

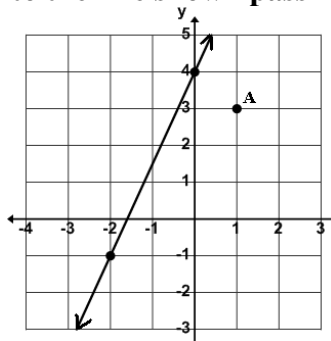
**TMSCA HIGH SCHOOL
MATHEMATICS
TEST #6 ©
DECEMBER 6, 2014**

GENERAL DIRECTIONS

1. About this test:
 - A. You will be given 40 minutes to take this test.
 - B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

1. Evaluate: $4! \times (8)^{-1} + 2 \div 4 \times 8 - (16)^{\frac{1}{2}}$
- (A) 2 (B) 3 (C) $5\frac{1}{8}$ (D) 6 (E) 24
2. Two million thirty-five thousand seven plus one hundred thousand four hundred sixty-eight is subtracted from one billion. How many times does the digit 9 appear in the answer?
- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
3. $(9 + 6) \times 3 = 27 + 18$ and $(9 - 6) \div 3 = 3 - 2$ are examples of the ____ properties of equality.
- (A) associative (B) commutative (C) distributive (D) addition (E) subtraction
4. Lotta Sense has 3 jars containing nickels, dimes, and quarters. The ratio of nickels to quarters is 4 to 1 and the ratio of quarters to dimes is 2 to 3. The total amount of money in the 3 jars is \$13.20. How many nickels does Lotta have?
- (A) 88 (B) 66 (C) 55 (D) 44 (E) 33

5. Find an equation of a line perpendicular to the line shown passing through point A.

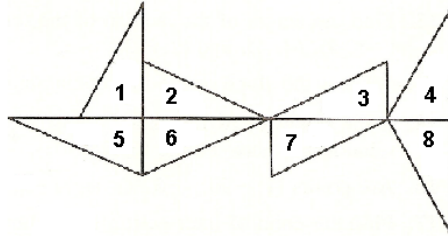


- (A) $2x + 5y = 11$ (B) $5x - 2y = -1$ (C) $-2x + 5y = 3$ (D) $5x - 2y = 13$ (E) $2x + 5y = 17$
6. The function $f(x) = 3x^2 - 5x - 7$ crosses the x-axis at two points. Find the distance between the two points. (nearest tenth)
- (A) 2.7 (B) 3.3 (C) 3.5 (D) 4.0 (E) 4.6
7. Soh Yung is three times as old as her sister, Tu Yung. Eight years ago Soh was seven times as old as Tu. What is the difference in their ages now?
- (A) 36 (B) 24 (C) 12 (D) 6 (E) 4
8. The ratio of the length to the width of a rectangle is 7:4. If 2 units are added to both the length and the width, then the ratio of the length to the width is now 8:5. What is the area of the original rectangle in square units?
- (A) 160 (B) 116 (C) 112 (D) 108 (E) 98

9. The point of intersection of the 3 altitudes of a triangle is called a(n) _____.

- (A) center (B) centroid (C) circumcenter (D) incenter (E) orthocenter

10. Mary Goround maps triangle 7 to triangle 1 by using which of the groups of three transformations in the order given?



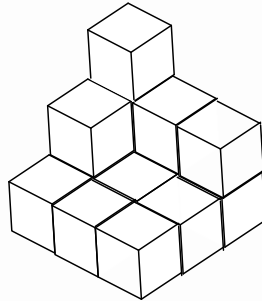
(w) half-turn (x) reflection (y) rotation (z) translation

- (A) y, x, & z (B) w, z, & x (C) y, x, & w (D) y, z, & y (E) w, x, & z

11. How many 7-letter code words can be formed using the letters in the word ALABAMA?

- (A) 1,260 (B) 840 (C) 288 (D) 210 (E) 144

12. One-centimeter cubes are glued together to form the object in the figure shown. How many more one-centimeter cubes are needed to create a cube with a volume of 64 cubic centimeters?



