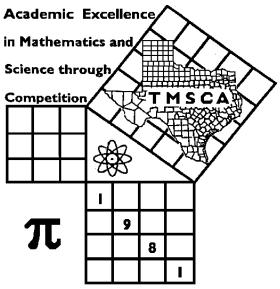


TMSCA HIGH SCHOOL MATHEMATICS TEST #9 ©

FEBRUARY 1, 2014

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2013 – 2014 TMSCA High School Mathematics Test 9

1. Evaluate $(1.818181...)^{-1}(0.51051051...)^{-1}$.

A) $\frac{3663}{3400}$ B) $\frac{5000}{459}$ C) $\frac{100000}{9231}$ D) $\frac{6660}{187}$ E) $\frac{459}{5000}$

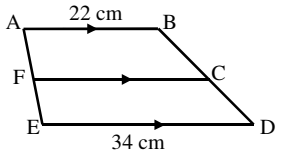
2. An electronics shop advertises a 25% off sale with an additional 10% off if purchases are made before noon. Carrie checked out at 10 am and used a coupon that gave her 15% off her total. What is the total percent discount on her items? (nearest %)

A) 57% B) 43% C) 50% D) 47% E) 53%

3. Events *A* and *B* are independent events. If $p(A \cap B) = 0.12$ and $p(A) = 3p(B)$. Calculate $p(A \cup B)$.

A) 80% B) 72% C) 68% D) 82% E) 52%

4. On the illustration shown right the area of quadrilateral ABCF is equal to the area of quadrilateral FCDE. Find FC. (nearest tenth).



A) 28.0 cm B) 27.3 cm C) 27.8 cm D) 29.3 cm E) 28.6 cm

5. Find the point of inflection for the graph of the function $f(x) = \frac{1}{3}x^3 - x^2$.

A) $(1,0)$ B) $(0,0)$ C) $\left(2,-\frac{4}{3}\right)$ D) $\left(1,-\frac{2}{3}\right)$ E) $(2,0)$

6. The point M is the midpoint of $\overline{AB}$. The coordinates of the three points are $A(x,7)$, $B(-9,y)$ and $M(1,14)$. Calculate $x+y$.

A) 32 B) 6.5 C) 14.5 D) 21 E) 16

7. A construction company pays \$131 per day for trained workers and \$99 per day for untrained workers. If 33 workers worked on a particular day and the payroll for the day was \$4227, how many of the workers were trained?

A) 3 B) 16 C) 30 D) 21 E) 12

8. $x = 37 + y$ and $xy = 920$. Calculate $x^2 + y^2$.

A) 2289 B) 449 C) 2749 D) 3209 E) 3658

9. The base of a right cone has a diameter of 18 cm and the vertex angle is 37°. What is the lateral surface area of the cone? (nearest cm^2)

A) 802 cm^2 B) 1604 cm^2 C) 423 cm^2 D) 846 cm^2 E) 1225 cm^2

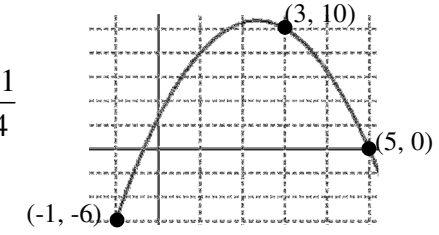
10. What is the area of the ellipse defined by the equation $4x^2 + y^2 - 8x + 4y - 8 = 0$.

A) 4π units^2 B) 16π units^2 C) 32π units^2 D) 8π units^2 E) 6π units^2

11. Given $f(x) = 2x + 5$ and $g(x) = x^2 - 1$, find $g(f(x))$.

A) $4x^2 + 24$ B) $4x^2 + 20x + 24$ C) $2x^2 + 3$ D) $4x^3 + 10x^2 + 6x + 15$ E) $2x^2 + 4$

12. Find the *x*-coordinate of the vertex on the parabola shown on the right.



A) $\frac{7}{3}$ B) $\frac{9}{4}$ C) $\frac{5}{2}$ D) 3 E) $\frac{11}{4}$

13. Triangle *ABC* is a right triangle, and $\overline{BD}$ is the altitude to the hypotenuse. Which of the following pairs are complementary?

