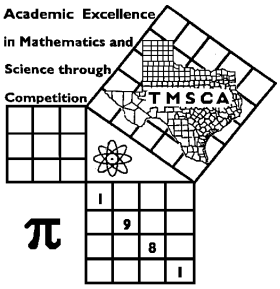

TMSCA HIGH SCHOOL MATHEMATICS TEST# 7 © JANUARY 16, 2016

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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2015-2016 TMSCA Mathematics Test 7

1. Evaluate: $\left(.\overline{6}\right)\left(6!\right)\div\left(.2\right)\left(7!\right)-10!$.

A. 7257600 B. -3193344 C. 8467200 D. -3628800 E. -3411072

2. Two million, three hundred eighteen thousand, three hundred one plus thirty-six thousand, fifty is subtracted from six million, six thousand, six. What is the sum of the digits in the difference when written out using digits instead of words?

A. 32 B. 40 C. 26 D. 29 E. 31

3. Bonnie plans to buy two dozen cupcakes that usually cost $1.25 each. She receives a 15% discount on the first dozen, a 20% discount on the second dozen, and pays 8.25% sales tax on the total. How much will her order cost?

A. $26.79 B. $24.75 C. $30.00 D. $26.85 E. $25.17

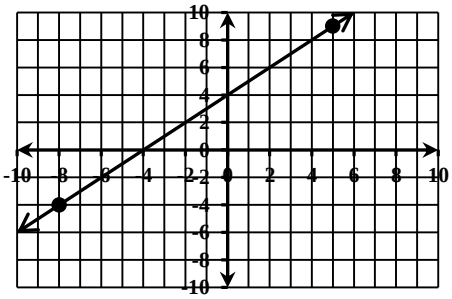
4. If $-3\left(4-x\right)=\left(x-4\right)\left(x-8\right)$, where $x\neq 4$, then $2x+8=$

A. 12 B. 30 C. 21 D. 11 E. 33

5. Simplify $\dfrac{\left(x+2\right)!}{\left(x-3\right)!}\div\dfrac{x!}{\left(x-1\right)!}$.

A. x^4+5x^2+4 B. $\dfrac{x}{x^2-3x+2}$ C. $x^6-5x^4+4x^2$ D. x^4-5x^2+4 E. $\dfrac{x^4-5x^2+4}{x^2}$

6. Find the equation of the line perpendicular to the one shown and through the point $\left(-2,3\right)$.



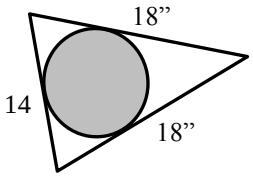
A. $x+y=1$ C. $x-y=1$ E. $x+y=-1$

B. $x-y=-5$ D. $x+y=-5$

7. Carla's mother drove her to an event at an average speed of 55 mph. She rode back with a friend at an average speed of 70 mph. If her total travel time was 2 hours 20 minutes, how far did she ride with her mother? (nearest mile)

A. 73 mi B. 72 mi C. 64 mi D. 68 mi E. 70 mi

8. The circle in the illustration is inscribed in the triangle. Find the area of the circle. (nearest tenth)



A. 67.7 in^2 B. 58.0 in^2 C. 62.8 in^2 D. 78.4 in^2 E. 63.6 in^2

9. The radius of a cone is 16 in. and the lateral surface area is 1106 in^2. Find the volume of the cone. (nearest tenth cubic foot)

A. 3.4 ft^3 B. 2.9 ft^3 C. 2.6 ft^3 D. 2.5 ft^3 E. 2.3 ft^3

10. If $\dfrac{A}{\left(x-8\right)}+\dfrac{B}{\left(3x-1\right)}=\dfrac{17x+2}{3x^2-25x+8}$ then $A+B=$

A. 3.5 B. 7 C. 6 D. 5 E. 5.5

11. The measure of an inscribed angle of a circle is 28°. What is the measure of the intercepted arc?

A. 62° B. 56° C. 14° D. 59° E. 38°

12. Carl has room for 8 travel books on his bookcase, but he has 11 travel books. In how many ways can he organize books on the shelf?

