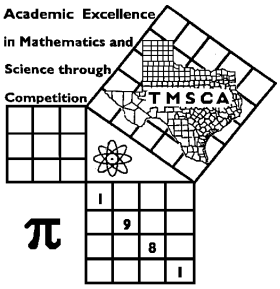

TMSCA HIGH SCHOOL MATHEMATICS TEST #3 ©

NOVEMBER 9, 2013

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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2013-2014 TMSCA Mathematics Test Three

1. Evaluate $\frac{11\bullet(7!)\div 8}{7+2^3}$.

(A) 998 (B) 462 (C) 770 (D) 66 (E) 149

2. At Hobby Stop the price of a tube of oil paint is \$7.85 and the price of a paint brush is \$3.95. Crafty Carl has a 30% off coupon to use for the paint and the brushes are on sale for 15% off. If Carl buys 7 tubes of paint and 4 brushes, what will his cost be after the 8.5% sales tax has been applied?

(A) \$56.31 (B) \$20.46 (C) \$51.90 (D) \$62.68 (E) \$57.77

3. On any workday the probability that Karen will remember her lunch is 90% and the probability that she will leave work on time is 70%. If these are independent events, what is the probability that on she will leave work late and forget her lunch on the same day?

(A) 63% (B) 30% (C) 37% (D) 3% (E) 6.3%

4. If $m\angle A+m\angle B+m\angle C=180^\circ$ and $m\angle C+m\angle D=180^\circ$, then $m\angle A+m\angle B+m\angle C=m\angle C+m\angle D$ is an example of _____ property.

(A) Distributive (B) Transitive (C) Associative (D) Commutative (E) Closure

5. Which of the following is a triangular number?

(A) 720 (B) 1440 (C) 1540 (D) 1080 (E) 2700

6. An equation of the line through $(-5,11)$ perpendicular to $2x+7y=35$ is

(A) $7x-2y=-46$ (B) $2x+7y=67$ (C) $7x-2y=24$ (D) $2x+7y=1$ (E) $7x-2y=-57$

7. Four workers can paint a wall in 20 minutes. How long will it take six workers at the same individual rate to paint a wall three times as long and three times as high?

(A) 60 min (B) 180 min (C) 40 min (D) 90 min (E) 120 min

8. If $x-y=7$ and $xy=3$, then $x^3-y^3=$

(A) 343 (B) 112 (C) 406 (D) 385 (E) 427

9. A little motorboat travelling with the current can make the 48 mile trip from A to B in 5 hours. The same boat takes 10 hours to make the return trip against the current. Find the speed of the current if the boat would have a constant speed in still water.

(A) 2.4 mh^{-1} (B) 3.6 mh^{-1} (C) 4.8 mh^{-1} (D) 7.2 mh^{-1} (E) 9.6 mh^{-1}

10. Simplify $\frac{2x^2+x-6}{x^2+4x-5}\bullet\frac{x^3-3x^2+2x}{4x^2-6x}$.

(A) $\frac{x^2-2x}{x+10}$ (B) $\frac{x^2-4}{x+5}$ (C) $\frac{x^2-4}{2x+10}$ (D) $\frac{x-2}{2x+10}$ (E) $\frac{x^2-2x}{x+5}$

11. Find the total surface area of a right cone given the radius of the base is 12 ft. and the vertex angle is 36°.

(A) 592 ft^2 (B) 1222 ft^2 (C) 718 ft^2 (D) 1916 ft^2 (E) 843 ft^2

12. Find the area of a circle defined by the equation $x^2+y^2-6x+4y-5=0$.

(A) $3\sqrt{2}$ (B) 18π (C) 36π (D) $3\pi\sqrt{2}$ (E) 9π

13. If $5^x\bullet 25^{2y}=1$ and $3^{5x}\bullet 9^y=\frac{1}{9}$, find the value of $x+y$.

(A) $\frac{4}{9}$ (B) $\frac{1}{9}$ (C) $-\frac{4}{9}$ (D) $\frac{1}{3}$ (E) $-\frac{1}{3}$

14. If $f(x)=x+1$ and $g(x)=\frac{3}{x}$, find $g\left(f^{-1}(x)\right)$.

(A) $\frac{x+3}{x}$ (B) $\frac{x+3}{x-1}$ (C) $\frac{3}{x+3}$ (D) $\frac{3}{x-1}$ (E) $\frac{x-1}{x+3}$

15. A hiker leaves position A and travels for 2 km on bearing 160° followed by 3 km on a bearing of 200°. How far is the final destination from position A?

