

## TMSCA HIGH SCHOOL MATHEMATICS TEST # 8 © JANUARY 19, 2013

## **GENERAL DIRECTIONS**

## 1. About this test:

- A. You will be given 40 minutes to take this test.
- B. There are 60 problems on this test.
- 2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
- 3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
- 4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
- 5. You may use additional scratch paper provided by the contest director.
- 6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
- 7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
- 8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
- 9. In case of ties, percent accuracy will be used as a tie breaker.

TMSCA 2012-2013 TMSCA High School Mathematics

1.	What is 36% of the s	sum of $\frac{1}{3}$ and $\frac{2}{9}$ ?	C			
		(B) $\frac{125}{81}$	(C) 81	(D) $\frac{81}{125}$	(E) $\frac{1}{5}$	
2.	Events A and B are i	independent such that	P(B) = 4P(A) and $P$	$P(A \cup B) = 0.84$ . Find	P(A).	
	(A) 0.46	(B) 0.80	(C) 0.24	(D) 0.20	(E) 0.96	
3.	Let <i>y</i> vary directly w (A) 34.99	with $x^2 + 11$ . If $y = 72$ (B) 151.88	2.9 when $x = 4$ , find y (C) 18.23	when $x = 1$ . (D) 32.40	(E) 151.46	
4.	Find the y-intercept	of the line containing	(7,3) and perpendicut	lar to $5x - 4y + 11 = 0$ .		
	(A) $(8.6,0)$	(B) (10.75,0)	(C) (0,10.75)	(D) $(0, 8.6)$	(E) $(0, -10.5)$	
5.	Given the sequence 2 (A) 1040	2, 6, 15, 32, 65,, 44 (B) 940	0, <i>k</i> , 1440 find the value (C) 1353	ue of <i>k</i> . (D) 801	(E) 678	
6.	If $\int_{2}^{k} \frac{1}{x+2} dx = \ln 2$ ,	find the value of k.				
	(A) 6	(B) 8	(C) 0	(D) 2	(E) 4	
7.	possible sums. The larger possible sum is					
0	(A) 153	(B) 117.188	(C) 187.500	(D) 46.875	(E) 55.147	
8.	2		gonal prism to ship a de		-	
	)		e longest support that w			
	(A) 25.89 in	(B) 129.44 in	(C) 144.66 in	(D) 89.40 in	(E) 30.61 in	
9.		6	ft in one hour. How lo leep if they each work (C) 2 hr 30 min	0	e	
10	. Find the area of the p	pentagon inscribed in	the circle on the right.			
	(A) $96 + 75\sqrt{3}$ (1)	B) $171\sqrt{3}$ (C)	the circle on the right. 171 $\sqrt{2}$ (D) 96+1	$00\sqrt{2}$ (E) 96+1	$10^{12}$	
11. A right conical tank with a base diameter of 2 yards and a height of 41 inches is filling with water at a rate of 3.1 gal/min. How long will it take to fill?						
	(A) 932.44 min	(B) 233.11 min	(C) 310.82 min	(D) 77.70 min	(E) 60.58 min	
12. Given that $(a+4i)(b+i)=17-31i$ , where $a,b\in\mathbb{Z}$ , find the value of $a+b$ .						
	(A) 3	(B) 4	(C) -7	(D) 7	(E) -10	
13	13. A regular pentagon is inscribed in a circle and the diagonals are drawn. What is the ratio of the lengths b to a?					
	(A) $\frac{\pi}{2}$ (B)	e (C) <i>q</i>	(D) $\pi$	(E) <i>ε</i>		

