T M S C A H I G H S C H O O L

M A T H E M A T I C S

T E S T # 5 ©

N O V E M B E R 2 1, 2 0 1 5

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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2015-2016 TMSCA Mathematics Test Five

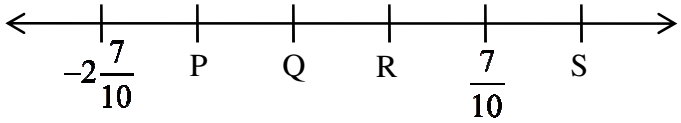
1. Evaluate: $6!+24(3-10)\div 40\times 15$.

   A. 719 B. 656 C. 657 D. 720 E. 678

2. Caroline had a rope that was 30 feet long. She cut off three pieces such that the ratio of lengths of the pieces were 2:5:10 with 20 inches of string left over. How long was the longest piece?

   A. 3 ft. 8 in. B. 16 ft. 8 in. C. 23 ft. D. 17 ft. 8 in. E. 21 ft. 4 in.

3. The distances between the hash marks (|) are equal. Find $P+Q+R+S$.



   A. -1.45 B. -3.32 C. -4 D. -2.3 E. -3.15

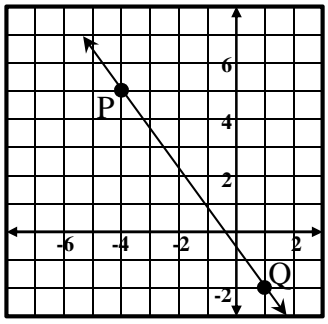
4. What is the harmonic mean of the first three perfect numbers? (nearest tenth)

   A. 43.7 B. 177 C. 30.7 D. 496 E. 14.7

5. Evaluate: $\dfrac{(x+1)!}{(x-2)!}\div\dfrac{x!}{(x-1)!}$.

   A. $x^5+2x^4-x^3-2x^2$ B. x^3+2x^2-x-2 C. x^4-5x^2+4 D. x^6+4x^2 E. x^2-1

6. Which of the following is the standard form of the equation of the perpendicular of $\overline{PQ}$ that passes through the point P?



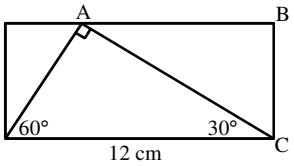
   A. $5x+7y=3$ C. $7x-5y=-10$ E. $7x+5y=15$

   B. $5x-7y=55$ D. $5x-7y=-18$

7. The point of intersection of the three angle bisectors of a triangle is called the_____.

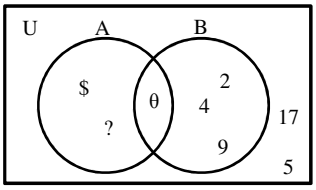
   A. Center B. Circumcenter C. Centroid D. Incenter E. Orthocenter

8. Find the area of the triangle ABC shown.



   A. 18 cm^2 B. $\dfrac{27\sqrt{3}}{2}$ cm^2 C. 12 cm^2 D. $24\sqrt{3}$ cm^2 E. $\dfrac{15\sqrt{3}}{2}$ cm^2

9. Use the Venn diagram to determine the set $A'\cap B$.



   A. $\{2,4,9\}$ C. $\{2,4,5,9,17\}$ E. $\{\$,?,\theta,2,4,9,\}$

   B. $\{\theta\}$ D. $\{\theta,2,4,5,9,17\}$

10. Karolyn invested $1000 for 4 years in a variable interest account. Her annual interest rates are shown in the table below. What was the average interest for the 4 years? (nearest hundredth of a percent)

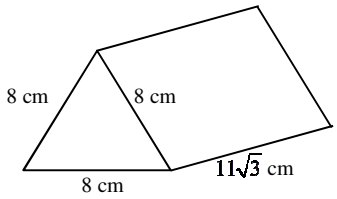
Year	1	2	3	4
Interest	4% gain	2.4% loss	3% loss	2.7% gain

   A. 0.33% B. 0.30% C. 2.97% D. 0.28% E. 3.02%

11. The student council consists of 8 girls and 5 boys. How many different 4-person delegations can the sponsor choose for the state convention if any delegation must have at least two girls?