(A) 4.71 km (B) 4.36 km (C) 3.61 km (D) 5 km (E) 4.03 km

16. A piece of wire 5.4 m long is bent to form the sides of a closed triangle. Find the largest possible area for such a triangle.

(A) $\frac{3\sqrt{3}}{5}\text{m}^2$ (B) $\frac{81\sqrt{3}}{100}\text{m}^2$ (C) $\frac{243\sqrt{3}}{25}\text{m}^2$ (D) $\frac{729\sqrt{3}}{100}$ (E) $\frac{27\sqrt{3}}{25}$

17. $\frac{\sin 2\theta}{1+\cos 2\theta}=$

(A) $\cot\theta$ (B) $\tan\theta$ (C) $\sec 2\theta$ (D) $\tan 2\theta$ (E) $\cot 2\theta$

18. Classify the graph of $x^2-6xy+9y^2-2y+1=0$.

(A) Circle (B) Cartoid (C) Hyperbola (D) Ellipse (E) Parabola

19. Evaluate $\sum_{k=0}^{\infty}\frac{2}{3}\left(-\frac{3}{5}\right)^{k+2}$.

(A) $\frac{3}{20}$ (B) $-\frac{1}{4}$ (C) $\frac{3}{5}$ (D) $\frac{5}{12}$ (E) $\frac{5}{3}$

20. What is the coefficient of the 5th term in the binomial expansion of $(2x-5)^8$?

(A) -224000 (B) 700000 (C) 1400000 (D) 224000 (E) -700000

21. $A=\begin{bmatrix}3 & 5\\ -2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}2 & 7\\ 3 & -4\end{bmatrix}$, so $\det AB=$

(A) -376 (B) 186 (C) -234 (D) -377 (E) 203

22. $\lim_{x\to 1/2}\frac{6x^2-17x+7}{2x^3-7x^2+3x}=$

(A) $\frac{1}{2}$ (B) 3 (C) $\frac{22}{5}$ (D) $\frac{13}{4}$ (E) does not exist

23. Which of the following has an amplitude of 4, period of $\frac{2}{3}$,phase shift of -1 and a displacement of -2?

(A) $-1+\frac{1}{4}\sin(3\pi x-2\pi)$ (C) $\frac{1}{3}+4\sin(-\pi+2\pi)$ (E) $\frac{1}{4}+3\sin(-\pi x+2\pi)$

(B) $-2+4\sin(3\pi x+3\pi)$ (D) $4-\sin(-2\pi x-3\pi)$

24. If $(3-5i)^2-(2+5i)^3=a+bi$, then $a+b=$

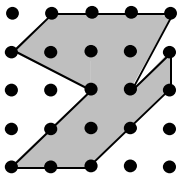
(A) 126 (B) -253 (C) -45 (D) 161 (E) -47

25. Let f be continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b). If $f(a)=f(b)$ then there is at least one number c such that $f'(c)=0$. The name of this theorem is

(A) Mean Value Theorem (C) Sandwich Theorem (E) Fundamental Theorem of Calculus

(B) Fundamental Theorem of Algebra (D) Rolle's Theorem

26. If the dots in the diagram shown right are 1 in apart both vertically and horizontally, find the area of the shaded region.



(A) 9 (B) 11 (C) 12 (D) 10 (E) 16

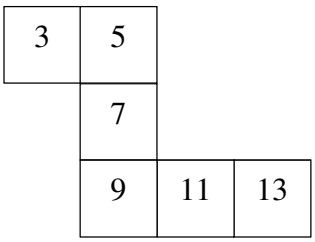
27. Find the surface area of a regular dodecahedron with a side length of 3 inches. (nearest in^2)

(A) 186 in^2 (B) 32 in^2 (C) 207 in^2 (D) 80 in^2 (E) 59 in^2

28. Given that the "1" at the top of Pascal's triangle is the 0^{th} row, _____ is in the 10^{th} row of Pascal's triangle?

