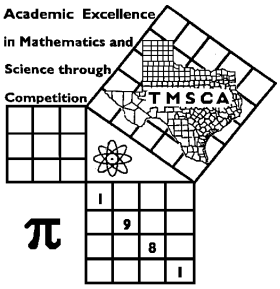


TMSCA HIGH SCHOOL MATHEMATICS TEST# 6 © DECEMBER 5, 2015

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA TMSCA

1. Evaluate: $1 - 2 \times 5 + 20 \div (1 - 5) \times 1 - 6$

(A) 0.25 **(B)** 0 **(C)** -9.75 **(D)** -16 **(E)** -20

2. Les Cash had $75 to spend. He spent 40% on clothes. Then he spent 60% of what he had left on shoes. He paid $9.50 for lunch. How much did he have left to spend at the arcade?

(A) $17.50 **(B)** $15.70 **(C)** $14.25 **(D)** $9.50 **(E)** $8.50

3. If F = {f,r,u,g,a,l}, L = {l,u,c,k,y}, and P = {p,o,l,i,t,e}, then $(F \cup L) \cap (L \cup P)$ contains how many distinct elements?

(A) 6 **(B)** 5 **(C)** 4 **(D)** 2 **(E)** 1

4. $(12 + 5) + 15 = (5 + 12) + 15$ and $12 \times (5 \times 16) = 12 \times (16 \times 5)$ are examples of the _____ properties of equality.

(A) associative **(B)** commutative **(C)** distributive **(D)** addition **(E)** subtraction

5. The order pairs $(-3, 5)$, $(3, -7)$, $(7, -15)$, $(9, -19)$ represent which of the following:

(A) a relation only **(B)** a function only **(C)** both a relation and a function

(D) neither a relation nor a function **(E)** a function that is not 1-1 function

6. Simplify: $\left(\frac{x^2 + 2x - 3}{x^2 - 4}\right)\left(\frac{3x - 6}{x^2 - 3x + 2}\right)$

(A) $\frac{3x + 9}{x^2 - 4}$ **(B)** $x + 3$ **(C)** $\frac{x + 3}{3x^2 - 12}$ **(D)** $3x^2 - 12$ **(E)** $\frac{3x + 9}{x + 2}$

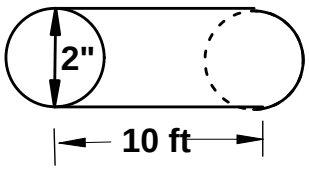
7. Find the greatest common divisor of $3^2 \times 5^3$, $2^3 \times 3 \times 5^2$, and $2^5 \times 5$.

(A) 5 **(B)** 30 **(C)** 15 **(D)** 6 **(E)** 120

8. Line $4x - ky = 5$ is perpendicular to line $2x - 3y = 6$. What is the value of k?

(A) $-\frac{2}{3}$ **(B)** $-1\frac{1}{2}$ **(C)** $-1\frac{2}{3}$ **(D)** $-2\frac{2}{3}$ **(E)** $-3\frac{1}{3}$

9. The diameter and length of the joint of PVC pipe is shown. Find the lateral surface area of the joint of pipe. (nearest in^2)



(A) 377 in^2 **(B)** 480 in^2 **(C)** 503 in^2 **(D)** 628 in^2 **(E)** 754 in^2

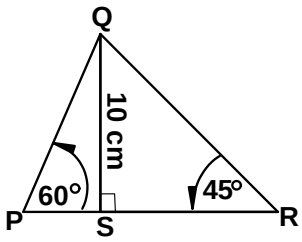
10. Perry Meeter has 200 feet of fencing. He wants to fence in his rectangular garden. What will the area of the garden be if the width is 45 feet and he uses all of the fencing?

(A) 3,025 ft^2 (B) 2,025 ft^2 (C) 3,262.5 ft^2 (D) 2,475 ft^2 (E) 3,487.5 ft^2

11. Let T_n be the nth triangular number, S_n be the nth square number, and P_n be the nth pentagonal number. Then $P_2 + S_2$ has the same value as:

(A) S_3 (B) T_2 (C) P_3 (D) T_4 (E) T_3

12. Find the area of $\triangle PQR$. (nearest tenth).



(A) 64.1 cm^2 (B) 75.0 cm^2 (C) 78.9 cm^2 (D) 95.7 cm^2 (E) 99.6 cm^2

13. The sum of the interior angles of a simple convex hexagon is:

(A) 720° (B) 800° (C) 1080° (D) 1,440° (E) 2,160°

14. Find the odds of randomly selecting a vowel from a box containing the letters from the name POCAHONTAS?

(A) 2:3 (B) 1:2 (C) 2:5 (D) 1:5 (E) 1:3

15. Let $3x + 2y = 1$ and $x - y = -2$. Find $x + y$.

(A) 2 (B) 1.2 (C) -0.6 (D) -1 (E) 0.8

16. If $\frac{5}{(x+2)(x-2)} - \frac{2}{(x-3)(x-2)} = \frac{k}{(x+2)(x-2)(x-3)}$, then k equals:

(A) $3x + 16$ (B) $7x - 11$ (C) $2x + 4$ (D) $3x - 19$ (E) $5x - 15$

17. The formulas that relate the coefficients of a polynomial to the sums and products of its roots is named after which of the following mathematicians?

