The number of combinations is $(3)(1)(5)=15$

49. Because the diameter of intersecting a chord is a perpendicular bisector of a chord, the 5 and 12 form the legs of a right triangle along with a radius of the circle, which has a length of 13, so the area of the circle is $\pi(13)^2\approx 531$.

56. The units digit repeats in a pattern of 7, 9, 3, 1, 7, 9, 3, 1, …, and the remainder of 2017 divided by 4 is 1, so the last digit will be 7.

57. This is the McClaurin series for $e^{1.5}\approx 4.4817$

60. $2(17)+3(4+1)=49$

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