A. 330 B. 2640 C. 6652800 D. 1663200 E. 1980

13. High-speed train A left Paris at 8:00 am travelling at 200 mph due south. Forty-five minutes later, Train B left the same station travelling at 185 mph due east. How far apart were the trains at 12:15 pm? (nearest mile)

A. 1053 mi B. 1362 mi C. 1158 mi D. 1069 mi E. 1468 mi

14. Which of the following statement(s) about function *f* is/are true?

I. Every function *f* has an inverse function f^{-1}.

II. The domain of *f* is the range of f^{-1}.

III. f^{-1} is the same as $\frac{1}{f}$.

A. I, II & III B. I & III C. II & III D. II only E. none of these

15. If the pattern of the sequence 2, 8, 16, 26, 38,…continues, find the 15th term.

A. 268 B. 236 C. 302 D. 284 E. 252

16. Determine the range of $f(x) = -7 + 3\cos(2\pi x - 3)$.

A. $[-9,-5]$ B. $[0,10]$ C. $[-10,-4]$ D. $[2,9]$ E. $[-9,-2]$

17. Find $m+n$ if $\begin{bmatrix} -2 & 3 \\ -1 & 8 \end{bmatrix}\begin{bmatrix} -3 \\ 4 \end{bmatrix} = \begin{bmatrix} m \\ n \end{bmatrix}$.

A. 53 B. 35 C. 22 D. 26 E. 48

18. Leslie started out at her cabin and hiked 2.5 miles on a bearing of 18°, then changed direction and hiked another 3.6 miles on a bearing of 118°. How far would Leslie have to hike to go straight back to the cabin? (nearest tenth)

A. 4.8 mi B. 4.0 mi C. 5.8 mi D. 2.3 mi E. 5.0 mi

19. Find the sum of all the critical points of $f(x) = -3x^3 - 11x^2 + 9x + 8$.

A. $\frac{11}{3}$ B. $\frac{9}{11}$ C. -1 D. $-\frac{22}{9}$ E. $\frac{8}{3}$

20. Use the Fibonacci characteristic sequence …, -2, *p*, 6, *q*, 20, *r*, … to find $p+q+r$.

A. 24 B. 48 C. 28 D. 36 E. 56

21. $\left[(1+2+3+4+...+17)\times 30\right] \div \left[(31+32+33+34+...+46+47)\times 60\right]$=?

A. $\frac{3}{52}$ B. $\frac{90}{13}$ C. $\frac{1}{4}$ D. $\frac{3}{26}$ E. $\frac{1}{26}$

22. Given $x = 7 - y$ and $xy = 14$ find $x^2 + y^2$.

A. 21 B. 35 C. 49 D. 42 E. 56

23. If $f''(x) = 18x - 10$, $f'(1) = 7$ and $f(-1) = -25$ find $f(2)$.

A. 2 B. 11 C. 20 D. 15 E. -9

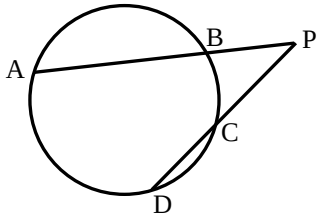
24. How many non-negative proper fractions in lowest terms have a denominator of 36?

A. 13 B. 29 C. 12 D. 10 E. 9

25. The length and width of a rectangle have a ratio of 15:8. The hypotenuse has a length of 3.4 ft. What is the area of the rectangle?

A. 2.4 ft^2 B. 4.8 ft^2 C. 3.6 ft^2 D. 5.4 ft^2 E. 5.1 ft^2

26. Points A, B, C and D lie on the circle shown. Find AB if BP = 4", DC = 9" and CP = 6". (not drawn to scale)



A. 18.5 in B. 13.5 in C. 12 in D. 15.5 in E. 21 in

27. How many distinct arrangements can be made using all of the letters in the word "THANKSGIVING"?

A. 479,001,600 B. 29,937,600 C. 79,833,600 D. 39,916,800 E. 59,875,200

28. Given the functions $f(x)=3x^2+1$, $g(x)=x^3$ and $h(x)=2x-3$ calculate the value of $h(f(g(-1)))$.

A. -8 B. 64 C. -7 D. -6 E. 5

29. Two fair cubic dice, each with the numbers 2, 4, 6, 8, 10 and 12 are rolled and the numbers on top are added. What is the expected value of the sum? (nearest tenth)