(A) Sophie Germain (B) Marin Mersenne (C) Zeno of Alea

(D) Franciscus Vieta (E) Freda Porter

18. If $a_1 = 1$ and $a_n = 2(a_{n-1}) - 3$, then a_5 equals:

(A) -55 (B) -29 (C) -26 (D) -21 (E) -15

19. Given the sequence 0, 3, 8, 15, 24, k, 48, 63, ... find k.

(A) 26 (B) 32 (C) 35 (D) 40 (E) 43

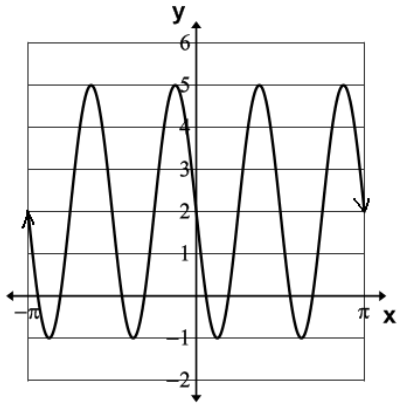
20. Mr. White's height is 6' 1". How long is Mr. White's shadow when the angle of elevation of the sun is 40 ° 15'? (nearest inch)

(A) 6' 0" (B) 7' 2" (C) 7' 6" (D) 8' 6" (E) 9' 0"

21. Determine the range of $f(x) = 2 - 3\cos(5x + 7)$.

(A) $[-1, 1]$ (B) $[-3, 2]$ (C) $[-3, 3]$ (D) $[-7, 5]$ (E) $[-1, 5]$

22. The equation $y =$ _____ will produce this graph.



(A) $2\sin(4x - \pi) + 3$ (B) $3\cos(2x - 2\pi) + 2$ (C) $3\cos(2x + \pi) + 4$

(D) $3\sin(4x + \pi) - 2$ (E) $3\sin(4x - \pi) + 2$

23. In the expansion of $(2x - 3y)^4$, the sum of the coefficients of the 2nd and the 3rd term is:

(A) 120 (B) 48 (C) 312 (D) 625 (E) 60

24. Use the Fibonacci characteristic sequence ..., p, — 2, 3, q, r, ... to Find $p + q + r$.

(A) 11 (B) 10 (C) 8 (D) 5 (E) 1

25. $f(x) = 2x^3 + 3x^2 + kx + 5$ is divided by $x - 1$ the remainder is 8. Find the value of k.

(A) 10 (B) 4 (C) 0 (D) — 1 (E) — 2

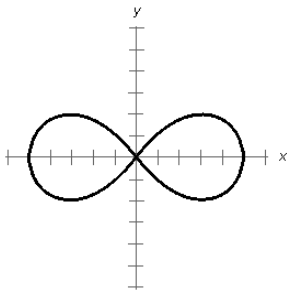
26. Let $f(x) = (2x + 1)^3$ Find $f'(-4)$.

(A) 486 (B) 384 (C) 343 (D) 294 (E) 147

27. What is the probability that a factor of 216 is a multiple of 3?

(A) $33\frac{1}{3}\%$ (B) 50% (C) $66\frac{2}{3}\%$ (D) 70% (E) 75%

28. The polar graph shown using rectangular coordinates is called a(n) _____.



(A) looped limacon (B) cardioid (C) eight rose (D) double circle (E) lemniscate

29. $\left[(1+2+3+4+...+13+14)\times 15\right] \div \left[(16+17+18+19+...+28+29)\times 30\right]$ **= ?**

(A) $\frac{1}{225}$ **(B)** $\frac{1}{900}$ **(C)** $\frac{1}{45}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{2}$

30. Coach Learner has 26 students in his math class class. Fourteen students play sports. Five are in the band. Ten compete in UIL academics. One is in band and UIL academics. Three are in sports and band. Four are in sports and academics. If no one does all three, how many students don't do any of the three things?

