



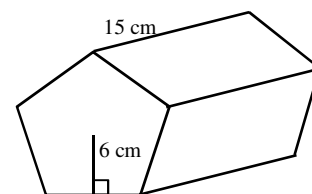
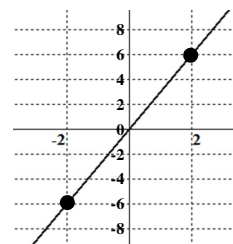
**TMSCA HIGH SCHOOL
MATHEMATICS
TEST # 7 ©
JANUARY 12, 2013**

GENERAL DIRECTIONS

1. About this test:
 - A. You will be given 40 minutes to take this test.
 - B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2012-2013 TMSCA High School Mathematics Test

1. Evaluate: $3 + 7 \times 2 - 4 \div 2 \cdot 16 + 17$
 (A) 145 (B) 2 (C) 5 (D) 55 (E) 145
2. A craft store offers a weekly coupon for 40% off of one regularly priced item. Joan buys baking supplies that are all on sale for 25% off for the week. She buys four cookie cutters with a regular price of \$1.99 each, two cans of colored icing with a regular price of \$4.59 each, a decorator kit with a regular price of \$22.50 and pays tax at a rate of 6.25% on her whole purchase. What is the smallest bill she can have if she can use the coupon on an item of her choice?
 (A) \$29.73 (B) \$23.78 (C) \$23.22 (D) \$28.00 (E) \$26.36
3. The distance driven on a road trip is directly proportional to the rate at which the car is moving. On Andy's vacation, the when the rate is 68 mph, the distance driven is 170 miles. How far will he drive if he raises his rate to 75 mph?
 (A) 225 miles (B) 150 miles (C) 187.5 miles (D) 262.5 miles (E) 175 miles
4. What is the slope of a line perpendicular to the line shown right?
 (A) $\frac{1}{3}$ (B) 3 (C) -3 (D) $-\frac{1}{3}$ (E) $\frac{3}{2}$
5. If $\frac{x^4 - 5x^2 + 4}{(x^2 + 4x + 4)(x^2 + 2x + 1)} = \frac{x^2 + ax + 2}{x^2 + bx + 2}$, find $\frac{a}{b}$.
 (A) 0.5 (B) -2 (C) 1 (D) -0.5 (E) -1
6. Triangle ABC is a right triangle, and \overline{BD} is the altitude to the hypotenuse. Which of the following pairs are complementary?
 (A) $\angle ADB, \angle CDB$ (B) $\angle BCD, \angle ABD$ (C) $\angle BAD, \angle ABD$ (D) $\angle ABC, \angle ACB$ (E) $\angle BAD, \angle CBD$
7. Find the lateral surface area of the regular, right pentagonal prism shown right.
 (A) 844 cm^2 (B) 1425 cm^2 (C) 520 cm^2 (D) 654 cm^2 (E) 640 cm^2
8. The third term of an arithmetic sequence is -2, and the sum of the first fourteen terms is 98. Find the common difference.
 (A) 4 (B) -2 (C) 2 (D) -6 (E) -4
9. How many faces does an icosidodecahedron have?
 (A) 10 (B) 32 (C) 12 (D) 36 (E) 20
10. Three books by Jane Austen need to be shelved together on a shelf containing ten books. How many arrangements of books are possible?
 (A) 120960 (B) 241920 (C) 40320 (D) 720 (E) 30240
11. If $g(x) = x - 1$ and $f(x) = x^3$, find $g(f(x+2))$.
 (A) $x^3 + 6x^2 + 12x + 8$ (B) $x^3 + 2x^2 + 4x + 7$ (C) $x^3 + 7$ (D) $x^3 + 6x^2 + 12x + 7$ (E) $x^3 + 3x^2 + 3x + 7$
12. A quadrilateral is inscribed in a circle. The measures of two angles opposite each other are $(x^2 - 12x)^\circ$ and $(3x - 10)^\circ$. Find the measure of the larger angle.
 (A) 160° (B) 90° (C) 47° (D) 20° (E) 133°



13. Mrs. Jones is fifteen years older than twice the age of her son. Twenty years ago, her age was nine times that of her son. Find her age.