14. Given  $y = -2\cos x$ , find the value of x for which  $\frac{dy}{dx} = \frac{dx}{dy}$ , where  $0 \le x \le \frac{\pi}{2}$ (A) 0(E)  $\pi$ (B)  $\frac{\pi}{4}$  (C)  $\frac{\pi}{3}$  (D)  $\frac{\pi}{6}$ 15. Given the pyramid at right, the fourth number in the 14<sup>th</sup> row is the sum of the first \_\_\_\_\_ 1 1 1 triangular numbers. 1 2 1 (A) 11 (B) 12 (C) 13 (D) 14 (E) 10 1 3 3 1 16. The measure of one interior angle of a regular dodecagon is (B) 144° (A) 108° (C) 136° (D) 150° (E) 156° 17. Given  $f(x) = 6x^3 - 11x^2 + kx + 105$ , find k if (x+3) is a factor of f(x). (C) -26 (A) -14 (B) -52 (D) -156 (E) -6 18. A fair six-sided die with sides labeled 1, 1, 2, 3, 3, 6 is thrown. What is the expected value of a single roll? (A) 2 (C) 3 (D)  $\frac{11}{6}$ (E)  $\frac{5}{2}$ (B)  $\frac{8}{2}$ 19. Find the total area of the two regions enclosed by the curves  $y = x^3 - 4x^2 + x - 12$  and y = 6x - 14. (B) 78.08 (C) 111.42 (D) 53.33 (E) 66.86 (A) 28.58 20.  $\cos x + \sin x \tan x =$ (A)  $\sec x$ (B)  $\cos x$ (C)  $\cot x$ (D)  $\csc x$ (E)  $\sin x$ 21. A circle is inscribed in a triangle. The center of the circle is the \_\_\_\_\_ of the triangle. (A) centroid (C) incenter (D) orthocenter (B) circumcenter (E) foci 22. If the equation of the function graphed below is  $y = a \sin bx + c$ , find the value of  $a \cdot b \cdot c$ . (B)  $-\pi$  (C)  $-\frac{\pi}{6}$  (D)  $\frac{\pi}{3}$ (E)  $\pi$ (A)  $\frac{\pi}{6}$ -2 10 23. Carrie's drives to work every weekday on the highway. Her average daily speeds for the week are 68 mph, 68 mph, 57 mph, 65 mph, and 66 mph. What is her average speed for the week?

(A) 63.36 mph

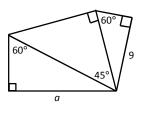
(B) 64.52 mph (C) 63.48 mph

(E) 63.75 mph

(D) 64.8 mph

24. Carries Ice Cream Shop has nine flavors of ice cream and three types of cones. How many distinct 2-scoop cones can be ordered? (D) 55 (E) 135

- (A) 108 (B) 165 (C) 45
- 25. Find the value of *a* on the picture to the right.
- (A)  $6\sqrt{6}$  (B)  $\frac{27\sqrt{2}}{2}$  (C)  $6\sqrt{3}$  (D)  $3\sqrt{6}$ (E)  $9\sqrt{2}$ 26.  $\sum_{i=1}^{10} [n(n+1)] =$ (A) 245 **(B)** 110 (C) 55 (D) 440



(E) 385

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27. Let 
$$a_1 = 3$$
,  $a_2 = -2$ , and  $a_n = 2(a_{n-1})(-a_{n-2})$ . Find  $a_5$ .  
(A) 284 (B) -576 (C) -288 (D) 1728 (E) -1152  
28. A regular pentagon is inscribed in a circle with a radius of 8 cm. Find the area of the shaded

region. (A)  $160.45 \text{ cm}^2$  (B)  $101.90 \text{ cm}^2$  (C)  $9.65 \text{ cm}^2$  (D)  $48.89 \text{ cm}^2$  (E)  $75.40 \text{ cm}^2$ 

29. A fish is reeled in at a rate of 2 feet per second from a point 7 feet above the surface of the water. How fast is the angle between the line and the water changing when there are 25 feet of line out?

- (A)  $\frac{7}{150}$  rad/sec (B)  $\frac{7}{300}$  rad/sec (C)  $-\frac{7}{600}$  rad/sec (D)  $\frac{7}{600}$  rad/sec (E)  $-\frac{7}{300}$  rad/sec
- 30. If a hiker travels 6 miles on a bearing of 12°, then another 5 miles on a bearing of 334°, what is the shortest distance back to his starting point?
  - (A) 10.00 miles (B) 2.26 miles (C) 10.06 miles (D) 13.06 miles (E) 10.41 miles
- 31. If  $f(x) = 2\cos^2 x$ ,  $g(x) = x^2 + 9$ , and h(x) = g(f'(x)), find  $h\left(\frac{\pi}{6}\right)$ . (A) 21.00 (B) 9.25 (C) 18.00 (D) 13.91 (E) 12.00
- 32. The total surface area of an octahedron is  $64\sqrt{3}$  cm<sup>2</sup>. Find the length of one edge? (A)  $8\sqrt{2}$  in (B)  $4\sqrt{3}$  in (C)  $4\sqrt{2}$  in (D)  $2\sqrt{3}$  in (E)  $8\sqrt{3}$  in