A. 13.6 B. 13.7 C. 14.0 D. 14.4 E. 14.7

30. The sum of the coefficients of the second and fifth terms in the expansion of $(2x-3)^8$ is

A. 90720 B. 93792 C. 87648 D. 51456 E. -51456

31. Given that the set of natural numbers continues in the triangular pattern shown below. Find the sum of the means of the fourth row and seventh row.

A. 64 C. 68 E. 39

B. 72 D. 78

			1				(row 0)
		2	3	4			(row 1)
	5	6	7	8	9		(row 2)
10	11	12	13	14	15	16	(row 3)
			…				(…)

32. Given triangle ABC with $AB=15$ cm, $BC=13$ cm and $m\angle A=60^\circ$. AC has two possible values. What is the product of these values?

A. 42 B. 56 C. 55 D. 48 E. 15

33. Bill and his brother paint at the same rate. Together, they can paint a wall of their house in 2.5 hours. How long would it take one of them to paint a wall twice as wide and twice as high?

A. 10 hours B. 5 hours C. 40 hours D. 20 hours E. 25 hours

34. If P, Q and R represent digits then $PRQ_5+RPQ_4-QRP_3$ has a numeric value in base 10 of:

A. $28P+18R-9Q$ B. $28P+24R-5Q$ C. $28P+18R-7Q$ D. $28P+24R-7Q$ E. $28P+18R-5Q$

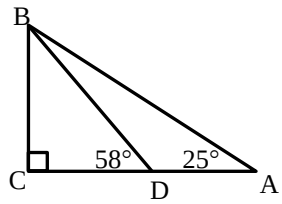
35. How many acute triangles with integral sides exist with sides 8", 14" and x"?

A. 5 B. 4 C. 8 D. 10 E. 12

36. Daily Delight Cupcakery sells 6 different flavors of cupcakes each day. How many different dozen cupcake boxes can they put together?

A. 924 B. 18564 C. 665280 D. 50388 E. 6188

37. Find AB if AD = 4,580 feet. (nearest foot)

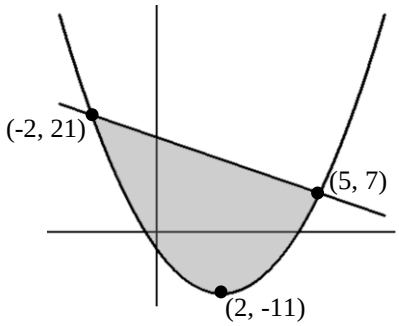


A. 8568 ft B. 9190 ft C. 7131 ft D. 4707 ft E. 9415 ft

38. If $\frac{x-5}{x+15}+\frac{x+15}{x-5}$ is equal to the mixed number $A\frac{B}{(x+15)(x-5)}$, then $B=$

A. 40 B. 400 C. 200 D. 225 E. 150

39. The graphs shown are a parabola and a line. Find the area of the shaded region. (nearest whole number)



A. 220 B. 44 C. 84 D. 99 E. 114

40. Simplify $\left(3-\sqrt{-75}\right)\left(2+\sqrt{-12}\right)$ to the form $a+bi$.

A. $-24+16\sqrt{3}i$ B. $36-4\sqrt{3}i$ C. $24+4\sqrt{3}i$ D. $-36-4\sqrt{3}i$ E. $-36+16\sqrt{3}i$

41. The square root of 331 in base 7 is:

A. 24_7 B. 12_7 C. 21_7 D. 13_7 E. 16_7

42. What is the sum of the solutions to the equation $\sin(2\theta)=\cos\theta$ if $0\le\theta<\frac{3\pi}{2}$.

A. π B. $\frac{2\pi}{3}$ C. $\frac{1}{2}$ D. 2π E. $\frac{3\pi}{2}$

43. Harold mixed 10 ounces of salt with 64 ounces of water. If he then added an additional 16 ounces of water, what percent of the new mixture would be water? (nearest tenth of a percent)

A. 88.9% B. 11.1% C. 12.5% D. 87.5% E. 90.0%

44. If $a_1=-3$, $a_2=-1$, and $a_n=(a_{n-1})(a_{n-2})-a_{n-2}$ for $a>2$, find the value of a_6.

A. 0 B. -36 C. 185 D. 125 E. -115

45. What is the digit in the millionths place in the sum of the series $4+\frac{16}{2!}+\frac{64}{3!}+\frac{256}{4!}+\frac{1024}{5!}+\ldots$?

