

## TMSCA HIGH SCHOOL MATHEMATICS TEST # 3 © NOVEMBER 3, 2012

## **GENERAL DIRECTIONS**

## 1. About this test:

- A. You will be given 40 minutes to take this test.
- B. There are 60 problems on this test.
- All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
- 3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
- 4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
- 5. You may use additional scratch paper provided by the contest director.
- 6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
- 7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
- 8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
- 9. In case of ties, percent accuracy will be used as a tie breaker.

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1.	What is 43.75% of 2	2.6÷1.4?				
	(A) $\frac{49}{30}$	(B) $\frac{50}{63}$	(C) $\frac{5}{6}$	(D) $\frac{15}{14}$	(E) $\frac{13}{4}$	
2.	Events A and B are independent such that $P(B) = 3P(A)$ and $P(A \cup B) = \frac{20}{27}$ . Find $P(A)$ .					
	(A) $\frac{2}{9}$	(B) $\frac{2\sqrt{5}}{3}$	(C) $\frac{2}{3}$	(D) $\frac{5}{27}$	(E) $\frac{2\sqrt{5}}{9}$	
3.	Let <i>x</i> vary inversely v (A) 70	with $y^2 + 1$ . If $x = 20$ (B) 2	when $y = 2$ , find x w (C) 4	when $y = 7$ . (D) 200	(E) 100	
4.	Find the y-intercept of the line containing (7,3) and parallel to $5x-2y+6=0$ .					
	(A) $(0, -14.5)$	(B) (20.5,0)	(C) $(0, 20.5)$	(D) $(5.8,0)$	(E) (0,14.5)	
5.	Given the sequence 2 (A) 1574	e, 2, 9, 16, 35760, <i>k</i> (B) 1347	, 2338, find the value (C) 1353	of <i>k</i> . (D) 2022	(E) 1512	
6.	If $\int_{3}^{k} \frac{1}{x-2} dx = \ln 7$ , f	ind the value of k.				
	(A) 6.5	(B) 12	(C) 11.5	(D) 9	(E) 23	
7.	The first term of an in possible sums. The l	nfinite geometric sequ arger possible sum is	ence is 18, while the	third term is 8. The sec	quence has two	
	(A) 10.8	(B) 32.4	(C) 54	(D) 108	(E) 27	
8.	Carl has a rope that is 1:2:3. What is the left	s two yards, one foot a ngth of the longest pie	and nine inches long.	He cuts it into three pi	eces in a ratio of	
	(A) 2 ft 7 in	(B) 1 yd 10.5 in	(C) 1 yd 1 ft 9 in	(D) 1 ft 3.5 in	(E) 1 yd 2 ft 2 in	
9.	Three brothers can di hole that is three time (A) 20 hr 25 min	ig a hole 6ft x 3ft x 2ft es as long, wide and d (B) 2 hr 25 min	t in one hour. How lo eep if they each work (C) 2 hr 15 min	ng would it take four b at the same rate as the (D) 6 hr 45 min	rothers to dig a new first three? (E) 20 hr 15 min	
10.	Find the value of <i>x</i> or (A) 1.71 (B)	n the diagram on the r 9 (C) 8	ight. (D) 9.74	(E) 6.16		
<ul> <li>11. A cylindrical pipe with a diameter of 17 in and a height of 1 yard is filling with water at a rate of 2.7 gal/min. How long will it take to fill?</li> <li>(A) 9.68 min (B) 3.08 min (C) 16.68 min (D) 4.37 min (E) 13.10 min</li> </ul>						
1.0					(1) 10110 1111	
12.	Given that $(a+i)(2-i)$	-bi) = 7 - i, where $a$ ,	$b \in \mathbb{Z}$ , find the value	of $a+b$ .		
	(A) 5.5	(B) -2	(C) 6.5	(D) 4	(E) 2	
13.	Given the pyramid at (A) 108 (B)	right, find the 3 <sup>rd</sup> nur ) 36 (C) 13	nber in the tenth row. 5 (D) 252	(E) 360	3 3 3 3 6 3	
14. If $(2i+3)+(3i-5)\div(1-i)=ai+b$ , then $a+b=$ 3 9 9 3						
	(A) 0	(B) 10	(C) -1	(D) 11	(E) 9	

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27. A 16 ft. ladder slides down a wall so that the base of the ladder is moving away from the base of the wall at a rate of 2 inches per second. How fast is the angle between the ladder and ground changing when the angle

is 
$$\frac{\pi}{6}$$
 radians?  
(A)  $-\frac{\sqrt{3}}{144}$  rad/sec (B)  $\frac{1}{48}$  rad/sec (C)  $\frac{\sqrt{3}}{144}$  rad/sec (D)  $-\frac{1}{48}$  rad/sec (E)  $-\frac{1}{96}$  rad/sec

28. If a hiker travels 6 miles on a bearing of 12°, then another 8 miles on a bearing of 334°, what is the shortest distance back to his starting point?