A. 280 B. 630 C. 560 D. 95 E. 72

12. The volume of the triangular prism shown is _____cm^3.

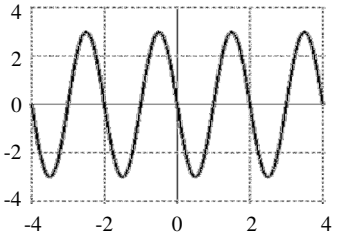


A. $352\sqrt{3}$ B. 704 C. 528 D. $348\sqrt{3}$ E. $328\sqrt{3}$

13. If $x-y=8$, and $xy=-15$, then $x^3-y^3=$

A. -360 B. -320 C. 392 D. 192 E. 152

14. The function $f(x)=$_____ will produce this graph.



A. $3\sin(\pi x+1)$ C. $3\sin(\pi(x-1))$ E. $\sin(\pi(x-1))+3$

B. $3\sin(\pi x-1)$ D. $3\sin(x+\pi)$

15. $A=\begin{bmatrix}3 & 5\\ 1 & -2\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1\\ 0 & 4\end{bmatrix}$. Calculate $\det(2A+B)$.

A. -18 B. -44 C. -11 D. -13 E. -8

16. What are the odds that a factor of 168 is a multiple of 4?

A. 1 to 2 B. 1 to 3 C. 1 to 1 D. 2 to 3 E. 5 to 12

17. $(3+2i)^5=$

A. $-597-122i$ B. $243-32i$ C. $-243-32i$ D. $-597+122i$ E. $-275+211i$

18. If *f* is continuous on the closed interval $[a,b]$ and *k* is any number between $f(a)$ and $f(b)$, then there is at least one number *c* in $[a,b]$ such that $f(c)=k$. This is the _____ .

A. Sandwich Theorem C. Rolle's Theorem E. Fundamental Theorem of Calculus

B. Intermediate Value Theorem D. Fundamental Theorem of Algebra

19. *P* and *Q* are the roots of $f(x)=2x^2+x-15$. Calculate $P^4-4P^3Q+6P^2Q^2-4PQ^3+Q^4$.

A. $\frac{1}{16}$ B. $\frac{28561}{625}$ C. $\frac{83521}{625}$ D. $\frac{5661}{16}$ E. $\frac{14641}{16}$

20. How many 3-digit numbers exist such that the sum of their digits equals 4?

A. 8 B. 9 C. 7 D. 10 E. 11

21. What is the digit in the millionths place for the sum of the sequence $1+3-\frac{3^2}{2!}-\frac{3^3}{3!}+\frac{3^4}{4!}+\frac{3^5}{5!}-\frac{3^6}{6!}-\frac{3^7}{7!}+\ldots$?

A. 2 B. 5 C. 7 D. 8 E. 3

22. How many distinct arrangements can be formed using all of the letters in the word "HOMECOMING"?

A. 40320 B. 907200 C. 151200 D. 20160 E. 3628800

23. A triangle with side lengths 12 cm, 12cm and 20 cm is a(n)_____triangle.

A. scalene acute B. isosceles obtuse C. isosceles acute D. scalene obtuse E. scalene right

24. If $\int_0^a \cos x dx = C$ then $\int_{-a}^a 4\cos x dx =$

A. 0 B. $2C$ C. $4C$ D. $8C$ E. $16C$

25. Given $a_0 = -1$, $a_1 = 2$ and $a_n = 2a_{n-1} + 3a_{n-2}$ find a_5.

A. 62 B. 181 C. 59 D. 122 E. 184

26. The Real value solution set of $|3x+1|+3<11$ is

A. $\left\{x\left|-\frac{7}{3}<x<3\right.\right\}$ B. $\left\{x\left|-3<x<-\frac{7}{3}\right.\right\}$ C. $\left\{x\left|\{x<-3\}\cup\left\{x>\frac{7}{3}\right\}\right.\right\}$ D. $\left\{x\left|-3<x<\frac{7}{3}\right.\right\}$ E. $\left\{x\left|\left\{x<-\frac{7}{3}\right\}\cup\{x>3\}\right.\right\}$

27. A triangle is inscribed in a circle. The center of the circle is the intersection of the _____ of the triangle.

A. Altitudes B. Medians C. Perpendicular Bisectors D. Angle Bisectors E. Sides

28. If $g(x) = x-1$ and $f(x) = x^3 - 1$, find $f(g(x+2))$.

A. x^3+3x^2+3x+2 C. x^3+x^2+x E. x^3+3x^2+3x+1

B. x^3+3x^2+3x D. x^3+3x^2+3x-1

29. The Beanery would like to market a mix of two of its most popular coffees. The Good Morning coffee sells for \$6 per pound, while the Dark Rich coffee sells for \$8 per pound. How much Dark Rich coffee should be used to create one pound of a mix that sells for \$6.50 per pound?

