T M S C A H I G H S C H O O L

M A T H E M A T I C S

T E S T # 4 ©

N O V E M B E R 1 4, 2 0 1 5

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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2015-2016 TMSCA Mathematics Test Four

1. Evaluate: $1\div(1+4)^{-1}\times 10-\frac{5}{8}+17\times(32)^{0}$.

   A. $18\frac{3}{8}$ B. $66\frac{3}{8}$ C. $36\frac{3}{8}$ D. $20\frac{3}{8}$ E. $20\frac{5}{8}$

2. $\angle A$ and $\angle B$ are supplementary. If the $m\angle A : m\angle B$ is 7:8, find the measure of the complement of $\angle A$.

   A. 12° B. 6° C. 15° D. 96° E. 84°

3. What is the least common multiple of 204, 510 and 646?

   A. 29070 B. 58140 C. 329460 D. 9690 E. 19380

4. Leroy walked from his house to the school to pick up his bicycle at an average rate of 5 mph. He rode his bike home at an average rate of 26 mph. The total trip took 33 minutes. How far does Leroy live from the school? (nearest tenth)

   A. 2.6 mi B. 2.8 mi C. 1.6 mi D. 2.9 mi E. 2.3 mi

5. At Hobby Stop the price of a tube of oil paint is $7.85 and the price of a brush is $3.95. Crafty Carl has a 30% off coupon to use for the brushes and the paints are on sale for 15% off. If Carl buys 8 tubes of paint and 4 brushes, what will his cost be after the 8.5% sales tax has been applied?

   A. $69.92 B. $50.27 C. $52.99 D. $57.39 E. $62.27

6. Simplify $a^{-3}\times b^{3}\div a^{-5}\times b^{-5}\div a^{-3}\times b^{5}$.

   A. $a^{5}b^{3}$ B. $\frac{b^{3}}{a^{5}}$ C. $a^{3}b^{5}$ D. $\frac{b^{7}}{a^{11}}$ E. $\frac{a}{b^{7}}$

7. Two parallel lines are cut by a transversal to form two alternate interior angles with measures $(x^{2}-26)°$ and $(3x+2)°$. What is the sum of the two angle measures?

   A. 180° B. 90° C. 92° D. 46° E. 23°

8. What is the distance between the point $(3,7)$ and the line $12x+5y=27$?

   A. $\frac{98}{169}$ B. $\frac{44}{169}$ C. $7\frac{7}{13}$ D. $4\frac{7}{12}$ E. $3\frac{5}{13}$

9. The graph of $f(x)=x^{4}-12x^{3}+48x^{2}-64x$ has critical points when $x=a$ and $x=b$ where $a<b$. What is the value of *b*?

   A. 1 B. 2 C. D. 4 E. 12

10. A right triangular prism has a height of $4\sqrt{2}$ inches and the base is a 30°- 60°- 90° triangle with a hypotenuse of 8 inches. Find the volume.

    A. $16\sqrt{6}$ B. $32\sqrt{6}$ C. 32 D. $16\sqrt{5}$ E. $32\sqrt{3}$

11. Gary gets a job where he earns $300 per week plus a 10% commission on all his sales. The best model for his gross earnings as a function of sales will be

    A. Linear B. Exponential C. Quadratic D. Logistic E. Logarithmic

12. Points *P* and *Q* have coordinates $(-5,6)$ and $(7,-10)$ respectively. Which of the following is an equation of the perpendicular bisector of $\overline{PQ}$?

    A. $3x-4y=5$ B. $4x+3y=-2$ C. $3x-4y=-39$ D. $3x-4y=61$ E. $3x-4y=11$

13. Jay's current age is ¼ of his mother's age. In 10 years, Jay's age will be 2 years less than ½ his mother's age. What is Jay's current age?

A. 7 B. 9 C. 8 D. 15 E. 12

14. The number of integers between 1 and 48 that are relatively prime to 48 is:

A. 14 B. 15 C. 17 D. 18 E. 20

15. The medians of triangle *KLM* intersect at point *N*. If the length of the median from *K* to $\overline{LM}$ is 14.4 cm then $KN =$ _____cm.