A) $\angle ADB$, $\angle CDB$ B) $\angle BCD$, $\angle ABD$ C) $\angle BCD$, $\angle DBC$ D) $\angle ABC$, $\angle ACB$ E) $\angle BAD$, $\angle CBD$

14. What is the amplitude of $f(x)=1.3+2.5\cos(3(x-5))$?

A) 1.3 B) 15 C) 2.5 D) $1.\overline{6}$ E) 5

15. Let $x+y=3$, $4x+7y=22$ and $11x+ky=53$. Find the value of *k* for which all three lines intersect in one point.

A) 14.8 B) 17 C) -49 D) 60 E) -19

16. Simplify $\frac{x^2-2x-15}{x^2+5x+6}\cdot\frac{x^2-x-6}{x^2+2x-15}$.

A) $\frac{x-5}{x+5}$ B) $\frac{x-5}{x+3}$ C) $\frac{x+3}{x-5}$ D) $\frac{x+2}{x+3}$ E) $\frac{x+3}{x+5}$

17. Adam and Barney start together at the starting line of a 500 m track. Adam runs clockwise at an average rate of 5 meters per second. Barney runs counter clockwise at a rate of 7.5 meters per second. How far will Adam have run when they meet?

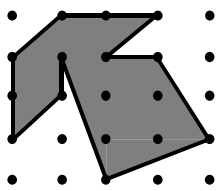
A) 300 m B) 250 m C) 240 m D) 200 m E) 320 m

18. Which of the following numbers appears in the 13th row of Pascal's triangle?

A) 330 B) 924 C) 3432 D) 1287 E) 1001

19. The dots in the diagram are 3 cm apart both vertically and horizontally. Calculate the area of the shaded region.

A) 9 units^2 B) 117 units^2 C) 81 units^2 D) 90 units^2 E) 27 units^2



20. Mr. Meredith set up a scavenger hunt for his pre-calculus class. Group one travelled 200 yards on a bearing of 212°, then 350 yards on a bearing of 97°, then 275 yards on a bearing of 325° to retrieve their clues. How far were they from their point of origin? (nearest yard)

A) 716 yd B) 97 yd C) 306 yd D) 85 yd E) 507 yd

21. Simplify $(\sin x+\cos x)^2$.

A) $2\sin x+1$ B) 1 C) $\cos 2x$ D) -1 E) $\sin 2x+1$

22. Given the Fibonacci-type sequence $8,a,6,b,c,d\ldots$. Calculate $a+b+d$.

A) -8 B) 10 C) -2 D) 14 E) 16

23. A fair coin is tossed 7 times. What is the probability of at least 5 tails in a row?

A) $\frac{3}{64}$ B) $\frac{1}{16}$ C) $\frac{1}{128}$ D) $\frac{5}{128}$ E) $\frac{1}{32}$

24. The point P has coordinates $(2,1)$. P is reflected over the line $y=-x$, translated 2 units horizontally in the positive direction, then 5 units vertically in the positive direction to point S. Find the coordinates of S.

A) $(4,4)$ B) $(1,3)$ C) $(3,7)$ D) $(-1,-2)$ E) $(4,0)$

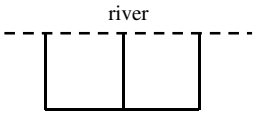
25. Two lines that are in the same plane but never intersect are_____.

A) Parallel B) Skew C) Perpendicular D) Concurrent E) Bisectors

26. Two numbers are in a ratio of 5:13. If the first is decreased by 5 and the second is increased by 5, the resulting numbers will be in a ratio of 2:7. Find the sum of the two numbers.

A) 63 B) 87 C) 112 D) 90 E) 74

27. Jonah has 250 m of fencing. He wants to build a rectangular enclosure with one central division and one side bounded by a river as shown. What is the maximum possible area for the enclosure?



A) 7813 m^2 B) 3906 m^2 C) 2604 m^2 D) 2500 m^2 E) 5208 m^2

28. The senior class sponsor wants to choose committee members from 18 athletes, 15 band members and 10 drama club members. The committee will include 4 athletes, 3 band members and 2 drama club members. How many possible committees could she form?