A. 2.5 oz. B. 12 oz. C. 8 oz. D. 4 oz. E. 9.5 oz.

30. Solve $\frac{3}{y}+\frac{5}{x+y}=4$ for x.

A. $\frac{8-4y}{3}$ B. $2-2y$ C. $\frac{2y^2-8y}{3-4y}$ D. $\frac{8y-4}{3}$ E. $\frac{4y^2-8y}{3-4y}$

31. In how many ways can a class of 18 students be split into three groups of 2, 6 and 10 students?

A. 2840292 B. 812323512 C. 6694974 D. 1225224 E. 15148224

32. The chord $\overline{AB}$ has a length of 14 cm and the circle has a diameter of 50 cm. How far is $\overline{AB}$ from the center of the circle?

A. 24 cm B. 48 cm C. 21 cm D. 45 cm E. 37 cm

33. A particle is moving along a straight line with a function $f(t) = 6t^3 - 5t - 11$, where $f(t)$ is the distance in in meters per second. Find the instantaneous rate of change at a time of 2 seconds.

A. 77 m/s B. 19 m/s C. 67 m/s D. 3 m/s E. 31 m/s

34. If $\log 8 = P$, and $\log 5 = Q$, then $\log 6.4 =$

A. $\frac{5P}{3Q}$ B. $\frac{5}{3}P+Q$ C. $\frac{5}{3}P-Q$ D. $\frac{3P}{5Q}$ E. $\frac{3P-Q}{5}$

35. If P, Q and R represent digits in $RPQ_5 + QRP_4 - PQR_3$ has a numeric value in base 10 of:

A. $-3P+20Q+30R$ C. $-3P+14Q+28R$ E. $-3P-12Q+30R$

B. $-3P+20Q+28R$ D. $-3P+14Q+30R$

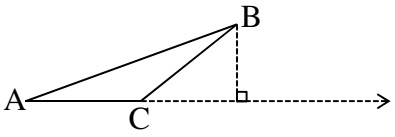
36. If P, Q and R are real numbers such that $P+Q+R=18$, $R^2 = P^2 + Q^2$ and $PQ = 6$, find the value of R.

A. $6\frac{1}{3}$ B. $8\frac{2}{3}$ C. $8\frac{1}{3}$ D. $9\frac{1}{3}$ E. $9\frac{2}{3}$

37. Express the complex number $2\left(\cos\frac{7\pi}{6}+i\sin\frac{7\pi}{6}\right)$ in rectangular form.

A. $-\frac{\sqrt{3}}{2}+\frac{i}{2}$ B. $-\sqrt{3}-i$ C. $-\sqrt{2}+i\sqrt{2}$ D. $-\sqrt{3}+i$ E. $-\sqrt{2}-i\sqrt{2}$

38. On triangle ABC shown, $m\angle BAC=\frac{\pi}{6}$ radians, $AB=12$ in. and $AC=9$ in. Find the area of triangle ABC.



A. $27\sqrt{3}$ in^2 B. 36 in^2 C. 54 in^2 D. $36\sqrt{3}$ in^2 E. 27 in^2

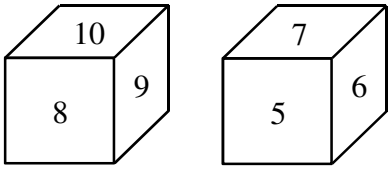
39. If $f(x)=ax^5+bx^3+cx+8$ and $f(7)=48$, then $f(-7)=$

A. -32 B. 41 C. 40 D. -31 E. 56

40. The number of calories in a particular candy varies directly with the mass. If a 10 g serving has 25 calories, how many calories will be in a 0.5 kg bag?

A. 1050 B. 200 C. 1250 D. 250 E. 1500

41. Chauncy created a pair of special dice which have only three numbers on each die. The side opposite each number is the same as the number. When the dice shown are rolled and the top numbers are added, what is the expected value of the sum?