- (A) 75 (B) 65 (C) 50 (D) 25 (E) 20

14. Determine the period of $y = \frac{2}{3} \sin(3x - 7) + 8$.

- (A) $\frac{2\pi}{3}$ (B) 2π (C) $\frac{3\pi}{2}$ (D) $\frac{\pi}{2}$ (E) $\frac{7\pi}{3}$

15. There are two values of k for which $\det \begin{bmatrix} k+1 & -5 \\ -2 & -k \end{bmatrix} = -22$. The sum of those two values is

- (A) 0 (B) 1 (C) 10 (D) -1 (E) -10

16. When $2x^2 - x + k$ is divided by $(x + 1)$ the remainder is m . Find the value of m in terms of k .

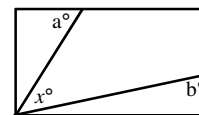
- (A) $m = k - 2$ (B) $m = k + 3$ (C) $m = k + 1$ (D) $m = k - 1$ (E) $m = k + 2$

17. On triangle ABC , $AB = 8$ cm, $BC = 7$ cm, and $m\angle A = 60^\circ$. There are two possible lengths for \overline{AC} . The shorter of the two lengths is

- (A) 1.2 cm (B) 5 cm (C) 2.3 cm (D) 3 cm (E) 12.7 cm

18. In the rectangle shown right, what is $a + b$ in terms of x ?

- (A) $90 + x$ (B) $90 - x$ (C) $180 + x$ (D) $270 - x$ (E) $360 - x$



19. Which of the following series converges?

- (A) $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$ (B) $\sum_{n=0}^{\infty} \frac{n+1}{2n+1}$ (C) $\sum_{n=1}^{\infty} \frac{n}{1000(n+1)}$ (D) $\sum_{n=0}^{\infty} \frac{3}{2^n}$ (E) $\sum_{n=1}^{\infty} \log n$

20. Let the region R be bounded in the first quadrant by the x -axis, y -axis and the graph of $f(x) = \cos 2x$. Find the volume of the solid generated by the complete revolution of R around the x -axis.

- (A) 0.5 (B) 1.23 (C) 2.47 (D) 0.39 (E) 3.14

21. Two lines in the same plane that never intersect are

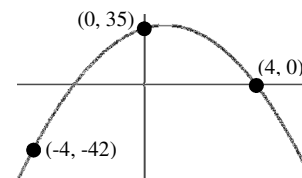
- (A) Bisectors (B) Skew (C) Perpendicular (D) Parallel (E) Collinear

22. Myrtle has eight different colors of markers. In how many ways can she package 4 markers to sell if she can repeat colors?

- (A) 495 (B) 220 (C) 210 (D) 70 (E) 330

23. What are the coordinates of the other zero of the parabola shown right?

- (A) $(-3, 0)$ (B) $(-2.5, 0)$ (C) $(-2.75, 0)$ (D) $(-2, 0)$ (E) $(3, 0)$



24. In a class with 88 students, 32 study economics, 28 study history and 39 do not study either subject. How many students study both subjects?

- (A) 49 (B) 80 (C) 11 (D) 21 (E) 39

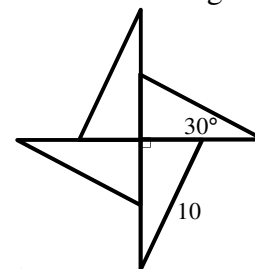
25. Let $f(x) = x^3 + 3x + 1$, and $g(x) = x^2$. Find $g(f'(3))$.

- (A) 1369 (B) 900 (C) 225 (D) 1225 (E) 1521

26. The octagon below is made up of four congruent triangles.

What is the perimeter of the octagon?

- (A) $20\sqrt{3} - 20$ (B) $40\sqrt{3}$ (C) $40 + 20\sqrt{3}$ (D) $20 + 20\sqrt{3}$ (E) 60



27. Find the total area of the two regions enclosed by the curves $y = x^3 - 9x + 13$ and $y = 3x + 11$.

- (A) 246.62 (B) 72.17 (C) 13.85 (D) 250.62 (E) 72.50

28. A 25 ft. ladder is sliding down a wall at a rate of 2 inches per second. When the top of the ladder is 7 feet from the floor, the angle the ladder makes with the wall is changing at a rate of _____ radians per second?