A. 9.6 B. 7.2 C. 4.8 D. 10.8 E. 12.0

16. What is the sum of the series $57-19+6.\overline{3}-2.\overline{1}+...$?

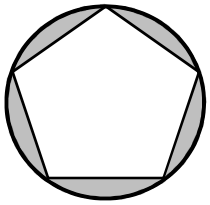
A. $\frac{494}{9}$ B. $\frac{171}{4}$ C. $\frac{172}{3}$ D. $\frac{380}{9}$ E. $\frac{171}{2}$

17. Which of the following has an amplitude of -3, period of $\frac{2}{5}$, phase shift 1 and displacement of -4?

A. $4-3\sin(5\pi x-1)$ C. $\frac{1}{5}-4\sin(3\pi x+1)$ E. $-4-3\sin(5\pi x-5\pi)$

B. $-3-4\sin(5\pi x+5\pi)$ D. $\frac{1}{5}+3\sin(4\pi x-4\pi)$

18. A dart lands randomly on the figure composed of a pentagon inscribed in circle. What are the odds it lands on the shaded region? (nearest hundredth)



A. 0.243 B. 0.321 C. 0.757 D. 0.127 E. 0.282

19. $\frac{\tan^2 t}{\sec t} =$

A. $\sec t+\cos t$ B. $\csc t+\sin t$ C. $\sec t-\cos t$ D. $\sec t-\sin t$ E. $\csc t-\sin t$

20. $\frac{4x^3-4x^2-9x+9}{3x^2-8x+5} \div \frac{2x^2+5x+3}{9x^2-30x+25} =$

A. $\frac{3x-5}{x+1}$ B. $\frac{6x^2-9x-15}{x+1}$ C. $\frac{2x-3}{x-1}$ D. $\frac{6x^2-19x+15}{x+1}$ E. $\frac{3x-5}{x-1}$

21. Find the average rate of change for $f(x)=3x^3-2x^2+5$ on the interval $[2,22]$

A. 8434 B. 4991 C. 1548 D. 3646 E. 480

22. Using Pascal's triangle as shown below, find the sum all the numbers in rows 0 through 10.

				1						(row 0)
			1		1					(row 1)
		1		2		1				(row 2)
	1		3		3		1			(row 3)
			…							…

A. 2047 B. 4095 C. 2048 D. 1023 E. 4096

23. How many distinguishable arrangements can be made from the letters in the word "PROBABILITY"?

A. 39916800 B. 19958400 C. 362880 D. 181440 E. 9979200

24. $867_9 = k_3$. Find the sum of the digits in *k*.

A. 9 B. 8 C. 7 D. 10 E. 6

25. Which of the following quadrant(s) does not contain a solution to $7x-5y\geq -49$?

A. QIV B. QI & QII C. QIII &QIV D. QIII E. None of These

26. A balloon rises at a rate of 4 meters per second from a point on the ground 30 meters from an observer. Find the rate of change of the angle of elevation of the balloon from the observer when the balloon is 30 meters above the ground.

A. $\frac{1}{30}$ rad/sec B. $\frac{1}{60}$ rad/sec C. $\frac{4}{15}$ rad/sec D. $\frac{1}{15}$ rad/sec E. $\frac{2}{15}$ rad/sec

27. Given $a_n=3a_{n-1}-2a_{n-2}$ where $a_1=1$ and $a_0=-1$ find a_4.

A. -82 B. -29 C. 82 D. 29 E. 773

28. Jill can tile a room alone in 12 hours, and her friend Kylie can tile a room alone in 9 hours. How long will it take the two of them to tile a room twice as long and twice as wide if they work together? (nearest minute)

A. 10 hr. 17 min. B. 21 hr. C. 9 hr. 6 min. D. 19 hr. 10 min E. 20 hr. 34 min.

29. If $5^x\bullet 25^{2y}=1$ and $3^{5x}\bullet 9^y=\frac{1}{9}$, then $x+y=$

A. $\frac{1}{3}$ B. $-\frac{1}{3}$ C. $-\frac{4}{9}$ D. $-\frac{5}{9}$ E. $\frac{4}{9}$

30. If $\int_3^8 f(x)\,dx=42$, then $\int_3^8\left[2+5f(x)\right]dx=$

A. 220 B. 212 C. 139 D. 218 E. 232

31. The equation $5x^2-12x+k=0$ has two positive solutions when which of the following is true?

A. $-7.2<k<7.2$ B. $k>7.2$ C. $k<7.2$ D. $-7.2<k<7.8$ E. $0<k<7.2$

32. There are 323 students in the senior class in Texas HS senior class. The ratio of boys to girls is 11:6. If the number of boys remains constant, how many new girls would have to enroll to change the ratio to 1:1?