- (A) 50 (B) 13 (C) 46 (D) 51 (E) 14

13. Let $f_0 = 0, f_1 = 1, f_2 = 1, f_3 = 2, f_4 = 3, \dots$ be the terms of the Fibonacci sequence. Find $\text{GCD}(f_{12}, f_8)$.

- (A) 5 (B) 4 (C) 3 (D) 2 (E) 1

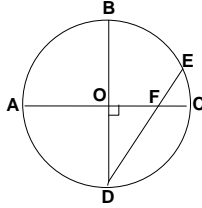
14. The range of the function $y = 5 - 3|x + 2|$ is:

- (A) {all reals} (B) $\{x : x \geq -3\}$ (C) $\{y : y \leq 5\}$
 (D) $\{y : y > 5\}$ (E) $\{x : -4\frac{2}{3} \leq x \leq -\frac{2}{3}\}$

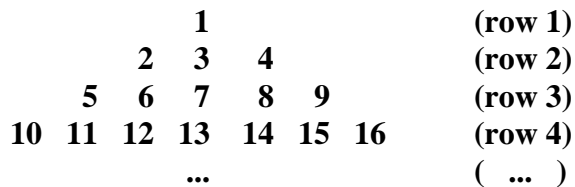
15. Find $f(g(a - 1))$ when $g(x) = 3x - 5a$ and $f(x) = 2x + 3a$.

- (A) $2(5a - 3)$ (B) $-(a + 1)$ (C) $3a - 5$ (D) $-(6 + a)$ (E) $10a - 1$

16. Given the circle O with perpendicular diameters and a chord, find AO if $EF = 4''$ and $DF = 5''$. (nearest tenth)



- (A) 6.4" (B) 6.0" (C) 5.3" (D) 4.7" (E) 4.5"
17. Simplify: $\frac{\tan(\theta) + \cot(\theta)}{\sec(\theta)} - \csc(\theta)$
- (A) 0 (B) 1 (C) $\sin(\theta)$ (D) $\cos(\theta)$ (E) $\sin(\theta) + \cos(\theta)$
18. Find the sum of the amplitude, the phase shift, and the period $y = 3 + \frac{1}{2}\cos(2x - \frac{\pi}{3})$. (nearest tenth)
- (A) 4.0 (B) 4.2 (C) 4.7 (D) 5.6 (E) 7.2
19. I. C. Smoak looks down from his tower and sees a campfire burning on the ground. The tower is 250 feet tall and the angle of depression from the observation point to the fire is 2.3° . How far away from the base of the tower was the campfire? (nearest foot).
- (A) 6,224 feet (B) 589 feet (C) 6,230 feet (D) 589 feet (E) 575 feet
20. The expansion of $(2x - 5)(3x + 2)(6x + 4)$ is $ax^3 + bx^2 + cx + d$. Find $a + b + c + d$.
- (A) -62 (B) -124 (C) -150 (D) -222 (E) -1,440
21. The coefficient of the 3rd term in the expansion of $(2x + 1)^5$ is:
- (A) 80 (B) 33 (C) 243 (D) 40 (E) 20
22. Simplify $(6 + \sqrt{-108})(5 - \sqrt{-75})$ to the form $a + bi$.
- (A) $30 - 30\sqrt{3}i$ (B) $120 + 0i$ (C) $-60 - \sqrt{3}i$ (D) $30 - 3\sqrt{30}i$ (E) $-60 + 0i$
23. Given that the set of natural numbers continue in the triangular pattern shown below, find the first number in row 8.



- (A) 64 (B) 60 (C) 56 (D) 50 (E) 48

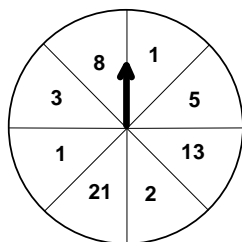
24. The directrix of the conic given by the equation $x^2 + 4x + 6y = 2$ is:
- (A) $y = -1.5$ (B) $y = -0.5$ (C) $y = 1$ (D) $y = 2$ (E) $y = 2.5$
25. Let $f(x) = 2x^3 - 12x^2 + 4x - 27$. The concavity of the curve is upward at which of the following values of x ?