- (A) 0.0069 (B) 0.00067 (C) 0.0833 (D) 1.0032 (E) 0.0803

29. Let $a_1 = 7$, $a_2 = -2$ and $a_n = a_{n-2} - 2a_{n-1}$. Find a_6 .

- (A) 59 (B) -24 (C) -142 (D) -66 (E) 25

30. Find the sum of the series to the nearest ten thousandth: $7 - \frac{7^3}{6} + \frac{7^5}{120} - \frac{7^7}{5040} \dots$

- (A) 0.7539 (B) 1.9459 (C) 0.8451 (D) 0.6570 (E) 0.1219

31. Two fair six-sided dice are rolled. What are the odds that the sum of the top faces is a deficient number?

- (A) 5:31 (B) 6:1 (C) 35:1 (D) 5:1 (E) 31:5

32. If $\frac{x+7}{x-7} + \frac{x-7}{x+7} = A + \frac{B}{(x+7)(x-7)}$, then $B =$

- (A) 0 (B) 14 (C) 28 (D) 49 (E) 196

33. The Real value solution set of $|3x+1|+3 < 11$ is

- (A) $\left\{x \mid -\frac{7}{3} < x < 3\right\}$ (B) $\left\{x \mid -3 < x < -\frac{7}{3}\right\}$ (C) $\left\{x \mid \{x < -3\} \cup \left\{x > \frac{7}{3}\right\}\right\}$ (D) $\left\{x \mid -3 < x < \frac{7}{3}\right\}$ (E) $\left\{x \mid \left\{x < -\frac{7}{3}\right\} \cup \{x > 3\}\right\}$

34. Two workers can paint a fence in three hours. How long would it take three workers to paint a fence twice as long and twice as high if they each paint at the same rate?

- (A) 4 hours (B) 8 hours (C) 6 hours (D) 12 hours (E) 9 hours

35. The chords \overline{AC} and \overline{BD} intersect inside $\odot O$ at P . If $AP = 9$, $CP = x^2$, $BP = x + 1$ and $DP = 6x$, find the positive value of x .

- (A) 4 (B) 6 (C) 2 (D) 3 (E) 5

36. Using the triangle and congruent semicircles shown right, find the area of the triangle in terms of the radius of the semicircles.

- (A) $8r^2\sqrt{6}$ (B) $16r^2\sqrt{6}$ (C) $20r^2$ (D) $5r^2$ (E) $2r^2\sqrt{6}$

37. Find the sum of the infinite series: $-1.2 - 0.9 - 0.675 - 0.50625 \dots$

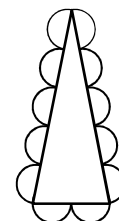
- (A) -3.28125 (B) -1.6 (C) -2.4 (D) -4.8 (E) 3.6

38. The area of the ellipse with the equation $9x^2 + 25y^2 - 36x + 150y = -36$ is

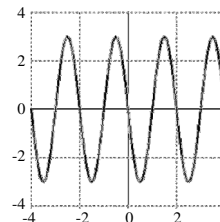
- (A) 25π (B) 15π (C) $\frac{25\pi}{9}$ (D) 9π (E) $\frac{9\pi}{25}$

39. Find the acute angle that the line $3x + 4y = 18$ forms with the x -axis to the nearest hundredth of a degree.

- (A) 41.41° (B) 48.59° (C) 33.69° (D) 36.87° (E) 12.52°



40. The function $f(x) = \underline{\hspace{2cm}}$ will produce this graph.



- (A) $3\sin(\pi x + 1)$ (B) $3\sin(x + \pi)$ (C) $\sin(\pi(x-1)) + 3$
 (D) $3\sin(\pi x - 1)$ (E) $3\sin(\pi(x-1))$

41. If $f(x) = \tan x$ then $\lim_{h \rightarrow 0} \frac{f(\pi + h) - f(\pi)}{h}$ is

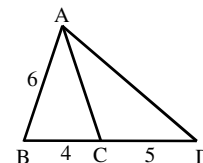
- (A) 0 (B) 1 (C) undefined (D) -1 (E) $\sqrt{3}$

42. If $(1-i)^9 = a + bi$, then $a + b =$

- (A) 32 (B) 0 (C) $32\sqrt{2}$ (D) -512 (E) $-32\sqrt{2}$

43. Given that $\overline{AB} \cong \overline{AC}$, find the area of triangle ABD .

- (A) $36\sqrt{2}$ (B) $18\sqrt{2}$ (C) 27 (D) 54 (E) $9\sqrt{2}$



44. If $\int_{-2}^4 f(x) dx = 10.5$ then $\int_{-2}^4 (f(x) + 3) dx =$

- (A) 21 (B) 13.5 (C) 22.5 (D) 28.5 (E) 16.5

45. Simplify: $(3 - 2\sqrt{-8})(1 + 3\sqrt{-50})$

- (A) $-117 + 41i\sqrt{2}$ (B) $-117 - 41i\sqrt{2}$ (C) $123 + 41i\sqrt{2}$ (D) 123 (E) $123 - 41i\sqrt{2}$