(A) 10.00 miles (B) 2.26 miles (C) 10.06 miles (D) 13.06 miles (E) 13.25 miles

29. Let 
$$a_1 = 12$$
,  $a_2 = 5$ , and  $a_n = 2a_{n-1} - 3a_{n-2}$ . Find  $a_5$ .  
(A) 147 (B) -56 (C) -339 (D) -159 (E) -33

30. If 
$$f(x) = 2\cos^2 x$$
,  $g(x) = x^2 + 9$ , and  $h(x) = g(f(x))$ , find  $h\left(\frac{\pi}{6}\right)$ .  
(A) 11.25 (B) 9.25 (C) 18.00 (D) 13.91 (E) 5.05

- 31. What is the surface area of a tetrahedron if the length of one edge is  $2\sqrt{2}$  in? (A)  $8\sqrt{2}$  in<sup>2</sup> (B)  $4\sqrt{3}$  in<sup>2</sup> (C)  $4\sqrt{2}$  in<sup>2</sup> (D)  $\frac{8\sqrt{3}}{3}$  in<sup>2</sup> (E)  $8\sqrt{3}$  in<sup>2</sup>
- 32. The point (2,7) is reflected over the *x*-axis, reflected over the line y = x, rotated 180° clockwise around the origin, then shifted down three units to the point (a,b). Find a+b. (A) 6 (B) 1 (C) 2 (D) -3 (E) 4

33. Which of the following is not a solution to  $f(x) \ge \left| \sqrt{x^2} - 9 \right|$ ? (A) (-3,5.75) (B) (-8,1) (C) (9,0.25) (D) (0,7.25) (E) (11,2.25)

34. If f(x) = x + 7, g(x) = 9 - 4x and  $h(x) = \frac{g(x)}{f(x)}$ , find  $h^{-1}(1)$ . (A) 5.33 (B) 3.20 (C) -0.67 (D) 0.40 (E) 4.00

35. The mean, median and mode of a set of four numbers are 2, 7.5, and 11 respectively. Find the range of the set.
(A) 11
(B) 29
(C) 5
(D) 25
(E) 4.5

36. If  $f(x) = x^3 + ax^2 + bx + c$  and f(x) has zeroes at -5, -2, and 3, find a + b + c. (A) -15 (B) 71 (C) -23 (D) 18 (E) -37

- 37. Which of the following is closed under addition?
- I. natural numbers II. irrational numbers III. negative integers IV. odd integers
  - $(A) I \& III \qquad (B) I, III \& IV \qquad (C) II \qquad (D) II \& IV \qquad (E) none of these$

38. What is the equation of a line through (-7, -2) that is normal to 2x + 3y = 9?

(A) 2x+3y+20=0 (B) 3x+2y+25=0 (C) 2x+3y+8=0 (D) 3x-2y+17=0 (E) 3x-2y+19=0

39. The school day at Austin Elementary School begins at 8:25 am and ends at 4:15 pm. How many degrees does the hour hand on the clock in the cafeteria move during one school day?

40. 
$$\int (\sin^2 x - \cos^2 x) dx =$$
\_\_\_\_\_+ C, where C is some arbitrary constant.  
(A)  $\frac{\sin^3 x}{3} - \frac{\cos^3 x}{3}$  (B)  $2(\sin x + \cos x)$  (C)  $-\sin x \cos x$  (D)  $-x$  (E)  $\frac{\sin 2x}{2}$ 

- 41. The average life of a color TV is 8 years with a standard deviation of 1.5 years before it breaks. Suppose that a company guarantees color TVs and will replace a TV that breaks while under guarantee with a new one. How long should the company set the guarantee if they do not want to replace more than 5% of TVs? (A) 10.9 years (B) 11.0 years (C) 5.5 years (D) 10.5 years (E) 8 years
- 42. Barbara is assembling a committee of students to help with freshman orientation. Four sophomore girls, 3 sophomore boys, 7 junior girls, 5 junior boys, 5 senior girls and 4 senior boys volunteer. If she wants the committee to include two girls and two boys from each upper grade, how many distinct committees could she form from the volunteers?
  - (A) 8400 (B) 113400 (C) 64 (D) 16800 (E) 226800