- (A) $\sqrt{2}$ (B) 1 (C) $\sqrt{5}$ (D) $-2\sqrt{2}$ (E) 2

26. Let $f(x) = \sqrt[3]{2x}$. Find $f'(4)$.

- (A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$

27. Willie Luze spins the Wheel of Fibs. The wheel consists of eight congruent sectors as shown. If he lands on a prime number he wins \$5.00. If he lands on a composite number he wins \$3.00. If he lands on any other number he loses \$4.00. What is the mathematical expectation on any one spin, assuming it cannot land on a line?



- (A) \$4.00 (B) \$2.25 (C) \$4.50 (D) \$3.20 (E) \$1.80

28. The Jim Naseum volleyball team has 5 setters, 6 blockers, and 3 roamers. How many 6-member teams can be formed if each team needs 2 setters, 3 blockers, and 1 roamer?

- (A) 3,003 (B) 1,001 (C) 900 (D) 600 (E) 450

29. Which of the following mathematicians is known for developing a division algorithm to find the greatest common divisor of two numbers?

- (A) Diophantus of Alexandria (B) Euclid of Alexandria (C) Archimedes of Syracuse
(D) Theano of Athena (E) Zeno of Alea

30. Let $F = \{ f, r, a, c, t, i, o, n \}$, $D = \{ d, e, c, i, m, a, l \}$, and $N = \{ n, u, m, b, e, r, s \}$. How many elements are in $(F \cap N) \cup (D \cap F)$?

- (A) 1 (B) 3 (C) 4 (D) 5 (E) 7

31. The Millersview scout troop purchased 1,000 boxes of cookies at a price of five boxes for \$4.00. They sell all of them at a price of two boxes for \$3.00. How much profit did the troop make?

- (A) \$250.00 (B) \$416.67 (C) \$583.33 (D) \$600.00 (E) \$700.00

32. The distance one travels varies directly with the amount of time traveling. If it takes 3 hours and 20 minutes to travel 200 miles, how long would it take to travel 90 miles?

- (A) 2 hrs 10 min (B) 2 hrs (C) 1 hr 50 min (D) 1 hr 30 min (E) 1 hr 18 min

33. Determine the least integer value of k so that $3x^2 - 8x - k = 0$ has two positive real roots.

- (A) 4 (B) -5 (C) 0 (D) 1 (E) -6

34. $\triangle ABC$ and $\triangle PQR$ exist such that $\angle BAC \cong \angle PRQ$, $\angle ACB \cong \angle RQP$, $AB = 52$, $CB = 20$, $PR = 13$, and $QR = 15$. Find $AC + PQ$.

- (A) 65 (B) 67 (C) 33 (D) 35 (E) 72

35. Find the shortest distance from the point $(5, 0)$ and the line $3x + 4y = 5$.

- (A) $4\sqrt{2}$ (B) 4 (C) $3\sqrt{2}$ (D) $2\sqrt{2}$ (E) 2

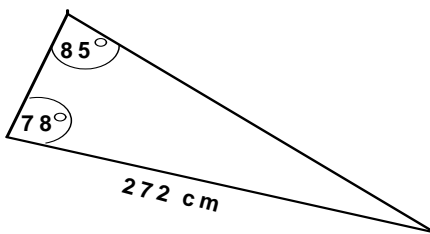
36. If $a_1 = 2$, $a_2 = -3$ and $a_n = (a_{n-2})(a_{n-1}) - (a_{n-1})$ for $n \geq 3$, then a_5 equals:

- (A) -48 (B) -16 (C) -12 (D) -9 (E) 12

37. Tater Farmur plants 4 spud eyes in the first row of his garden. He plants 6 spud eyes in the second row, 8 in the third row, 10 in the fourth row, and continues planting in this manner until he finishes his 20 rows. How many spud eyes did Tater plant in his garden?