33. The point (2, -7) is reflected over the *x*-axis, reflected over the line y = x, rotated 270° clockwise around the origin, then shifted down three units to the point (a,b). Find a+b. (A) 2 (B) -12 (C) -7 (D) -5 (E) -9

34. Which of the following is not a solution to  $f(x) \ge \left|25 - \sqrt{x^2}\right|$ ?

- (A) (-3,22) (B) (-8,18) (C) (9,14) (D) (0,30) (E) (4,21)
- 35. If f(x) = x + 7, g(x) = 9 4x and  $h(x) = \frac{f(x)}{g(x)}$ , find  $h^{-1}(1)$ . (A) -0.25 (B) 1.48 (C) -0.67 (D) -0.58 (E) 0.4
- 36. The probability of rolling each number one through six on a weighted die is shown in the table below. Find the expected value of a single roll.

	x	1	2	3	4	5	6
	p(x)	1	1	k	5	1	1
	1 ( )	$\overline{3}$	12		24	12	$\overline{6}$
	(A) 1	(B) $\frac{7}{2}$	()	C) $\frac{9}{2}$	(D) $\frac{11}{4}$	(E	$)\frac{25}{8}$
37. If $f(x) = 2x^3 + ax^2 + bx + c$ and $f(x)$ has zeroes at -5, -7, and 3, find $a + b + c$ .							
	(A) -95	(B) -190	) ()	C) -23	(D) -194	(E	) -97

38. Which of the follow I. natural numbers (A) I & III	-		umbers IV. c (D) IV	odd integers (E) none of these		
39. What is the equation	of a line through $(5,1)$	4) that is normal to $x^2$	$x^2 - y = 11?$			
	(B) $x + 10y - 145 = 0$			(E) $x - 10y + 19 = 0$		
40. The school day at Austin Elementary School begins at 8:25 am and ends at 4:15 pm. How many degrees does the minute hand on the clock in the cafeteria move during one school day?						
(A) 2650°	(B) 235°	(C) 1290°	(D) 2580°	(E) 2820°		
$41. \int \left(\cos^2 x - \sin^2 x\right) dx$			ry constant.			
(A) $\frac{\cos^3 x}{3} - \frac{\sin^3 x}{3}$	(B) $-2(\sin x + \cos x)$	$)  (C)  \sin x \cos x$	(D) <i>x</i>	(E) $-\frac{\sin 2x}{2}$		
	antees color TVs and wild the company set the	will replace a TV that	breaks while under gr not want to replace m	arantee with a new		
43. There are seven boy three girls to attend a (A) 350	s and five girls on the s a community luncheon (B) 45		t groups could the spo			
44. A seasoned basketba game. What is the p (A) 0.241	all player makes 45% c probability that she mal (B) 0.368			times during a Friday (E) 3.223		
45. $f(x) = (x-2) - \frac{(x-2)}{(x-2)}$	$\frac{1}{3!} + \frac{1}{5!} - \frac{1}{7}$	Find the 10 <sup>-8</sup> p	place of $f(4)$ .			
(A) 0		(C) 9		(E) 2		
46. A furniture store is running a sale on dining chairs. The original price of each chair is \$128. During the sale, customers receive a 10% discount on the first chair they buy, 20% on the second, 30% on the third and so on with a limit of six per customer. How much would a customer pay for a set of six chairs if the tax is 8.25%?						
(A) \$563.42	(B) \$580.75	(C) \$503.32	(D) \$623.52	(E) \$540.38		
47. How many numbers (A) 2	in the form $a^4$ , where (B) 3	$a \in \mathbb{Z}^+$ divide $3! \times 4!$ (C) 5	× 7! ? (D) 6	(E) none		
48. What is the area of a (A) $8.37 \text{ in}^2$	triangle with side leng (B) 44.00 in <sup>2</sup>	gths 8 inches, 11 inche (C) 35.50 in <sup>2</sup>	es, and 17 inches? (D) 26.00 in <sup>2</sup>	(E) 39.75 in <sup>2</sup>		
49. The faces of an icos (A) hexagons	ahedron are regular (B) squares	(C) pentagons	(D) triangles	(E) octagons		
50. Express $\log(100\sqrt{ab})$ in terms of P and Q if $P = \log a$ and $Q = \log b$ .						
(A) 20 <i>PQ</i>	(B) $100P + \frac{1}{2}Q$		(D) <i>PQ</i>	(E) $\frac{1}{2}(P+Q)+2$		