A. 132 B. 102 C. 95 D. 143 E. 85

33. Find *C* if the remainder of $2x^3-7x^2+3x+C$ divided by $x-5$ is 16.

A. 106 B. 456 C. -74 D. -424 E. 428

34. The chord $\overline{AB}$ has a length of 40 cm and the circle has a diameter of 50 cm. How far is $\overline{AB}$ from the center of the circle?

A. 48 cm B. 30 cm C. $10\sqrt{21}$ cm D. 37 cm E. 15 cm

35. Use the function on the right. Find $f(2)+f(-4)+f(5)$.

A. 19.5 B. 1.5 C. 7.5 D. 9.5 E. 5.5

$$f(x)=\begin{cases} x-5, & x<0 \\ (2x)^2, & 0\leq x\leq 4 \\ \frac{x}{2}, & x>4 \end{cases}$$

36. Solve $\frac{3}{x}+\frac{5}{2x+y}=7$ for *y*.

A. $\frac{14x^2-11x}{3-7x}$ B. $\frac{x^2-2x}{x-5}$ C. $\frac{-14x^2+11x}{7x-1}$ D. $\frac{14x^2-11x}{7x-3}$ E. $\frac{2x^2-11x}{x-3}$

37. A circle is inscribed in a triangle. The center of the circle is the intersection of the _____ of the triangle.

A. Perpendicular Bisectors B. Altitudes C. Medians D. Angle Bisectors E. Sides

38. If y varies directly with x and $y = 340$ when $x = 200$, calculate y when $x = 117$.

A. 68.9 B. 581.1 C. 300.9 D. 198.9 E. 147.3

39. John's Ice Cream carries 12 gourmet flavors of home-made ice cream every day. If they also carry four choices of cones, how many different 2-scoop cone choices are there every day?

A. 264 B. 45 C. 312 D. 180 E. 78

40. P and Q are the roots of $f(x) = 4x^2 + 4x - 15$. Calculate $P^4 - 4P^3Q + 6P^2Q^2 - 4PQ^3 + Q^4$.

A. 625 B. 256 C. 243 D. 81 E. -64

41. The graph of the parametric equations $x = 5\cos t$ and $y = 3\sin t$ is a(n) _____.

A. Circle B. Ellipse C. Hyperbola D. Parabola E. Line

42. $4\log a^3 b - 2\log \frac{a}{b^2} + \frac{1}{2}\log ab =$

A. $\log a^9 b$ B. $\frac{1}{2}\log a^{11} b^9$ C. $\frac{1}{2}\log a^5 b$ D. $\frac{1}{2}\log a^{21} b^{17}$ E. $\log\left(a^{10}\sqrt{ab}\right)$

43. Calculate $4 - \frac{3^2}{2!} - \frac{3^3}{3!} + \frac{3^4}{4!} + \frac{3^5}{5!} - \frac{3^6}{6!} - \frac{3^7}{7!} + \ldots$ to the nearest ten-thousandth.

A. -0.8489 B. -0.8295 C. -1.0464 D. 1.0510 E. -0.1424

44. Let a, b and c be real numbers such that $c = a + b + 12$, $c^2 = a^2 + b^2$ and $ab = 12$. Find the value of $12c$.

A. 5 B. 24 C. 60 D. 7 E. 84

45. Regular hexagon ABCDEF is inscribed in a circle and all the diagonals are drawn. What is $m\angle AEC$?

A. 120° B. 45° C. 60° D. 90° E. 72°

46. Which of the following is an equation of the tangent line to $f(x) = \sin^2(2x)$ when $x = \frac{\pi}{6}$?

A. $y - \frac{1}{4} = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$ C. $y - \frac{\sqrt{3}}{2} = \frac{1}{4}\left(x - \frac{\pi}{6}\right)$ E. $y - \frac{1}{4} = \sqrt{3}\left(x - \frac{\pi}{6}\right)$