(A) 1 (B) 2 (C) 3 (D) 5 (E) 7

31. Let p and q be the roots of $x^2 - 5x + 6 = 0$ **. Find** $p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4$.

(A) 1 (B) 16 (C) 81 (D) 256 (E) 625

32. $\overleftrightarrow{AB}$ **intersects** $\overleftrightarrow{CD}$ **at point E such that m∠AEC = m∠BED. ∠AEC and ∠BED are considered to be _____ angles.**

(A) complementary (B) vertical (C) alternate interior (D) supplementary (E) right

33. Find the sum of the coefficients of the quotient: $(x^4 - 2x^3 - 3x + 6) \div (x - 2)$

(A) -2 **(B)** -1 **(C) 0 (D) 1 (E) 2**

34. Simplify: $(\sec^2\theta - 1)(1 - \sin^2\theta)$.

(A) $-\sec^2\theta$ **(B)** $-\csc^2\theta$ **(C) 1 (D)** $\sin^2\theta$ **(E)** $\cos^2\theta$

35. Which of the following statements about $f(x) = \frac{1}{2}x^3 - 2$ **is/are true?**

I. $f(x)$ **is a function.**
II. $f(x)$ **is a one to one function on its domain.**
III. $f(x)$ **has an inverse function on its domain.**

(A) I & II (B) I only (C) I & III (D) all of them (E) none of them

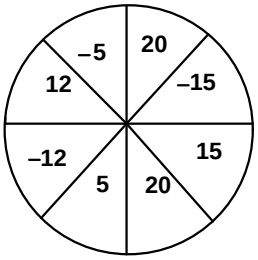
36. How many positive 3-digit numbers exist such that the sum of their digits equals 9?

(A) 27 **(B) 54** **(C) 36** **(D) 63** **(E) 45**

37. Let $f(x) = (x + 3)^2 - 7$ and $g(x) = \sqrt{x + 7} - 3$. Find $(f \circ g)(x + 1)$.

(A) x **(B) $x - 3$** **(C) $x + 1$** **(D) $x + 4$** **(E) $x - 1$**

38. Betty Wensom spins the wheel. The wheel consists of eight congruent sectors as shown. What is the mathematical expectation on any one spin?



(A) 2.5 **(B) 5** **(C) 8.5** **(D) 20** **(E) 400**

39. If the pattern of the sequence 1, 5, 11, 19, 29, 41, ... continues, find the 18th term.

(A) 343 **(B) 341** **(C) 324** **(D) 307** **(E) 305**

40. *U Rent All* rents trucks for \$8.00 an hour plus 30¢ a mile. *I Rent Some* rents the same type of truck for \$7.00 an hour plus 40¢ a mile. A truck needs to be rented for 5 hours that will travel 100 miles. How much would be saved by renting from *U Rent All* instead of *I Rent Some*?

(A) \$1.00 **(B) \$4.00** **(C) \$5.00** **(D) \$7.00** **(E) \$9.00**

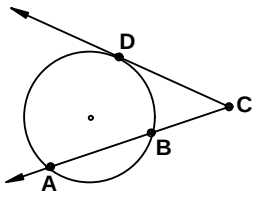
41. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. If $f_{15} = 610$, Find f_{16}.

(A) 976 **(B) 981** **(C) 987** **(D) 993** **(E) 998**

42. Tu Oad is 3 years older than Soh Yung. The sum of three times Tu's age and twice Soh's age is equal to Tan Gram's age. If Tan is 44 years old, what is the sum of Soh's age and Tu's age?

(A) 10 **(B) 13** **(C) 17** **(D) 19** **(E) 22**

43. If CD = 8 cm and CB = 5 cm then AC = ?



(A) $12\frac{4}{5}$ cm **(B) $11\frac{1}{8}$ cm** **(C) 13 cm** **(D) $14\frac{3}{5}$ cm** **(E) $8\frac{1}{8}$ cm**

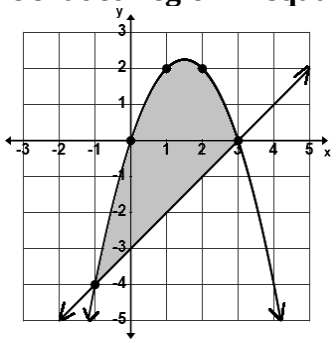
44. Saul T. Water bought some neon tetras and some fantail guppies for his aquarium. Tetras cost $2.10 each and guppies cost $1.80 each. He paid $28.50 for 14 fish, not including tax. How many tetras did he buy?

(A) 13 (B) 11 (C) 10 (D) 7 (E) 3

45. There are 3 judges, 5 policemen, and 9 office workers in the Painted Rock Courthouse. A county planning committee containing 1 judge, 3 policemen, and 4 office workers is to be created. How many different committees can be created?