- 51. The sum of the first ten terms of an arithmetic sequence is 665 and the sum of the first twenty terms is 2430. Find the common difference of the sequence.
  - (A) 13 (B) 17 (C) 15 (D) 11 (E) 12

52. At which of the following x-values is the graph of  $f(x) = x^4 - 12x^3 + 48x^2 - 64x$  concave down? (A) 3 (B) 4 (C) 2 (D) 1 (E) 5

53. If  $\cos \theta = -\frac{7}{25}$  and  $\pi \le \theta \le 2\pi$ , then  $\tan \theta =$ (A)  $-\frac{24}{25}$  (B)  $\frac{24}{7}$  (C)  $-\frac{24}{7}$  (D)  $\frac{7}{24}$  (E)  $-\frac{7}{24}$ 

54. The line 3x + 2y = 39 forms a chord with the circle  $x^2 + y^2 + 4x - 6y - 156 = 0$ . Find the length of the chord. (A) 12

(A) 13 (B)  $2\sqrt{39}$  (C)  $\sqrt{195}$  (D)  $4\sqrt{13}$  (E) 12

55. An investment is made in a fund that pays an annual percentage rate of 6%, compounded monthly. How long (to the nearest tenth of a year) will it take for the investment to double?
(A) 11.9
(B) 11.3
(C) 11.6
(D) 11.0
(E) 11.7

56. In town A, during a one week period, the probability that it is cloudy on any particular day is 0.35. If it is cloudy, the chance of rain is 0.72. What is the probability that it will rain at least once during the week?
(A) 0.131
(B) 0.869
(C) 0.999
(D) 0.855
(E) 0.900

57. What is the area of the largest isosceles triangle that can be inscribed in a circle with the equation  $x^2 + y^2 + 6x + 10y - 87 = 0$ ?

(A) 
$$\frac{121\sqrt{3}}{16}$$
 (B)  $\frac{363\sqrt{3}}{16}$  (C)  $\frac{121\sqrt{3}}{4}$  (D)  $\frac{363}{4}$  (E)  $\frac{363\sqrt{3}}{4}$ 

58. Find the constant term in the expansion of  $\left(2x^2 - \frac{3}{x}\right)^2$ . (A) 19683 (B) -145152 (C) 979776 (D) 489888 (E) -326592

- 59. How many distinct arrangements are there of three letters chosen from the word COMBINATION? (A) 165 (B) 990 (C) 399 (D) 133 (E) 495
- 60. If  $\frac{x-8}{x+3} + \frac{x+3}{x-8}$  is equal to the mixed number  $A \frac{B}{(x+3)(x-8)}$ , then B =(A) 64 (B) 8 (C) 121 (D) 24 (E) 9

## 2012-2013 TMSCA High School Mathematics Test 8 Key

1. E	21. C	41. C
2. D	22. B	42. A
3. D	23. B	43. A
4. D	24. E	44. D
5. D	25. E	45. E
6. A	26. D	46. E
7. C	27. E	47. D
8. A	28. D	48. C
9. D	29. B	49. D
10. A	30. E	50. E
11. D	31. E	51. D
12. E	32. C	52. A
13. C	33. A	53. B
14. D	34. C	54. D
15. A	35. E	55. C
16. D	36. E	56. B
17. B	37. D	57. E
18. B	38. C	58. D
19. E	39. B	59. C
20. A	40. E	60. C

2012 – 2013 TMSCA Solutions Mathematics Test Eight

1. 
$$0.36\left(\frac{3}{9}+\frac{2}{9}\right) = \frac{9}{25} \cdot \frac{5}{9} = \frac{1}{5}$$
  