B. $y - \frac{3}{4} = \sqrt{3}\left(x - \frac{\pi}{6}\right)$ D. $y - \frac{3}{4} = \frac{1}{4}\left(x - \frac{\pi}{6}\right)$

47. Chance is going to flip a fair coin 6 times. What is the probability that he will get exactly 4 tails in a row?

A. $\frac{1}{16}$ B. $\frac{3}{64}$ C. $\frac{7}{64}$ D. $\frac{1}{8}$ E. $\frac{5}{64}$

48. Find the angle between the vectors $v_1 = \langle -8, 3\rangle$ and $v_2 = \langle 11, 9\rangle$. (nearest degree)

A. 30° B. 23° C. 150° D. 67° E. 120°

49. $\csc\theta < 0$ and $\tan\theta < 0$. Where will θ terminate?

A. QI B. QII C. QIII D. QIV E. x - axis

50. Given triangle ABC such that $m\angle A = 60°$, $AC = 26$ cm and $BC = 25$ cm there are two possible lengths for the side $\overline{AB}$. Find the sum of these two lengths.

A. 25 B. 26 C. 51 D. 17 E. 34

51. Solve $e^{2x} - 10e^x + 21 = 0$.

A. $0,\ \log 21$ B. $\log 3, \log 7$ C. $\ln 3, \ln 7$ D. $0,\ \ln 21$ E. $3,\ 7$

52. Find the value of $A+B+C$, where *A*, *B* and *C* are positive integers such that: $\frac{13}{10}=A+\left(\frac{1}{B+\frac{1}{C+1}}\right)$

A. 12 B. 7 C. 15 D. 6 E. 9

53. Let $f_0=0$, $f_1=1$, $f_2=1$, $f_3=2$, $f_4=3$, …be the terms of the Fibonacci sequence. Find $(f_9)^2-(f_8)^2$.

A. 1868 B. 715 C. 272 D. 610 E. 55

54. If $f(x)=ax^5+bx^3+cx+7$ and $f(9)=48$, then $f(-9)=$

A. 41 B. 48 C. -34 D. -48 E. 55

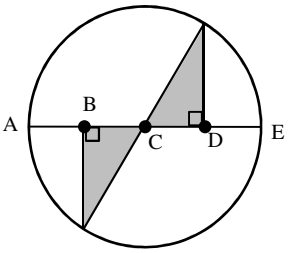
55. Quadrilateral ABCD has vertices $(-7,3)$, $(-4,6)$, $(5,5)$ and $(9,-2)$ respectively. What is the area of ABCD?

A. 77 B. 45 C. 67 D. 71 E. 61

56. Given the set of integers in ascending order $\{a,b,c,d,e\}$ has a median of 14 a mean 16.4, mode 11 and range of 16. Find the value of *d*.

A. 27 B. 21 C. 17 D. 14 E. 19

57. The radius of circle *C* is 9 in, *B* is the midpoint of $\overline{AC}$ and *D* is the midpoint of $\overline{CE}$. Find the area of the shaded region. (nearest in^2)



A. 18 in^2 B. 35 in^2 C. 20 in^2 D. 40 in^2 E. 27 in^2

58. Given $f(x)=2x^2$, $g(x)=\frac{1}{x}$ and $h(x)=\sqrt{x}$ evaluate $h^{-1}(g(f(2)))$.

A. 8 B. $\frac{1}{64}$ C. $2\sqrt{2}$ D. 64 E. $\frac{1}{2\sqrt{2}}$

59. $1331_b=$ _____$_{10}$ when $b>3$.

A. $b^2+3b+3+\frac{1}{b}$ B. $(b^2+1)(b+1)$ C. $b(b+1)(b+2)$ D. $(b+1)^3$ E. $(b^2+1)(b-1)$

60. $\sum_{k=0}^{15}k(k^2+1)=$

A. 11130 B. 14520 C. 74400 D. 1300 E. 3390

2015-2016 TMSCA Mathematics Test Four Answers

1. B	21. C	41. B
2. B	22. A	42. D
3. E	23. E	43. A
4. E	24. A	44. C
5. A	25. E	45. C
6. A	26. D	46. B
7. D	27. D	47. E
8. E	28. E	48. E
9. D	29. B	49. D
10. B	30. A	50. B
11. A	31. E	51. C
12. E	32. C	52. D
13. A	33. C	53. B
14. B	34. E	54. C
15. A	35. D	55. E
16. B	36. A	56. E
17. E	37. D	57. B
18. B	38. D	58. B
19. C	39. C	59. D
20. D	40. B	60. B