(A) 210 (B) 84 (C) 56 (D) 252 (E) 70

29. A cube is formed by folding the net shown and the numbers on opposite faces are multiplied. What is the sum of the products?

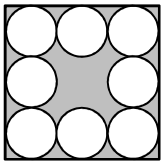


(A) 161 (B) 157 (C) 169 (D) 173 (E) 159

30. A bag of marbles contains 6 black, 8 red and 11 yellow marbles. Three are chosen one at a time without replacement. What is the probability that they will be yellow, yellow and black in that order?

(A) $\frac{121}{2300}$ (B) $\frac{132}{3125}$ (C) $\frac{11}{230}$ (D) $\frac{22}{345}$ (E) $\frac{726}{15625}$

31. A dart lands at random within the picture shown to the right. What are the odds the dart will land in the shaded region?



(A) 0.3019 (B) 0.4324 (C) 0.3581 (D) 0.6981 (E) 0.5579

32. $(0.1818...)^{-1}+(0.444...)^{-1}-(0.833...)^{-1}=$

(A) $1\frac{91}{198}$ (B) $6\frac{11}{20}$ (C) $-\frac{41}{198}$ (D) $8\frac{19}{20}$ (E) $6\frac{109}{180}$

33. P and Q are the roots of $x^3+2x^2-4x-8=0$. Evaluate $P^4-4P^3Q+6P^2Q^2-4PQ^3+Q^4$.

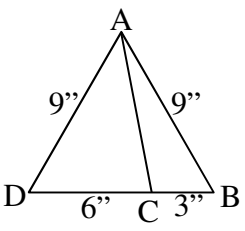
(A) 0 (B) 512 (C) 16 (D) 64 (E) 256

34. Two fair 7-sided dice are thrown and the numbers on the bottoms of both dice are added together. What is the probability that the sum will be less than 12?

(A) $\frac{43}{49}$ (B) $\frac{6}{7}$ (C) $\frac{46}{49}$ (D) $\frac{6}{49}$ (E) $\frac{3}{49}$

35. Find AC. (nearest ¼ inch)

(A) $6\frac{3}{4}$" (B) $9\frac{1}{2}$" (C) 8" (D) $8\frac{3}{4}$" (E) $7\frac{1}{2}$"



36. 400 gallons of 89 octane gasoline is obtained by mixing 87 octane and 92 octane gasoline. How much of the 87 octane gasoline is used?

(A) 200 gal (B) 240 gal (C) 250 gal (D) 260 gal (E) 280 gal

37. The slope of the tangent to $4x^2 - 9y^2 = 19$ at $(-5,3)$ is

(A) $-\frac{20}{9}$ (B) $\frac{20}{27}$ (C) $-\frac{59}{54}$ (D) $-\frac{20}{27}$ (E) $\frac{59}{54}$

38. The area of triangle ABC is 109 m^2. Given that $m\angle A = 50^\circ$ and $AB = 13$ m find AC. (nearest $\frac{1}{10}$ m)

(A) 10.9 m (B) 16.8 m (C) 15.5 m (D) 26.1 m (E) 21.9 m

39. How many elements are in $\{\theta \mid 6\cos^2\theta - \cos\theta = 2, \theta \in (-\pi, \pi)\}$?

(A) 0 (B) 6 (C) 3 (D) 2 (E) 4

40. The polar graph of $r = 6\cos 5\theta$ has _____ petals.

(A) 3 (B) 4 (C) 5 (D) 6 (E) 12

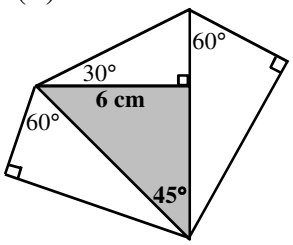
41. $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \frac{1}{81} \ldots =$

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{5}{6}$ (D) $\frac{3}{4}$ (E) $\frac{5}{8}$