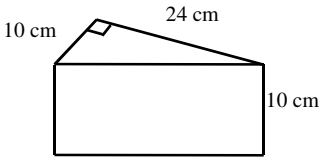


A. 14.5 B. 15 C. $15.\overline{3}$ D. $15.\overline{6}$ E. $16.\overline{3}$

42. $(2x+7)(5x-1)=2x(5x-1)+7(5x-1)$ is an example of _____ property.

A. Transitive B. Associative C. Commutative D. Distributive E. Substitution

43. Calculate the total surface area of the triangular prism shown.



A. 1200 cm^2 B. 644 cm^2 C. 924 cm^2 D. 600 cm^2 E. 840 cm^2

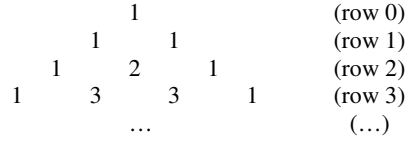
44. $\frac{\cot^2 t}{\csc t}=$

A. $\sec t+\cos t$ B. $\sec t-\cos t$ C. $\csc t+\sin t$ D. $\sec t-\sin t$ E. $\csc t-\sin t$

45. $f''(x)=2$, $f'(2)=5$ and $f(2)=10$. Evaluate $f(7)$.

A. 52 B. 62 C. 54 D. 60 E. 56

46. Given the triangular pattern shown below, find the third number in row 15.



A. 105 C. 136 E. 162

B. 171 D. 120

47. The ratio of length to width of a rectangle is 7:3 and the area is 75.81 in^2. What is the perimeter of the rectangle?

A. 38 in B. 55 in C. 19 in D. 40 in E. 302 in

48. If $\frac{2x-7}{3x-2}-\frac{3x+2}{2x-7}=\frac{Ax^2+Bx+C}{Px^2+Qx+R}$, then $\frac{A+B+C}{P+Q+R}=$

A. $-2\frac{2}{5}$ B. $1\frac{1}{3}$ C. $\frac{2}{5}$ D. -4 E. $-4\frac{4}{5}$

49. Adam is four years younger than Bill. Three years ago, Charles was twice as old as Adam. Charles is five years older than Bill. What is the sum of their current ages?

A. 40 B. 49 C. 38 D. 43 E. 51

50. Calculate the slope of the normal line to $2x^3+3x^2y-y^2=-2$ at the point $(-1,3)$.

A. -4 B. -3 C. $\frac{1}{4}$ D. $\frac{1}{3}$ E. 3

51. The Bears and the Lions are going to play a 3-game tournament. Based on their records, the probability that the Bears will win a particular game is 0.5, while the probability that the Lions will win a particular game is 0.3. What is the probability that the tournament will end in a tie?

A. 0.908 B. 0.008 C. 0.188 D. 0.180 E. 0.900

52. The afternoon train from Seattle, WA to Salem, OR is scheduled to take 5 hours and 12 minutes travelling at an average speed of 43 mph. Due to tracks warping in hot weather, the train must slow to an average of 25 mph for 3 hours of the scheduled trip on a summer day. How late will the arrival be in Salem? (nearest minute)

A. 1 h 46 min B. 1 h 15 min C. 2 h 12 min D. 2 h 23 min E. 2 h 32 min

53. $\int_{-3}^{7} f(x)\,dx=9$. Calculate $\int_{-3}^{7}(6+5f(x)\,dx)$.

A. 105 B. 51 C. 87 D. 110 E. 150

54. Let $f(x)=\frac{2x^2-4x+5}{x-3}$, and $s(x)$ be the slant asymptote of f. Find the value of $s(5)$.

A. 10 B. 0 C. 12 D. 8 E. 15

55. Find the area of the triangle with vertices $(1,2)$, $(6,12)$ and $(4,-1)$.

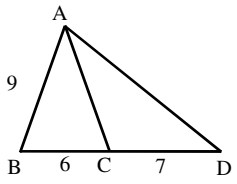
A. 36.5 B. 46.5 C. 15.5 D. 42.5 E. 22.5

56. Meredith set out to row on a lake. She rowed 750 m on a bearing of 75°, then 280 m on a bearing of 338°, then 800 m on a bearing of 210°. How far is she from her original starting point? (nearest meter)