A) 1392300 B) 137700 C) 3240 D) 77760 E) 626535500

29. Each of four intgers -3, -2, -1 and 1 are written on a separate card. Two cards are randomly selected. What are the odds that the product of the numbers is negative?

A) 2:1 B) 1:1 C) 1:3 D) 1:2 E) 3:1

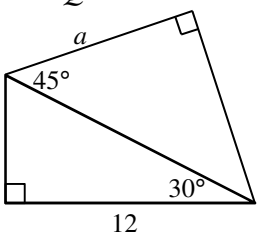
30. A pizza shop has 8 choices of toppings and two types of crust. They run a fall special offering a large, three-topping pizza for $12.00. How many possible pizza orders are there if a topping can be repeated?

A) 165 B) 240 C) 112 D) 120 E) 330

31. If $\log 9 = P$, and $\log 5 = Q$, then $\log 0.6 =$

A) $\frac{PQ}{2}$ B) $2PQ$ C) $\log\left(\frac{\sqrt{P}}{Q}\right)$ D) $\frac{P-2Q}{2}$ E) $\frac{\sqrt{P}}{Q}$

32. What is the length of side *a* on the diagram shown right?



A) 24 B) $4\sqrt{6}$ C) $8\sqrt{6}$ D) $12\sqrt{2}$ E) $12\sqrt{6}$

33. Find the sum of all 2-digit numbers whose digits differ by 5 and reversing the digits results in a square number.

A) 220 B) 94 C) 125 D) 155 E) 172

34. Find the sum of the mean, median and mode for the numbers 3, -4, 8, 7, 1, 9, -6 and 3.

A) $\frac{45}{8}$ B) $\frac{69}{8}$ C) $\frac{93}{8}$ D) $\frac{21}{8}$ E) $\frac{101}{8}$

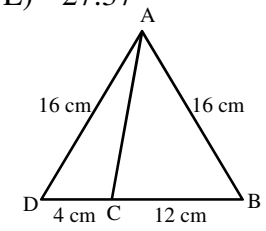
35. Write the repeating decimal 0.424242… base five as a fraction in base five.

A) $\frac{11}{220_5}$ B) $\frac{11}{22_5}$ C) $\frac{21}{220_5}$ D) $\frac{21}{202_5}$ E) $\frac{21}{22_5}$

36. The harmonic mean of A and B is 26.8 and the contraharmonic mean is 27.9. What is the arithmetic mean of A and B?

A) 27.34 B) 27.25 C) 27.33 D) 27.35 E) 27.37

37. Find AC. (nearest tenth centimeter)

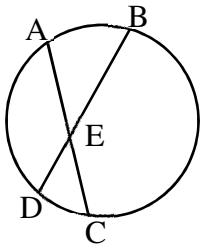


A) 9.9 cm B) 13.8 cm C) 15.3 cm D) 14.9 cm E) 14.4 cm

38. Given $y=\ln(2x-1)$, find the value of x for which $\frac{dy}{dx}=\frac{dx}{dy}$.

A) 1 B) 2 C) 0 D) 0.5 E) 1.5

39. Given $m\overset{\frown}{DC}=46^\circ$ and $m\angle AEB=52^\circ$, calculate $m\overset{\frown}{AB}$.



A) 49° B) 58° C) 50° D) 48° E) 47°

40. Let $a_1=12$, $a_2=5$ and $a_n=2a_{n-1}-3a_{n-2}$. Find a_5.

A) 147 B) -56 C) -339 D) -159 E) -33

41. What is Real value solution set to the inequality $3|2x+1|-2<13$?

A) $(-\infty,-3)\cup(2,\infty)$ B) $(-3,2)$ C) $(-\infty,-2)\cup(3,\infty)$ D) $(-2,3)$ E) $(-\infty,-3)\cup(-2,\infty)$