42. Ranger Bob walks out 70 ft. from the base of a large tree. If the angle of elevation from the ground where Bob stands to the top of the tree is 76°, how tall is the tree? (nearest foot)

(A) 281 ft (B) 66 ft (C) 68 ft (D) 90 ft (E) 103 ft

43. What is the area of the **unshaded** region on the illustration shown? (nearest cm^2)



(A) 45 cm^2 (B) 66 cm^2 (C) 68 cm^2 (D) 90 cm^2 (E) 132 cm^2

44. If $A = \ln 2$, $B = \ln 3$ and $C = \ln 5$ then $\ln 18.75 =$

(A) $2BC - 2A$ (B) $\frac{BC^2}{A^2}$ (C) $\frac{B + C^2}{A^2}$ (D) $B + 2C - 2A$ (E) $\frac{B + 2C}{2A}$

45. If $y = 9 - x$ and $xy = 16$ then $|x - y| =$

(A) $3\sqrt{7}$ (B) $\sqrt{17}$ (C) 4 (D) 7 (E) $\sqrt{33}$

46. Simplify $a^{-3} \div b^3 \times a^{-5} \div b^{-5} \times a^{-3} \div b^5$.

(A) $\frac{b^3}{a^{11}}$ (B) $\frac{1}{a^{11}b^3}$ (C) $\frac{1}{ab^7}$ (D) $\frac{b^7}{a^{11}}$ (E) $\frac{a}{b^7}$

47. Find the sum of the solutions to the equation $|x^2 - 3x| = -4x + 6$.

(A) 6 (B) 0 (C) -2 (D) 4 (E) -1

48. Carol invested \$7000 in an account for five years. Her returns are shown in the table below. What was her average interest rate for the 5 years?

Year	1	2	3	4	5
Interest	3.5% gain	3.8% loss	2.7% gain	1.5% loss	5.9% gain

(A) 2.40% (B) 1.36% (C) 6.66% (D) 1.43% (E) 1.30%

49. Given $f(x)=ax^6+bx^4+cx^2+x$ and $f(4)=87$ find $f(-4)$.

(A) 83 (B) 79 (C) 91 (D) 87 (E) 95

50. A circle with a radius of 24 cm has a center at the point Q. How far from Q is a chord of the circle that has a length of 10 cm?

(A) $\sqrt{119}$ (B) $4\sqrt{11}$ (C) $\sqrt{551}$ (D) $8\sqrt{11}$ (E) $2\sqrt{119}$

51. Find the angle between the vectors $v_1=\langle -17,3\rangle$ and $v_2=\langle 11,9\rangle$. (nearest degree)

(A) 49° (B) 41° (C) 131° (D) 147° (E) 33°

52. Find the area of the region defined by the inequalities $x\ge 0$, $y\ge 0$ and $y\le 3\cos(2x)$.

(A) 4.5 (B) 0 (C) 6 (D) 3 (E) 1.5

53. The function $f(x)=x^3+3x^2+3$ has a point of inflection ____.

(A) $(0,3)$ (B) $(1,1)$ (C) $(-1,-1)$ (D) $(2,-1)$ (E) $(-1,5)$

54. Classify the graph of $3x^2+8xy+4y^2-7=0$.

(A) Circle (B) Ellipse (C) Parabola (D) Hyperbola (E) Cartoid

55. $\tan\theta>0$ and $\sin\theta<0$. Where will θ terminate?

(A) QI (B) QII (C) QIII (D) QIV (E) y - axis

56. Find the area of the convex quadrilateral with vertices $(2,7)$, $(4,1)$, $(1,-5)$ and $(-6,2)$.

(A) 24 (B) 60.5 (C) 27.5 (D) 59.5 (E) 30.5

57. How many solution (x,y) are there to the equation $5x+4y=256$ where x and y are both positive integers?

(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

58. Given $y=x^{x^2}$. Find $D_x y$.

(A) $x^{x^2+1}(1+2\ln x)$ (B) $x+2x\ln x$ (C) $2x^{x^2+2}$ (D) $x^2+2\ln x$ (E) $x+\ln(2x)$