- (A) 500 (B) 480 (C) 460 (D) 440 (E) 420

38. Find the area of the triangle shown (nearest cm^2).



- (A) 133 cm^2 (B) $16,490 \text{ cm}^2$ (C) $3,313 \text{ cm}^2$ (D) $11,015 \text{ cm}^2$ (E) $10,619 \text{ cm}^2$

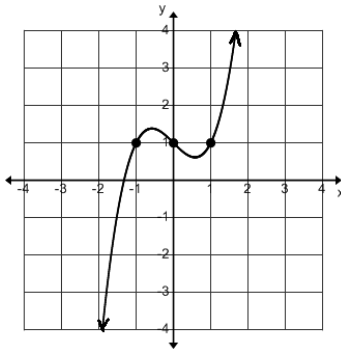
39. If the two-digit number $3Q$ is subtracted from the two-digit number $P4$ the difference is 16. Find $P + Q$, where P and Q are single digits.

- (A) 18 (B) 16 (C) 13 (D) 7 (E) 6

40. The graph of $f(x) = \sqrt{2x - 3}$ is a(n) _____ function over its domain.

- (A) linear (B) constant (C) decreasing (D) increasing (E) step

41. Which of the following is true about the relation graphed below?



- (A) It is an odd function. (B) It is an even function. (C) It is not a function.
 (D) It is neither an even nor an odd function (E) It is a one-to-one function.

42. The Brite-Lite Bulb Company shipped a dozen bulbs to the Indy Dark Company. The bulb company statistics show that the probability the bulbs will work is 85%. What are the odds that they don't work?

- (A) $\frac{3}{20}$ (B) $\frac{5}{34}$ (C) $\frac{1}{5}$ (D) $\frac{17}{20}$ (E) $\frac{3}{17}$

43. Eratosthenes developed a "sieve" for finding prime numbers. Which of the following types of numbers are found by using a similar type "sieve"?

- (A) lucky (B) perfect (C) happy (D) polite (E) primeval

44. If the roots of $x^3 + bx^2 + cx + d = 0$ are $-2, -1,$ and $4,$ then $b + c + d$ equals:

- (A) 1 (B) -1 (C) -3 (D) -17 (E) -19

45. Let $f(x) = x^2 + bx + c$. If $f(x)$ is divided by $x + 2$ the remainder is 5 and if $f(x)$ is divided by $x - 3$ the remainder is 15. Find $b + c$.

- (A) 10 (B) 8 (C) 6 (D) 5 (E) 4

46. How many solutions does $\sin(2\theta) = 2\sin(\theta)$, where $0 \leq \theta \leq \frac{5\pi}{2}$.

- (A) 5 (B) 3 (C) 2 (D) none (E) infinite

47. An infinite geometric sequence has a common ratio of $\frac{1}{4}$ and a sum of 44. What is the first term of the sequence?

- (A) 4 (B) 11 (C) 16 (D) 20 (E) 33

48. Let $a, b,$ and c be real numbers such that $a + b + c = 8, c^2 = a^2 + b^2,$ and $ab = 8.$ Find the numerical value of c .