42. How many distinct arrangements can be made from the letters "PARALLEL"?

A) 720 B) 3360 C) 6720 D) 40320 E) 20160

43. Find the range, or ranges, of values of K for which $f(x)=Kx^2-4x+5-K$ has two distinct real roots.

A) $1<K<4$ B) $K<1$ and $K>4$ C) $K<-6$ and D) $K<\frac{4}{5}$ E) $-6<K<2$

44. Given that $(a+i)(2-bi)=7-i$, where $a,b\in\mathbb{Z}$, find the value of $a+b$.

A) 5.5 B) -2 C) 6.5 D) 4 E) 2

45. Simplify $a^4\times b^3\div a^{-1}\times b^{-4}\times a^2\div b^2\div b^{-3}$.

A) $\frac{a^5}{b^2}$ B) a^5b^2 C) a^7b D) a^7 E) a^5

46. Let A and B be the roots of $3x^2-4x-7=0$. Find the value of $A^4+4A^3B+6A^2B^2+4AB^3+B^4$.

A) $\frac{10000}{81}$ B) $\frac{2401}{81}$ C) $\frac{10000}{2401}$ D) $\frac{256}{81}$ E) $\frac{256}{2401}$

47. If $f(a)=g(a)=0$, $f'(a)$ and $g'(a)$ exist and $g'(a)\neq 0$ then $\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right)=\frac{f'(a)}{g'(a)}$. This is_____.

(A) Definition of Derivative (C) Rolle's Theorem (E) Fundamental Theorem of Calculus

(B) Intermediate Value Theorem (D) L'Hopital's Rule

48. Given $GCF(36,k)=12$ and $LCM(36,k)=252$, find the value of k.

A) 126 B) 48 C) 42 D) 63 E) 84

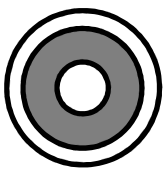
49. If $\frac{x-8}{x+8}+\frac{x+8}{x-8}=A\frac{B}{C}$, then $B=?$

A) 64 B) 32 C) 88 D) 256 E) 128

50. If $y^5=122+597i$ and $y^4=-119+120i$ and $y=a+bi$ then calculate $a+b$.

A) -57 B) -17 C) -1 D) 0 E) 17

51. The diameters of the concentric circles on the right are 14 cm, 12 cm and 6 cm. If a dart hits the figure at random, what is the probability that it will land in the shaded area? (nearest %)



A) 45% B) 27% C) 59% D) 55% E) 73%

52. The points $A(-8,0), B(1,12), C(12,3)$ and $D(3,-5)$ are plotted and connected to form the convex quadrilateral *ABCD*. What is the area of this quadrilateral?

A) 153 units^2 B) 173 units^2 C) 213 units^2 D) 170 units^2 E) 87 units^2

53. Let $f(x) = ax^5 + bx^3 + cx - 8$ and $f(-6) = 27$. Calculate $f(6)$.

A) -35 B) 19 C) -27 D) -43 E) 51

54. $11_2 + 11_3 + 11_4 + 11_5 + 11_6 + 11_7 + 11_8 + 11_9 =$ _____$_{10}$.

A) 88 B) 44 C) 52 D) 66 E) 72

55. $\int_{-2}^{4} (kx^2 + 5x + 2)\,dx = 114$. What is the value of *k*?

A) 3 B) 2 C) -3 D) -1 E) 1

56. What is the area of a regular hexagon in terms of the length, *s*, of one side?

A) $\frac{3s^2\sqrt{3}}{4}$ B) $\frac{s^2\sqrt{3}}{6}$ C) $\frac{s^2\sqrt{3}}{4}$ D) $\frac{3s^2\sqrt{3}}{2}$ E) $\frac{2s^2\sqrt{3}}{3}$

57. If $\begin{pmatrix} 2 & a & b \\ 1 & 0 & -3 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ b \end{pmatrix}$, find the value of *a*.

A) -5 B) $\frac{10}{3}$ C) 7 D) $\frac{8}{3}$ E) -6

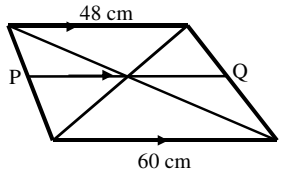
58. Simplify to the nearest ten-thousandth place: $1 + (1.5) + \frac{(1.5)^2}{2!} + \frac{(1.5)^3}{3!} + \frac{(1.5)^4}{4!} + \ldots$.