(A) 68,040 (B) 24,310 (C) 3,780 (D) 473 (E) 136

46. Find the area of the shaded region in square units.



(A) $13\frac{1}{3}$ (B) $10\frac{2}{3}$ (C) 10 (D) $9\frac{2}{3}$ (E) $7\frac{1}{3}$

47. Find the determinant of the 3 x 3 matrix $\begin{bmatrix} 1 & 0 & -3 \\ -2 & 3 & 0 \\ 0 & -1 & 2 \end{bmatrix}$ **.**

(A) 12 (B) 6 (C) 0 (D) — 3 (E) — 6

48. Using the following array, determine the value of the mean of row 8.

2					(row 1)
4	6				(row 2)
8	10	12			(row 3)
14	16	18	20		(row 4)
22	24	26	28	30	(row 5)
	...				(...)

(A) 65 (B) 75 (C) 63 (D) 77 (E) 81

49. $567_8 - 432_8 =$ _____ $_2$

(A) 1101101 (B) 1010101 (C) 1011101 (D) 1110101 (E) 1011011

50. Points (— 4 , 2), (3 , — 5), and (x , y) are collinear. Which of the following could be (x , y).

(A) (— 3 , — 4) (B) (— 5 , 1) (C) (1 , 1) (D) (1, — 3) (E) (— 2, — 3)

51. (2, 3) and (3, 1) are opposite vertices of a parallelogram. If (0, 0) is the third vertex, then the fourth vertex is:

(A) (1, — 1) **(B) (— 1, 2)** **(C) (5, 4)** **(D) (2.5, 2)** **(E) (6, 3)**

52. Solve: $\log_2(8x) - \log_2(x^2 - 1) = \log_2(3)$

(A) 3 **(B) 8** **(C) 2** **(D) $\frac{1}{3}$** **(E) $\frac{1}{8}$**

53. $\triangle XYZ$ exists such that XZ = 20 cm, XY = 12 cm and $m\angle XYZ = 45°$. Find YZ. (nearest tenth)

(A) 16.7 cm **(B) 18.4 cm** **(C) 23.6 cm** **(D) 25.5 cm** **(E) 26.6 cm**

54. If $[(2 + 3i)(4 - 5i)] \div (i) = a + bi$, then $a + b = ?$

(A) — 25 **(B) — 21** **(C) — 1** **(D) 5** **(E) 9**

55. If $f'(x) = 12x^2 - 6x + 2$ and f(1) = 2, find f(0).

(A) — 1 **(B) 0** **(C) 1** **(D) 2** **(E) 8**

56. Rose Thorn creates flower baskets consisting of five flowers in each basket. How many different baskets can Rose make if she has roses, irises, carnations, lilies, geraniums, and snapdragons?

(A) 2,772 **(B) 720** **(C) 462** **(D) 252** **(E) 30**

57. A number M is randomly chosen from the set {3, 6, 8, 11, 13}. A number N is randomly chosen from the set {15, 16, 19, 23}. What is the probability that the product of M and N is an odd number?

(A) 30% **(B) 40%** **(C) 45%** **(D) 50%** **(E) 55%**

58. The length of the sides of $\triangle PQR$ are the roots of $f(x) = x^3 - 11x^2 + 38x - 40$. The perimeter of $\triangle PQR$ is 11. Find the area in square units of $\triangle PQR$. (nearest tenth)

(A) 54.2 **(B) 46.8** **(C) 45.1** **(D) 40.0** **(E) 32.2**

59. Which of the following statements about $f(x) = x^2 -$ x is/are true?

I. $f(x)$ is defined at 1 **II. $\lim_{x \to 1} f(x)$ exists**

III. $f(x)$ is continuous at 1 **IV. $f(x)$ is differentiable at 1**

(A) I, II, & III **(B) I & II** **(C) II & III** **(D) all of them** **(E) none of them**

60. The repeating decimal 0.2111... in base 3 can be written as which of the following fractions in base 3 in simplified form?

(A) $\frac{12}{20}_3$ **(B) $\frac{1}{11}_3$** **(C) $\frac{11}{22}_3$** **(D) $\frac{12}{100}_3$** **(E) $\frac{3}{12}_3$**

2015-16 TMSCA HS Math Test #6
Answer Key

1. E	**21. E**	**41. C**
2. E	**22. E**	**42. C**
3. B	**23. A**	**43. A**
4. B	**24. B**	**44. B**
5. C	**25. E**	**45. C**
6. A	**26. D**	**46. B**
7. A	**27. E**	**47. C**
8. D	**28. E**	**48. A**
9. E	**29. D**	**49. C**
10. D	**30. D**	**50. D**
11. A	**31. E**	**51. C**
12. C	**32. B**	**52. A**
13. A	**33. A**	**53. E**
14. A	**34. D**	**54. B**
15. E	**35. D**	**55. A**
16. D	**36. E**	**56. D**
17. D	**37. C**	**57. C**
18. B	**38. B**	**58. B**
19. C	**39. B**	**59. D**
20. B	**40. C**	**60. A**