59. What is the measure of one interior angle of a regular dodecagon?

(A) 144° (B) 120° (C) 140° (D) 136° (E) 150°

60. Given $b\in\mathbb{Z}^+$, $bbb_{b+1}=$ ____$_{10}$

(A) $b(b+2)$ (B) $(b+1)^3$ (C) $b(b^2+3b+3)$ (D) $b(b+1)(b+2)$ (E) $(b+1)^2$

2013-2014 TMSCA Mathematics Test Three Answers

1. B	21. D	41. D
2. A	22. C	42. A
3. D	23. B	43. A
4. B	24. D	44. D
5. C	25. D	45. B
6. E	26. D	46. B
7. E	27. A	47. C
8. C	28. A	48. E
9. A	29. C	49. B
10. C	30. C	50. C
11. D	31. B	51. C
12. B	32. B	52. E
13. E	33. E	53. E
14. D	34. A	54. D
15. A	35. C	55. C
16. B	36. B	56. B
17. B	37. D	57. D
18. E	38. E	58. A
19. A	39. E	59. E
20. B	40. C	60. C

2013-2014 TMSCA Mathematics Test Three Select Solutions

8. $x^3 - y^3 = (x-y)\left[(x-y)^2 + 3xy\right]$
$= 7\left(49 + 3(3)\right) = 406$

13. $5^x \bullet 5^{4y} = 5^0$ and $3^{5x} \bullet 3^{2y} = 3^{-2}$ so
$x + 4y = 0$ and $5x + 2y = -2$
added together the equations yield
$6x + 6y = -2 \rightarrow x + y = -\frac{1}{3}$

15. $v_1 = \langle 2\cos 160, \sin 160 \rangle = \langle x_1, y_1 \rangle$
$v_2 = \langle 3\cos 200, 3\sin 200 \rangle = \langle x_2, y_2 \rangle$
$d = \sqrt{(x_1 + x_2)^2 + (y_1 + y_2)^2}$

17. $\frac{\sin 2\theta}{1 + \cos 2\theta} = \frac{2\sin\theta\cos\theta}{1 + \cos^2\theta - 1} = \frac{\sin\theta}{\cos\theta} = \tan\theta$

18. Given $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$
If $B^2 - 4AC = 36 - 4(1)(9) = 0$
The graph is a parabola

26. I = # of interior points
P = # of perimeter points
$A = \frac{2I + P}{2} - 1 = \frac{2(4) + 14}{2} - 1 = 10$

33. $x^2(x+2) - 4(x+2) = (x+2)^2(x-2)$
Roots are 2 and -2.
$(P - Q)^4 = (-2 - 2)^4 = 256$

38. $109 = \frac{1}{2}(13)(AC)\sin 50$
$AC \approx 21.9$

39. $6\cos^2\theta - \cos\theta - 2 = 0$
$(3\cos\theta - 2)(2\cos\theta + 1) = 0$
$\cos\theta = \frac{2}{3}$ and $\cos\theta = -\frac{1}{2}$ each have 2 solutions in $(-\pi, \pi)$ so 4 solutions

40. $r = a\cos b\theta$ has b petals if b is odd and $2b$ petals if b is even.

47. $x^2 - 3x = -4x + 6$ and $x^2 - 3x = 4x - 6$
Solutions are -3, 2, 1 and 6, but only -3 and 1 work in the original equation. $-3 + 1 = -2$

49. Because $g(x) = ax^6 + bx^4 + cx^2$,
$g(4) = g(-4)$.
$f(4) = 87 = g(4) + 4$, so $g(4) = 83$
$f(-4) = g(4) - 4 = 83 - 4 = 79$

58. $\ln y = x^2 \ln x$
$\frac{1}{y}\frac{dy}{dx} = x^2 \bullet \frac{1}{x} + \ln x \bullet 2x$
$\frac{dy}{dx} = (x + 2x\ln x) \bullet x^{x^2} = x^{x^2+1}(1 + 2\ln x)$

60. $bbb_{b+1} = (b+1)^3 - 1$ in base 10.
$(b+1)^3 - 1 = b^3 + 3b^2 + 3b + 1 - 1$
$= b(b^2 + 3b + 3)$