A. 5 B. 8 C. 0 D. 3 E. 6

46. An archer consistently hits her target 4 out of every 7 shots. If she shoots at her target 9 times, what is the probability that she will make at least 7 shots? (nearest hundredth)

A. 0.82 B. 0.36 C. 0.02 D. 0.18 E. 0.22

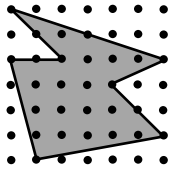
47. Let *a*, *b* and *c* be real numbers such that $c=a+b+9$, $c^2=a^2+b^2$ and $ab=8$. Find the numerical value of $18c$.

A. 65 B. 81 C. -81 D. 63 E. -65

48. $\lim_{h\to 0}\frac{\sin\left[2\left(\frac{\pi}{3}+h\right)\right]-\sin\left[2\left(\frac{\pi}{3}-h\right)\right]}{2h}=$

A. $\sqrt{3}$ B. -1 C. $-\sqrt{3}$ D. 1 E. does not exist

49. The horizontal and vertical distance between the dots is 5 cm. What is the area of the heptagon?



A. 19.5 cm^2 B. 97.5 cm^2 C. 975 cm^2 D. 512.5 cm^2 E. 487.5 cm^2

50. The Bulldogs and the Pirates are going to play a three game series. The probability that the Bulldogs will win any single game is 0.25. The probability that the Pirates will win any single game is 0.65. What is the probability that the 3-game series will be a tie?

A. $\frac{1}{1000}$ B. $\frac{197}{2000}$ C. $\frac{69}{4000}$ D. $\frac{13}{800}$ E. $\frac{39}{400}$

51. If *w* is 10% larger than *x*, *x* is 40% larger than *y*, and *y* is 50% smaller than *z*, by what percentage is *w* smaller than *z*?

A. 70% B. 27% C. 73% D. 23% E. 35%

52. The odds of drawing a pink raffle ticket at random from a bucket 495 tickets are 4:7. How many pink tickets would have to be removed from the bucket to reduce the odds to 1:3?

A. 15 B. 75 C. 65 D. 105 E. 95

53. If $\int_0^k a\cos\theta d\theta = 29$, then $\int_{-k}^k a\cos\theta d\theta =$.

A. 0 B. 58 C. 29 D. 116 E. 841

54. Given $(f \circ g)(x) = 2x+6$ and $g(x) = 2x+1$, calculate $f(9)$.

A. 14 B. 24 C. 23 D. 13 E. 29

55. The slant height of a particular cone is equal to the diameter of the base. What is the volume of the cone in terms of the radius?

A. $V = \pi r^3\sqrt{3}$ B. $V = \frac{\pi r^3\sqrt{2}}{3}$ C. $V = 2\pi r^2$ D. $V = \frac{\pi r^3}{3}$ E. $V = \frac{\pi r^3\sqrt{3}}{3}$

56. The length of a rectangular picture is three times the width. The picture is surrounded by a frame which is 5 inches wide. If the perimeter of the outside of the frame is 96 inches, what is the length of the picture in inches?