46. Solve: $\log_3 x + \log_3(x-6) = 3$

- (A) 9 (B) -0.464 (C) 6 (D) -3 (E) 6.46

47. The volume of an octahedron with an edge length of one centimeter?

- (A) $\frac{\sqrt{3}}{2} \text{ cm}^2$ (B) $\frac{8\sqrt{2}}{3} \text{ cm}^2$ (C) $\frac{\sqrt{2}}{3} \text{ cm}^2$ (D) $2\sqrt{3} \text{ cm}^2$ (E) $\frac{8\sqrt{3}}{2} \text{ cm}^2$

48. Set $S = \{1, 2, 3, 4, 5, 6, 7\}$. How many 4-element subsets of set S are there?

- (A) 16 (B) 35 (C) 128 (D) 49 (E) 28

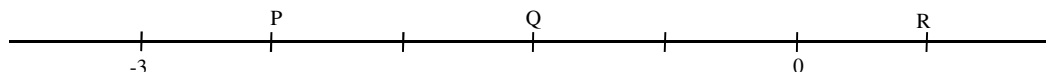
49. If $y^2 = 5 - 12i$ and $y^3 = -9 - 46i$ where $y = a + bi$ then $a + b =$

- (A) -38 (B) 5 (C) -62 (D) 1 (E) 6

50. At which of the following values of x is the function at right discontinuous?

- (A) -3 (B) -1 (C) 0 (D) 1 (E) 2
- $$f(x) = \begin{cases} x+5, & x \leq -3 \\ |x|-1, & -3 < x < 2 \\ x^2+2, & x \geq 2 \end{cases}$$

51. The distances between the hash marks (|) are equal. Find $P + Q + R$.



- (A) -2.4 (B) -3 (C) -1.2 (D) 1.2 (E) 0.6

52. If $\frac{7x+13}{x^2+2x-3} = \frac{A}{x+3} + \frac{B}{x-1}$, then $AB =$

- (A) 7 (B) -6 (C) 10 (D) -3 (E) 6

53. The letters in the word TUESDAY are arranged in a line. How many of different arrangements are possible that begin with D and end with S?

- (A) 21 (B) 35 (C) 24 (D) 120 (E) 720

54. $\frac{\cot^2 t}{\csc t} =$

- (A) $\sec t + \cos t$ (B) $\sec t - \cos t$ (C) $\csc t + \sin t$ (D) $\sec t - \sin t$ (E) $\csc t - \sin t$

55. How many petals does the graph of the curve $r = 2 \cos 6\theta$ have?

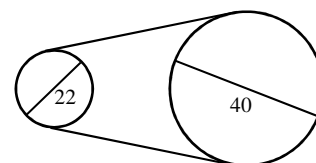
- (A) 3 (B) 12 (C) 24 (D) 2 (E) 6

56. $\frac{x^2 + 3x - 10}{x^2 + 4x + 4} \div \frac{x^2 + 10x + 25}{x + 2} =$

- (A) $\frac{x-2}{x^2+7x+10}$ (B) $\frac{x-2}{x^2+4x+4}$ (C) $\frac{x^2+3x-10}{x^2+7x+10}$ (D) $\frac{x+2}{x^2+7x+10}$ (E) $\frac{x^2+3x+10}{x^2+7x+10}$

57. A belt joins two pulleys shown. If the smaller pulley rotates at 72 rpm, then the larger pulley is rotating at _____ rpm.

- (A) 12.22 (B) 130.91 (C) 10.28 (D) 39.60 (E) 32.29



58. Over time, it is observed that the arrival time for people attending an exhibition is normally distributed with a mean of 3 hours and 48 minutes after the door opens and the standard deviation was 52 minutes. If the doors open at 9 am, at what time will 90% of the people have arrived?