2.  $P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B) =$   
 $P(A) + 4P(A) - P(A) \cdot 4P(A) = 0.84$   
graph to solve  $P(A) = 0.20$   
3.  $72.9 = k(4^2 + 11), k = 2.7, \text{ so}$   
 $y = 2.7(1^2 + 11) = 32.4$   
4.  $\pm m = -0.8, y - 3 = -0.8(0 - 7), y = 8.6$   
5.  $1(2), 2(3), 3(5), 4(8) ...9(89)$ , each term is  
the term number times a Fibonacci number,  
 $9(89) = 801$   
6.  $\left[\ln(x + 2)\right]_{2}^{k} = \ln(k + 2) - \ln 4 = \ln 2$   
 $\ln(k + 2) = 3\ln 2 = \ln 8, k + 2 = 8, k = 6$   
7.  $75r^2 = 27, r = \pm 0.6, S = \frac{75}{1 - 0.6} = 187.5$   
8.  $P = 80$  in,  $s = 16$  in, to find longest diagonal,  
 $d^2 = 16^2 + 16^2 - 2(16)(16)\cos 108^{\circ}, d \approx 25.89$   
9. let  $r$  be the rate of one digger.  $3r \cdot 1 = 1$  hole.  
 $r = \frac{1}{3}, 6\left(\frac{1}{3}\right)t = 27, t = 13.5$  hours  
10. one right triangle with legs 12 and 16 and 3  
equilateral triangles with sides of 10.  
 $A = \frac{12(16)}{2} + 3\left(\frac{10^2\sqrt{3}}{4}\right) = 96 + 75\sqrt{3}$   
11.  $t = \frac{36^2\pi(41)}{3} \div 231 \div 3.1 \approx 77.70$  min  
12.  $ab + ai + 4bi - 4 = 17 - 31i$ , so  
 $ab - 4 = 17, and a + 4b = -31$ , solve  
system and  $b = -7, a = -3$ , so  $a + b = -10$   
13. golden ratio  
14.  $\frac{dy}{dx} = 2\sin x = \frac{dx}{dy}$  for  $2\sin x = \pm 1$ , so  
 $\sin x = \pm \frac{1}{2}$  only at  $x = \frac{\pi}{6}$  on domain  
15. The fourth number any *n*th row is the sum  
of the first  $n - 3$  triangular numbers (11)

$$\frac{90^{\circ}(12-2)}{12} = 150^{\circ}$$

$$\frac{90^{\circ}(12-2)}{12} = 150^{\circ}$$

$$\frac{1}{12} = 12^{\circ}$$

$$\frac{1}{12} = 12^{\circ}$$

$$\frac{1}{12} = 12^{\circ}$$

$$\frac{1}{12} = 12^{\circ}$$

$$\frac{1}{10} = 11^{\circ}$$

$$\frac{1}$$

54. 
$$y_1 = \frac{39 - 3x}{2}$$
, substitute to obtain  
 $x^2 + y_1^2 + 4x - 6y_1 - 156 = 0$ , graph to solve  
 $x = 3,11$  yields points (3,15) and (11,3)  
 $d = \sqrt{8^2 + 12^2} = 4\sqrt{13}$   
55.  $2 = \left(1 + \frac{0.06}{12}\right)^{12t}$ ,  $t \approx 11.6$   
56. p(rain one day) =  $0.35(0.72) \approx 0.252$   
p(at least one rain) = 1-no rain  
 $1 - 0.748^7 \approx 0.869$   
57.circle radius 11,equilateral triangle  $s = 11\sqrt{3}$   
 $A = \frac{\left(11\sqrt{3}\right)^2 \sqrt{3}}{4} = \frac{363\sqrt{3}}{4}$   
58.  $\binom{9}{3} (2x^2)^3 \left(-\frac{3}{x}\right)^6 = 489888$   
59. all different =  $8 \cdot 7 \cdot 6 = 336$   
two "O"s =  $7 \cdot 3 = 21$ , same for "T" and "N"  
 $336 + 3(21) = 399$   
60.  $(-8 - 3)^2 = 121$