- (A) 5 (B) 3 (C) 2.5 (D) 2 (E) 1.125

49. Which of the following surfaces is generated by $x^2 + 0y^2 + 4z^2 - 4 = 0$?
- (A) elliptic cone (B) elliptic cylinder (C) circular paraboloid
- (D) hyperbolic cylinder (E) hyperboloid of one sheet
50. Mark DeKard has eleven 3×5 index cards. He numbers the cards 1 through 11. He places them in a bag, shakes them up, and randomly selects two cards without replacement. What is the probability that both are even or both are odd?
- (A) $\frac{2}{11}$ (B) $\frac{10}{19}$ (C) $\frac{5}{11}$ (D) $\frac{21}{22}$ (E) $\frac{5}{6}$
51. If $A > B$ and $B > 3$, then $A - 3 > \underline{\hspace{2cm}}$
- (A) -3 (B) -1 (C) 0 (D) 2 (E) 3
52. The graphs of the lines $y = x - 2$ and $y = mx + 3$, where m is a constant, intersect at point P. Point P is in quadrant I if and only if which of the following is true?
- (A) $m = 1$ (B) $m < 1$ (C) $m > -1.5$ (D) $-1.5 < m < 1$ (E) $-1 < m < 1$
53. If $(\log x)^2 = \log(x^2)$ then the sum of all possible solutions is:
- (A) 1 (B) 2 (C) 3 (D) 101 (E) infinity sum
54. If $\cos(\theta)\cos(2\theta) = \frac{1}{4}$, where $0^\circ < \theta < 90^\circ$, then $\sin(\theta)\sin(2\theta)$ equals: (nearest tenth)
- (A) 1.5 (B) 0.5 (C) 1.3 (D) 0.75 (E) 0.6
55. Harry Brush can paint a house in 10 days. He painted one day by himself, then his brother, Sandy, joins him to finish the job. They finish it in 3 days instead of 10. How many days would it have taken Sandy to do the whole job alone?
- (A) 9 (B) 8 (C) 6 (D) 5 (E) 4
56. A right triangle has a hypotenuse of length 12". If one of the acute angles is decreasing at the rate of 10° per second, how fast is the area of the triangle decreasing when this acute angle is 8° ? (nearest tenth)
- (A) $3.5 \text{ in}^2/\text{sec}$ (B) $12.1 \text{ in}^2/\text{sec}$ (C) $12.4 \text{ in}^2/\text{sec}$ (D) $1.7 \text{ in}^2/\text{sec}$ (E) $10.7 \text{ in}^2/\text{sec}$
57. The probability that statement P is true is $\frac{5}{8}$, and the probability that statement Q is true is $\frac{3}{5}$. Determine the probability that $P \rightarrow Q$ is false.
- (A) $\frac{1}{4}$ (B) $\frac{2}{5}$ (C) $\frac{3}{8}$ (D) $\frac{3}{4}$ (E) $\frac{3}{20}$

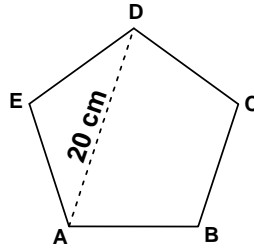
58. The square root of 301 in base 4 is:

- (A) 11_4 (B) 13_4 (C) 21_4 (D) 23_4 (E) 31_4

59. How many solutions are there for the equation $3x + 4y = 2015$ such that both x and y are positive integers?

- (A) 165 (B) 166 (C) 167 (D) 168 (E) 169

60. Given the regular pentagon shown, find AB . (nearest tenth)



- (A) 14.4 cm (B) 14.2 cm (C) 13.0 cm (D) 12.6 cm (E) 12.4 cm

**2014-15 TMSCA HS Math Test #6
Answer Key**

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|-------|-------|-------|
| 1. B | 21. A | 41. D |
| 2. C | 22. B | 42. E |
| 3. C | 23. D | 43. A |
| 4. A | 24. E | 44. E |
| 5. E | 25. C | 45. E |
| 6. C | 26. A | 46. B |
| 7. B | 27. B | 47. E |
| 8. C | 28. D | 48. B |
| 9. E | 29. B | 49. B |
| 10. A | 30. D | 50. C |
| 11. D | 31. E | 51. C |
| 12. A | 32. D | 52. D |
| 13. C | 33. B | 53. D |
| 14. C | 34. A | 54. E |
| 15. D | 35. E | 55. D |
| 16. D | 36. A | 56. B |
| 17. A | 37. C | 57. A |
| 18. B | 38. E | 58. B |
| 19. A | 39. C | 59. D |
| 20. C | 40. D | 60. E |