A. 21 in B. 24 in C. 7 in D. 8 in E. 30 in

57. If p and q are the zeros of the function $f(x) = 28x^2 + 3x - 135$ then $pq^2 + p^2q =$

A. $\frac{405}{784}$ B. $\frac{405}{1568}$ C. $\frac{3}{56}$ D. $\frac{69}{14}$ E. $\frac{1215}{1568}$

58. How many distinct arrangements are there of three letters chosen from the word "CALCULATOR"?

A. 210 B. 228 C. 264 D. 120 E. 90

59. Classify the graph of with the equation: $9x^2 + 6xy + 2y^2 + 2x - 3y + 5 = 0$

A) Circle B) Centroid C) Parabola D) Hyperbola E) Ellipse

60. If $9^{x-y} = 243$ and $27^{x+y} = 243$ then $x^2 - y^2 =$ _____?

A. $\frac{25}{12}$ B. $\frac{25}{6}$ C. $-\frac{25}{12}$ D. $\frac{5}{6}$ E. $-\frac{5}{12}$

2015-2016 TMSCA Mathematics Test Seven Answers

1. C	21. D	41. E
2. E	22. A	42. E
3. A	23. B	43. A
4. B	24. C	44. C
5. D	25. B	45. C
6. A	26. A	46. D
7. B	27. E	47. A
8. A	28. E	48. B
9. E	29. C	49. D
10. D	30. C	50. B
11. B	31. D	51. D
12. C	32. B	52. B
13. D	33. D	53. B
14. E	34. C	54. A
15. A	35. A	55. E
16. C	36. E	56. A
17. A	37. C	57. A
18. B	38. B	58. C
19. D	39. E	59. E
20. E	40. B	60. B

2015-2016 TMSCA Test Seven Select Solutions

8. $r = \frac{b}{2}\sqrt{\frac{2a-b}{2a+b}} = \frac{14}{2}\sqrt{\frac{36-14}{36+14}} \approx 4.64$ and the area is about $\pi(4.64)^2 \approx 67.7$

12. $({}_{11}C_8)(8!) = 6652800$

21. $\frac{\frac{17}{2}(18)(30)}{\frac{17}{2}(78)(60)} = \frac{18}{78(2)} = \frac{3}{26}$

27. There are 12 letters total with 2-Ns, 2-Gs and 2-Is, so the total number of distinct arrangements will be $\frac{12!}{(2!)(2!)(2!)} = 59875200$.

32. Using the Law of Cosines:
$13^2 = 15^2 + x^2 - 2(15x)\cos 60$ or $0 = x^2 - 15x + 56$ and the product of the roots is 56.

34. In base 10,
$25P + 5R + Q + 16R + 4P + Q - 9Q - 3R - P =$
$28P - 7Q + 18R$.

35. All the possible side lengths are 7, 8, 9, 10, 11, **12**, **13**, **14**, **15**, **16**, 17, 18, 19, 20 and 21, but $a^2 + b^2 > c^2$, where *c* is the longest side length for the triangle to be acute which is true of the triangles where the last side length is a bold value.

36. ${}_{6+12-1}C_{12} = 6188$

41. $331_7 = 169_{10}$ and the square root will be $13_{10} = 16_7$.

47. $(c-9)^2 = (a+b)^2$, so
$c^2 - 18c + 81 = a^2 + 2ab + b^2$ and $-18c + 81 = 2ab$ for $-18c + 81 = 16$ and $18c = -65$.

48. This is the definition of the derivative of $y = \sin 2\theta$ when $x = \frac{\pi}{3}$ which is -1.

49. If *P* is the number or points on the perimeter and *I* is the number of points on the interior, then the area without the 5 cm scaling factor is $\frac{2I + P - 2}{2} = \frac{2(16) + 11 - 2}{2} = 20.5$, then multiply by 25 for the area considering the 5 to get 512.5.

50. For a tied series, the teams can either play 3 tie games, or they can each win a game and tie the 3rd game. There are 6 arrangements of the second possibility, so the probability of a tie series is: $(0.1)^3 + 6(0.65)(0.25)(0.1) = \frac{197}{2000}$.

53. Because $f(\theta) = a\cos\theta$ is an even function, it will be symmetric about the *y*-axis and the new integral will have a value of 58.

57. $pq^2 + p^2q = pq(p+q)$ or for a quadratic function $f(x) = ax^2 + bx + c$,
$\left(\frac{c}{a}\right)\left(-\frac{b}{a}\right) = \left(\frac{-135}{28}\right)\left(\frac{-3}{28}\right) = \frac{405}{784}$.