43. A seasoned basketball player makes 40% of her free throws. She shoots free throws 3 times during a Friday game. What are the odds that she makes at least 2 of them?
(A) 0.237 (B) 0.310 (C) 0.543 (D) 0.763 (E) 3.223

44. Find the digit in the  $10^{-8}$  place of the series:  $-3 + \frac{3}{2!} - \frac{3}{4!} + \frac{3}{6!} \dots$ (A) 1 (B) 6 (C) 9 (D) 5 (E) 8

45. Carla inherited a windfall of *x* dollars. After putting half into savings, she gave one-third of the remaining money to her sister, forty dollars to her friend, then sixty percent of the remaining to charity. She was left with eighty dollars to spend for fun. What was the amount of the original windfall?
(A) \$520
(B) \$1280
(C) \$720
(D) \$900
(E) \$660

46. How many numbers in the form  $a^4$ , where  $a \in \mathbb{Z}^+$  divide  $3! \times 4! \times 6!$ ? (A) 2 (B) 3 (C) 5 (D) 6 (E) none

47. How many possible obtuse triangles are there with two side lengths of 6 and 11 where the length of the third side is an integer?(A) 3 (B) 7 (C) 4 (D) 8 (E) 11

48. On the diagram shown right,  $m \angle Q = 35^{\circ}$ , QR = 9 cm, and 2QS = SR. Find QS. (A) 3.29 cm<sup>2</sup> (B) 4.26 cm<sup>2</sup> (C) 7.37 cm<sup>2</sup> (D) 3.69 cm<sup>2</sup> (E) 2.58 cm<sup>2</sup>



50. Express  $\log(10ab^2)$  in terms of *P* and *Q* if  $P = \log a$  and  $Q = \log b$ . (A)  $10PQ^2$  (B) 10P + 2Q (C)  $10P + Q^2$  (D) P + 2Q (E) P + 2Q + 1

- 51. The sum of the first ten terms of an arithmetic sequence is 120 and the sum of the first twenty terms is 840. Find the sum of the first thirty terms.
  - (A) 1560 (B) 2250 (C) 2205 (D) 2160 (E) 1910
- 52. A body moves in a straight line. At time t seconds its acceleration is given by a = 6t + 1. When t = 0, the velocity v of the body is 2 m/s and its displacement s from the origin is 1 meter. Find an expression for displacement in terms of time.

(A) 
$$3t^2 + t + 2$$
  
(B)  $t^3 + \frac{t^2}{2} + 2t + 1$   
(C)  $6t^2 + t + 2$   
(D)  $t^3 + t^2 + 4t$   
(E)  $\frac{t^3}{3} + \frac{t^2}{2} + t + 1$   
53.  $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 =$   
(A)  $\sin^2\theta - \cos^2\theta$   
(B) -2  
(C) 1  
(D) -1  
(E) 2

54. The line 3y = x + 5 forms a chord with the circle  $x^2 + y^2 - 6x - 2y - 15 = 0$ . Find the length of the chord. (A)  $15\sqrt{10}$  (B)  $3\sqrt{10}$  (C)  $3\sqrt{5}$  (D)  $5\sqrt{2}$  (E)  $5\sqrt{3}$ 

55. An investment is made in a fund that pays an annual percentage rate of 6%, compounded quarterly. How long (to the nearest tenth of a year) will it take for the investment to double?
(A) 11.9
(B) 11.3
(C) 11.6
(D) 11.0
(E) 11.7

56. A department store has two burglar systems, A and B. In the event of an attempted break-in, the systems function properly with probabilities p(A) = 0.95 and p(B) = 0.90. The two systems function independently. When an attempt is made to break in, what is the probability of at least one of the alarm systems functioning properly?

(A) 0.995 (B) 0.885 (C) 0.9925 (D) 0.855 (E) 0.955

57. A company that manufactures dog food wishes to pack food in closed right cylindrical cans. What should the height be if each can is to have a volume of  $128\pi$  cm<sup>3</sup> and the minimum possible surface area? (A) 4 cm (B) 16 cm (C)  $4\sqrt{2}$  cm (D)  $8\sqrt{2}$  cm (E) 8 cm