A. 196 m B. 325 m C. 185 m D. 61 m E. 239 m

57. Given that $\overline{AB}\cong\overline{AC}$, find the area of triangle *ABD*.

A. $\frac{117}{2}$ B. $\frac{39\sqrt{5}}{2}$ C. $39\sqrt{2}$ D. $78\sqrt{2}$ E. $\frac{78\sqrt{5}}{2}$



58. Abigail, Bonny and Carla can plant a flower bed for Ms. Daisy in 3, 4 and 8 hours respectively. How long would it take them to plant Ms. Daisy's flower bed working together? (nearest hundredth)

A. 1.41 hr. B. 1.67 hr. C. 1.53 hr. D. 1.61 hr. E. 2.13 hr.

59. What is the constant term in the binomial expansion of $\left(3x^5-\frac{1}{x}\right)^{12}$?

A. 66 B. 792 C. 28 D. 8052 E. 594

60. If $\frac{6x+51}{x^2+3x-10}=\frac{A}{x-2}+\frac{B}{x+5}$, then $AB=$

A. 18 B. -27 C. 12 D. 15 E. -36

2015-2016 TMSCA Mathematics Test Five Answers

1. C	21. A	41. B
2. B	22. B	42. D
3. A	23. B	43. E
4. E	24. D	44. E
5. E	25. A	45. D
6. B	26. D	46. A
7. D	27. C	47. A
8. B	28. B	48. D
9. A	29. D	49. B
10. D	30. E	50. C
11. B	31. D	51. C
12. C	32. A	52. B
13. E	33. C	53. A
14. C	34. C	54. C
15. A	35. C	55. E
16. C	36. B	56. B
17. D	37. B	57. C
18. B	38. E	58. A
19. E	39. A	59. E
20. D	40. C	60. B

2015-2016 TMSCA Mathematics Test Five Select Solutions

10. The average interest will be $\sqrt[4]{(1.04)(0.976)(0.97)(1.027)} \approx 1.0028$ or a 0.28% gain.

11. The number of delegations with at least 2 girls is $({}_8C_2)({}_5C_2)+({}_8C_3)({}_5C_1)+({}_8C_4)({}_5C_0)$ which equals 630.

13. $x^3-y^3=(x-y)\left((x-y)^2+3xy\right)=$ $8\left(64+3(-15)\right)=152$.

20. Listed in order of their first digit:
103, 130, 121, 112,
202, 220, 211
301, 310
400

21. This is the power series for the function $f(x)=\sin x+\cos x$ when $x=3$
$\sin 3+\cos 3 \approx -0.8488724885$. The digit in the millionths place is a 2.

22. There are 10 total letters. "O" and "M" are each appear twice, so the total number of distinct arrangements is $\frac{10!}{(2!)(2!)}=907200$.

31. $({}_{18}C_2)({}_{16}C_6)({}_{10}C_{10})=1225224$.

35. $(25R+5P+Q)+(16Q+4R+P)-(9P+3Q+R)$ equals $-3P+14Q+28R$.

39. $f(7)=40+8=48$,
$f(-7)=-40+8=-32$.

41. $\begin{bmatrix}13 & 14 & 15\\ 14 & 15 & 16\\ 15 & 16 & 17\end{bmatrix}$ This array shows the possible sums with just the faces that are showing. These sums would just be repeated 4 times in a 6x6 array if we wrote out all of the sums, but the mean would be the same, 15.

46. The triangular pattern shown is Pascal's triangle, and the third number on any row beginning with row 2 is always the $(n-1)^{th}$ triangular number. For row 15, $\frac{14(15)}{2}=105$.

50. $6x^2+3\left(x^2\frac{dy}{dx}+y\cdot 2x\right)-2y\frac{dy}{dx}=0$
at $(-1,3)$, $6+3\left(\frac{dy}{dx}-6\right)-6\frac{dy}{dx}=0$.
Solve for $\frac{dy}{dx}=-4$, so the slope of the normal line will be $-\frac{1}{4}$.

51. The tournament could end in a tie if the results are TTT or all 6 arrangements of TLB, so the probability will be $0.2^3+6(0.2)(0.3)(0.5)=0.188$.

59. ${}_{12}C_2\left(3x^5\right)^2\left(-\frac{1}{x}\right)^{10}=594$