2015-2016 TMSCA Mathematics Test Four Selected Solutions

4. $\frac{d}{5}+\frac{d}{26}=\frac{33}{60}$, $d\approx 2.3$ mi

16. This is an infinite geometric series with an initial term 57 and a ratio of $-\frac{1}{3}$. The sum will be $\frac{57}{1-\left(-\frac{1}{3}\right)}=57\div\frac{4}{3}=\frac{171}{4}$

21. The average rate of change for the function will be the slope of the secant line connecting the two endpoints of the function on the given interval or $m=\frac{f(22)-f(2)}{22-2}=1548$.

22. The sum the numbers in each row of Pascal's triangle is 2 to the power of the row number. So the sum of the first 10 rows is going to be the sum of the geometric series $1+2+4+8+....+1024$ or $2^{n+1}-1=2047$.

23. There are 11 letters with "B" and "I" each repeated twice, so the number of distinct arrangements will be $\frac{11!}{(2!)(2!)}=9979200$.

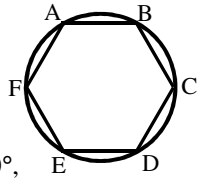
24. $867_9=8(9)^2+6(9)+7=$
$6(3^2)^2+2(3^2)^2+6(3^2)+6+1=$
$2\bullet 3(3^2)^2+2(3^2)^2+2\bullet 3(3^2)+2\bullet 3+1=$
$2\bullet 3^5+2\bullet 3^4+2\bullet 3^3+2\bullet 3+1=222021_3$, so the sum of the digits is 9.

26. Let θ be the angle of elevation and x be the height of the balloon at any particular time. The relationship between the two will be $\tan\theta=\frac{x}{30}$. Taking the derivatives of each side with respect to time $\sec^2\theta\frac{d\theta}{dt}=\frac{1}{30}\bullet\frac{dx}{dt}$. When $x=30$, $\theta=\frac{\pi}{4}$, so $2\frac{d\theta}{dt}=\frac{1}{30}\bullet 4$. $\frac{d\theta}{dt}=\frac{1}{15}$ rad/sec.

39. . If there are 12 types and 2 scoops, the number of distinct ice cream orders will be ${}_{12+2-1}C_2=78$, but that is just the ice cream. With the cone choices, there are $78\bullet 2=156$ order choices.

43. This is the power series for $f(x)=\sin x+\cos x$ when $x=3$.
$\sin 3+\cos 3\approx -0.8489$.

45. The arcs marked by consecutive letters are each 60°. The inscribed angle AEC intercepts an arc with measure 120°, so the measure of the angle is 60°.



48. Using the two definitions of scalar product
$\cos\theta=\frac{(-8)(11)+(3)(9)}{\sqrt{(64+9)(121+81)}}\approx 120^\circ$.

50. Set up a quadratic using the law of cosines: $625=676+x^2-52x\cos 60$ where $x=AB$. Arranged so that there is a 0 on one side, $0=x^2-26x+51$. The roots of this equation are the two possible lengths of AB so the sum of the lengths is 26.

51. $e^{2x}-10e^x+21=(e^x-7)(e^x-3)=0$, so $e^x=7$ or $e^x=3$ then take the natural log of both sides of each equation for $x=\ln 7$ or $x=\ln 3$.

52. $\frac{13}{10}=1+\frac{3}{10}=1+\frac{1}{\frac{10}{3}}=1+\frac{1}{3+\frac{1}{2+1}}$
so $A+B+C=6$.

54. If $f(9)=48=41+7$, then $f(-9)=-41+7=-34$.

57. Each of the small, shaded triangles is a 30°-60°-90° triangle with a hypotenuse length of 9 inches. The shaded region will have an area $A=2\bullet\frac{1}{2}(4.5)(4.5\sqrt{3})$ which is approximately 35 in^2.

59. $1331_b=1b^3+3b^2+3b+1$ which factors to $(b+1)^3$.