- (A) 11:49 am (B) 1:55 pm (C) 1:15 pm (D) 12:09 pm (E) 2:15 pm

59. The Platonic solid with pentagonal faces is the

- (A) Tetrahedron (B) Octahedron (C) Dodecahedron (D) Cube (E) Icosahedron

60. How many positive perfect cubes are factors of $(3!)(4!)(7!)$?

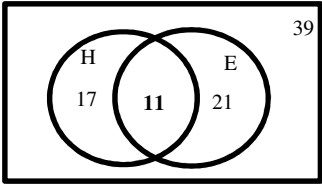
- (A) 5 (B) 7 (C) 4 (D) 6 (E) 8

2012-2013 TMSCA High School Mathematics Test 7 Key

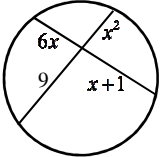
1. B	21. D	41. B
2. D	22. E	42. B
3. C	23. B	43. B
4. D	24. C	44. D
5. E	25. B	45. C
6. C	26. D	46. A
7. D	27. E	47. C
8. C	28. A	48. B
9. B	29. C	49. D
10. B	30. D	50. E
11. D	31. D	51. B
12. E	32. E	52. C
13. B	33. D	53. D
14. A	34. B	54. E
15. D	35. C	55. B
16. B	36. A	56. A
17. D	37. D	57. D
18. A	38. B	58. B
19. D	39. D	59. C
20. B	40. E	60. D

2012 – 2013 TMSCA Solutions Mathematics Test Seven

- $3+14-2\cdot 16+17=17-32+17=2$
- $[0.75(4\cdot 1.99+2\cdot 4.59)+0.6(22.5)]\cdot 1.0625=28.00$
- $170=68k \rightarrow k=2.5 \rightarrow d=75\cdot 2.5=187.5$ miles
- $m=\frac{-6-6}{-2-2}=3, \perp m=-\frac{1}{3}$
- $\frac{(x^2-1)(x^2-4)}{(x-1)^2(x+2)^2}=\frac{(x+1)(x-1)(x-2)(x+2)}{(x-1)^2(x+2)^2}$
 $=\frac{(x-1)(x-2)}{(x+1)(x+2)}=\frac{x^2-3x+2}{x^2+3x+2}, \frac{a}{b}=\frac{-3}{3}=-1$
- ABD is a right triangle. $m\angle BAD+m\angle ABD=90^\circ$
- $p=6\tan 36^\circ\cdot 10, LSA=ph\approx(43.59)(15)\approx 654$
- Solve system $a+2d=-2$ and $\frac{14}{2}(2a+13d)=98$.
 $a=-6, d=2$
- There are 32 faces. 12 pentagons and 20 triangles
- Treat 3 books as one item. $a=8!(3!)=241920$.
 The 3! is the possible arrangements of the 3.
- $(x+2)^3=x^3+3x^2\cdot 2+3x\cdot 2^2+2^3$, so
 $g((x+1)^3)=x^3+6x^2+12x+8-1$
- Opposite angles are supplementary, so
 $x^2-12x+3x-10=180 \rightarrow (x-19)(x+10)=0$
 $19^2-12(19)=133$
- solve system: $2s+15=j, 9(s-20)=j-20$.
 son's age = 25, Mrs. Jones' age = 65.
- period $\frac{2\pi}{b}$, where b is the coefficient of x .
- $-k^2-k-10=-22, (k+4)(k-3)=0,$
 $-4+3=-1$
- $2(-1)^2-(-1)+k=k+3=m$
- $7^2=8^2+x^2-2\cdot 8x\cos 60, 0=x^2-8x+15$
 $0=(x-5)(x-3), x=3,5$
- $90-b+90-a+x=90 \rightarrow a+b=90+x$
- $\sum_{n=0}^{\infty} \frac{3}{2^n}$ is a geometric series where $|r|<1$.
- $\pi \int_0^{\pi/4} \cos^2(2x) dx \approx 1.23$