A) 4.4817 B) 0.4055 C) 0.0707 D) 0.9975 E) 0.0262

59. How many solutions are there to $6x + 8y = 218$ such that $x, y \in \mathbb{Z}^+$.

A) 8 B) 27 C) 7 D) 36 E) 9

60. Find PQ.



A) $\frac{160}{3}$ cm B) 54 cm C) $\frac{164}{3}$ cm D) $24\sqrt{5}$ cm E) $36 + 8\sqrt{5}$ cm

2013-2014 TMSCA Mathematics Test Nine Answers

1. A	21. E	41. B
2. B	22. E	42. B
3. C	23. B	43. B
4. E	24. B	44. D
5. D	25. A	45. D
6. A	26. D	46. D
7. C	27. E	47. D
8. D	28. E	48. E
9. A	29. B	49. D
10. D	30. B	50. C
11. B	31. D	51. D
12. A	32. B	52. B
13. B	33. D	53. D
14. C	34. B	54. C
15. B	35. E	55. A
16. A	36. D	56. D
17. D	37. E	57. D
18. D	38. E	58. A
19. C	39. B	59. E
20. D	40. B	60. A

2013-2014 TMSCA Mathematics Test Nine Select Solutions

4. The root-mean-square of the lengths of the bases of a trapezoid is the length of the segment that bisects the area.

$PQ = \sqrt{\frac{34^2 + 22^2}{2}} \approx 28.6$ cm

8. $x - y = 37$, so

$(x-y)^2 = x^2 - 2xy + y^2 = 1369$

$x^2 + y^2 = (x-y)^2 + 2xy = 3209$

19. $A = \frac{8+12}{2} - 1 = 9$ units^2 in the diagram, and since the linear scaling is 1:3, the scaling for area will be 1:9. So the area of the shaded region will be $9(9) = 81$ cm^2.

23. With 7 coin tosses, each possibility has a probability of $\left(\frac{1}{2}\right)^7 = \frac{1}{128}$. Let H be a group of 5 heads, all the possibilities for at least 5 heads in a row are:

Htt	tHt	ttH
Hth	htH	Hht
thH	hhH	

So, probability of at least 5 heads in a row is $8\left(\frac{1}{128}\right) = \frac{1}{16}$.

30. If toppings can be repeated, then the number of possible topping choice combinations is ${}_{8+3-1}C_3 = 120$. Multiply by the 2 types of crust for 240 choices.

36. The arithmetic mean of two numbers is also the arithmetic mean of their harmonic and contraharmonic means. So the arithmetic mean of A and B is 27.35.

38. $\frac{dy}{dx} = \frac{dx}{dy}$ when $\frac{dy}{dx} = \pm 1$.

$\frac{dy}{dx}\left(\ln(2x-1)\right) = \frac{2}{2x-1}$.

$\frac{2}{2x-1} = \pm 1$ when $x = 0.5$, or 1.5, and 0.5 is outside the domain of the original function.

42. PARALLEL has 8 letters, the A occurs twice and the L occurs three times, so the number of distinct arrangements is $\frac{8!}{2!\cdot 3!} = 3360$.

49. For numbers $\frac{a}{b} + \frac{b}{a} = A\frac{B}{C}$, $B = (a-b)^2$. For this problem, $(x-8-x-8)^2 = 256$.

53. Let $g(x) = f(x) + 8$, $g(x)$ is an odd function, so $g(-6) = -g(6)$.

$g(-6) = 27 + 8 = 35$ and $-35 = f(6) + 8$, so $f(6) = -43$.

58. This is the MacClaurin series expansion of $f(x) = e^x$. $e^{1.5} \approx 4.4817$.

60. The length of the segment parallel to the bases that passes through the intersections of the diagonals of a trapezoid is the harmonic mean of the lengths of the bases. So,

$PQ = \frac{2(48)(60)}{(48+60)} = \frac{160}{3}$ cm.