58. Find 
$$\frac{d}{dx} \left( \frac{2x+1}{x^2-2} \right)$$
.  
(A)  $-\frac{2x^2+2x+4}{x^4+4}$  (B)  $-\frac{2x^2+2x+4}{x^4-4x^2+4}$  (C)  $\frac{2x^2+2x+4}{x^4-4x+4}$  (D)  $\frac{1}{x}$  (E)  $\frac{2x^2+2x+4}{x^4+4}$ 

59.  $2_3 + 33_4 + 444_5 + 5555_6 =$ \_\_\_\_\_7 (A) 6034 (B) 4121 (C) 3645 (D) 4323 (E) 1436

60. Random variable x is normally distributed such that P(x > 6.2) = 0.9474 and P(x < 9.8) = 0.6368. Find the mean of the distribution. (A) 1.83 (B) 8 (C) 9.16 (D) 8.84 (E) 8.12

## 2012-2013 TMSCA High School Mathematics Test 3 Key

1. C	21. E	41. C
2. A	22. A	42. E
3. B	23. A	43. C
4. A	24. C	44. D
5. C	25. E	45. C
6. D	26. E	46. D
7. C	27. D	47. D
8. B	28. E	48. A
9. E	29. B	49. B
10. E	30. A	50. E
11. E	31. E	51. D
12. D	32. C	52. B
13. A	33. B	53. E
14. A	34. D	54. B
15. E	35. B	55. C
16. B	36. E	56. A
17. B	37. A	57. E
18. E	38. D	58. B
19. A	39. B	59. B
20. A	40. C	60. C

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$$\begin{array}{c|c} 32. (2,7) \rightarrow (2,-7) \rightarrow (-7,2) \rightarrow (7,-2) \rightarrow (7,-5) \\ a+b=2 \\ 33. graph \\ 34. 1= \left(\frac{9-4y}{y+7}\right) \rightarrow 5y=2 \ , \text{ so } y=0.4 \\ 35. a, b, 11, 11. \frac{b+11}{2}=7.5 \ , \text{ so } b=4 \\ (a+4+11+1)=4(2) \ , \text{ so } a=-18 \\ \text{Range}=11-(-18)=29 \\ 36. \text{ product } (x+5)(x+2)(x-3)= \\ x^{2}+4x^{2}-11x-30, a+b+c=-37 \\ 37. definition of closed \\ 38. \perp m=1.5, y+2=1.5(x+7), \text{ so } \\ 3x-2y+17=0 \\ 39. 30\left(7+\frac{5}{6}\right)=235 \\ 40. -\int \cos 2xdx=-\frac{1}{2}\sin 2x+c=-\sin x\cos x+c \\ 41. z\text{ score } =1.6449, -1.6449=\frac{x-8}{1.5}, \\ x=5.5 \\ 42. \left(\frac{4}{2}\right) \binom{3}{2} \binom{7}{2} \binom{5}{2} \binom{5}{2} \binom{4}{2} = 226800 \\ 43. p=3(0.4)^{2}(0.6)+(0.4)^{3}=0.352 \\ \text{ odds } = \frac{0.352}{1-0.352}=0.543 \\ 44. -3\cos 1=-1.620906918 \\ 45. 80\left(\frac{5}{2}\right) \rightarrow 200 \rightarrow 240 \rightarrow 360 \rightarrow 720 \\ 46. 2^{2}\cdot^{3}\cdot^{3} \text{ sh s factors } \frac{1}{2}\cdot^{2}, \frac{4}{3}\cdot^{4}, \frac{4}{3}\cdot^{4}, \frac{1}{2}\cdot^{4}, \frac{4}{3}\cdot^{4}, \frac{1}{3}\cdot^{4}, \frac{4}{3}\cdot^{4}, \frac{1}{2}\cdot^{4}, \frac{4}{3} \\ 45. 80\left(\frac{5}{2}\right) \rightarrow 200 \rightarrow 240 \rightarrow 360 \rightarrow 720 \\ 46. 2^{2}\cdot^{3}\cdot^{3} \text{ sh s factors } \frac{1}{2}\cdot^{2}, \frac{4}{3}\cdot^{4}, \frac{4}{3}\cdot^{4}, \frac{1}{3}\cdot^{4}, \frac{4}{3}\cdot^{4}, \frac{1}{3}\cdot^{4}, \frac{4}{3}\cdot^{4}, \frac{1}{3} \\ 50. = \log 10 + \log a + 2\log b = 1+P + 2Q \\ 51. \frac{10}{2}(2a+9d) =120, \frac{20}{2}(2a+19d) = 840 \\ \text{ solve system: } a=-15, d=6, \\ S_{y_{0}}=15(2(-15)+29(6))2160 \end{array}$$