- definition of parallel
- $\binom{8+4-1}{4}=\binom{11}{4}=330$
- quadratic regression
- 
- $f'(x)=3x^2+3$, so $f'(3)=27+3=30$
 $g(30)=30^2=900$
- Four sides are 10 and the other for sides are the long leg minus the short leg of a 30-60-90 triangle.
 $4\cdot 10+4(5\sqrt{3}-5)=20+20\sqrt{3}$
- Graph for intersections, then
 $\int_{-3.545}^{3.378} |x^3-9x+13-3x-1| dx \approx 72.5$
- $\cos \theta = \frac{x}{25}, -\sin \theta \frac{d\theta}{dt} = \frac{1}{25} \frac{dx}{dt}$
 When $x=7, \theta \approx 1.287, \frac{dx}{dt} = -\frac{2}{12} \frac{ft}{sec}$,
 substitute and solve $\frac{d\theta}{dt} \approx 0.0069$
- Substitution: 7, -2, 11, -24, 59, -142
- series expansion of $\sin \theta$, set mode to radians and evaluate $\sin 7 \approx 0.6570$
- all rolls are deficient except 6 and 12
 $p=1-(p(6)+p(12))=1-\left(\frac{5}{36}+\frac{1}{36}\right)=\frac{5}{6}$
 So odds are 5:1
- $\frac{a}{b}+\frac{b}{a}=2+\frac{(a-b)^2}{ab}, b=14^2=196$
- solve:

$3x+1>-8$ $x>-3$	$3x+1<8$ $x<\frac{7}{3}$
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- $2r\cdot 3=1$ fence, $r=\frac{1}{6}$ fence/worker/hour
 New fence has $4x$ the area: $3\left(\frac{1}{6}\right)t=4$
- $6x(x+1)=9x^2$
 $0=3x(x-2)$
 So $x=2$

- $h^2=(10r)^2-(2r)^2$, so $h=4r\sqrt{6}$
 $A=\frac{1}{2}(4r)(4r\sqrt{6})=8r^2\sqrt{6}$
- geometric with $u_1=-1.2$ and $r=0.75$
 $S=\frac{-1.2}{1-0.75}=-4.8$
- standard form $\frac{(x-2)^2}{25}+\frac{(y+3)^2}{9}=1$
 $a=5, b=3, A=ab\pi=15\pi$
- intercepts (0,4.5) and (6,0),
 $\theta=\arctan\left(\frac{4.5}{6}\right)$
- amplitude = 3, vertical shift=0, period=2, phase shift = 1.
- limit is the definition of $f'(x)$ @ $x=\pi$,
 $f'(\pi)=\sec^2 \pi = \frac{1}{\cos^2 \pi} = \frac{1}{(-1)^2}=1$
- $(1-i)=\sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right]$
 $(1-i)^9=(\sqrt{2})^9\left[\cos\left(-\frac{9\pi}{4}\right)+i\sin\left(-\frac{9\pi}{4}\right)\right]=$
 $16-16i$, so $a+b=0$
- $h^2+2^2=6^2, h=4\sqrt{2}$,
 $A=\frac{1}{2}(9)(4\sqrt{2})=18\sqrt{2}$
- $\int_{-2}^4 f(x) dx + [3x]_{-2}^4 = 10.5+18=28.5$
- $(3-4i\sqrt{2})(1+15i\sqrt{2})=$
 $3+45i\sqrt{2}-4i\sqrt{2}-60i^2\cdot 2=123+41i\sqrt{2}$

- $\log_3(x(x-6))=3, 27=x^2-6x,$
 $0=(x-9)(x+3), 9$ is the solution in the domain
- $V=\frac{1}{3}a^3\sqrt{2}=\frac{\sqrt{2}}{3}$
- $\binom{7}{4}=35$
- $\frac{-9-46i}{5-12i}=3-2i, a+b=1$
- $|2|-1\neq 2^2+2$
- $\frac{0-(-3)}{5}=0.6, 3(-3)+10(0.6)=-3$
- $7x+13=A(x-1)+B(x+3)$
 if $x=1, 20=4B, B=5$
 if $x=-3, -8=-4A, AB=10$
- 5 letters to arrange, $5!=120$
- $\frac{\csc^2 t-1}{\csc t}=\csc t-\frac{1}{\csc t}=\csc t-\sin t$
- $r=\cos a\theta$ has $2a$ petals if a is even and a petals if a is odd. $2\cdot 6=12$
- $\frac{(x+5)(x-2)}{(x+2)^2}\cdot\frac{(x+2)}{(x+5)^2}=\frac{(x-2)}{(x+2)(x+5)}$
- $\frac{22(72)}{40}=39.6$
- $z \approx 1.28155, 1.28155=\frac{x-3.8}{13/15}$
 $x \approx 4.911 \rightarrow 4$ hr and 55 min after 9 am
- $2^8\cdot 3^4\cdot 5\cdot 7$, cube factors 1, 2, 3, 4, 6, 12

