



**TMSCA HIGH SCHOOL  
MATHEMATICS  
TEST # 2 ©  
OCTOBER 31, 2015**

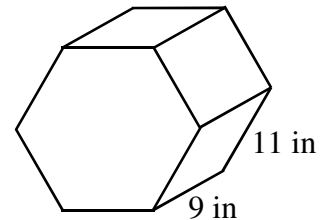
**GENERAL DIRECTIONS**

1. About this test:
  - A. You will be given 40 minutes to take this test.
  - B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.



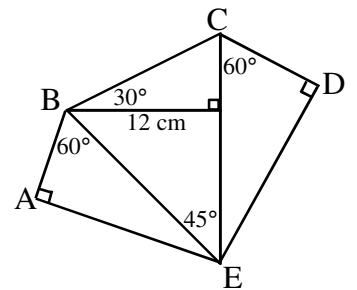
2015-2016 TMSCA Mathematics Test Two

- Evaluate:  $1 \div (1+3)^{-1} \times 4 - \frac{5}{8} + 17 \times (32)^0$ .  
 A.  $13\frac{3}{8}$       B.  $32\frac{5}{8}$       C.  $32\frac{3}{8}$       D.  $20\frac{3}{8}$       E.  $20\frac{5}{8}$
- $\angle A$  and  $\angle B$  are supplementary. If the  $m\angle A : m\angle B$  is 5:7, find the measure of the complement of  $\angle A$ .  
 A.  $24^\circ$       B.  $21^\circ$       C.  $15^\circ$       D.  $18^\circ$       E.  $36^\circ$
- What is the greatest common factor of 204, 510 and 646?  
 A. 17      B. 34      C. 19      D. 51      E. 13
- Leroy walked from his house to the school to pick up his bicycle at an average rate of 6 mph. He rode his bike home at an average rate of 26 mph. The total trip took 32 minutes. How far does Leroy live from the school?  
 A. 2.6 mi      B. 2.8 mi      C. 1.6 mi      D. 1.9 mi      E. 2.4 mi
- At Hobby Stop the price of a tube of oil paint is \$7.85 and the price of a paint brush is \$3.95. Crafty Carl has a 30% off coupon to use for the paint and the brushes are on sale for 15% off. If Carl buys 8 tubes of paint and 4 brushes, what will his cost be after the 8.5% sales tax has been applied?  
 A. \$35.01      B. \$50.27      C. \$52.99      D. \$57.39      E. \$62.27
- Simplify  $a^{-3} \div b^3 \times a^{-5} \div b^{-5} \times a^{-3} \div b^5$ .  
 A.  $\frac{b^3}{a^{11}}$       B.  $\frac{1}{a^{11}b^3}$       C.  $\frac{1}{ab^7}$       D.  $\frac{b^7}{a^{11}}$       E.  $\frac{a}{b^7}$
- Two parallel lines are cut by a transversal to form two consecutive interior angles with measures  $(x^2 - 26)^\circ$  and  $(5x + 2)^\circ$ . What is the measure of the acute angle?  
 A.  $62^\circ$       B.  $17^\circ$       C.  $50^\circ$       D.  $87^\circ$       E.  $57^\circ$
- The value of  $y$  varies inversely with  $5x^2$ . If  $y = 1.2$  when  $x = 2$ , what is the value of  $y$  when  $x = 4$ ?  
 A. 1.2      B. 0.15      C. 4.8      D. 2.4      E. 0.30
- The graph of the function  $f(x) = x^4 - 12x^3 + 48x^2 - 64x$  has points of inflection when  $x = a$  and  $x = b$  where  $a < b$ . What is the value of  $b$ ?  
 A. -1      B. 4      C. -2      D. 3      E. 1
- Find the lateral surface area of the regular hexagonal prism shown.  
 (nearest square inch)  
 A.  $495 \text{ in}^2$       B.  $594 \text{ in}^2$       C.  $1015 \text{ in}^2$       D.  $2314 \text{ in}^2$       E.  $1157 \text{ in}^2$



- Find the sum of the arithmetic mean, median, mode and range of 3, 19, 5, 2, 10, 35, & 3.  
 A. 49      B. 48      C. 55      D. 52      E. 53
- Which of the following is an equation of the line passing through  $(-3.2, 1.5)$  parallel to  $2x + 3y = 9$ ?  
 A.  $20x + 30y = 141$       C.  $20x - 30y = -19$       E.  $20x - 30y = -141$   
 B.  $20x + 30y = -19$       D.  $20x - 30y = 19$

13. A piggy bank contains a total of 82 dimes and quarters worth \$16.45. How many dimes are in the bank?  
 A. 26                      B. 54                      C. 27                      D. 41                      E. 55
14. The number of integers between 1 and 54 that are relatively prime to 54 is:  
 A. 15                      B. 17                      C. 18                      D. 20                      E. 24
15. Classify the triangle with side lengths 17 cm, 20 cm and 32 cm.  
 A. Isosceles Acute    B. Isosceles Obtuse    C. Scalene Acute    D. Scalene Obtuse    E. None of these
16. What is the sum of the arithmetic series  $11+8.5+6+\dots+(-26.5)$  ?  
 A. -116                      B. -124                      C. 48.5                      D. -270                      E. -288
17. Solve  $\sin 2x = -\cos x$ , where  $0 \leq x < 2\pi$ .  
 A.  $\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$     B.  $\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$     C.  $\left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\right\}$     D.  $\left\{\frac{\pi}{2}, \frac{\pi}{3}, \frac{3\pi}{2}, \frac{2\pi}{3}\right\}$     E.  $\left\{\frac{\pi}{3}, \frac{2\pi}{3}\right\}$
18. Find the perimeter of pentagon ABCDE. (nearest tenth)



- A. 80.1 cm    B. 68.2 cm    C. 62.9 cm    D. 71.0 cm    E. 74.8 cm
19.  $\frac{d}{d\theta} \sin(3\theta^2) =$   
 A.  $6\theta \cos(3\theta^2)$     B.  $\cos(6\theta)$     C.  $-3\theta \cos(6\theta)$     D.  $3\theta \cos(3\theta^2)$     E.  $-3\theta \cos(3\theta^2)$
20. Simplify  $\frac{2x^2 + x - 6}{x^2 + 4x - 5} \times \frac{x^3 - 3x^2 + 2x}{4x^2 - 6x}$ .  
 A.  $\frac{x^2 - 4}{2x + 10}$     B.  $\frac{x^2 - 2x}{x + 10}$     C.  $\frac{x^2 - 4}{x + 5}$     D.  $\frac{x - 2}{2x + 10}$     E.  $\frac{x^2 - 2x}{x + 5}$
21. Find the average rate of change for  $f(x) = 3x^3 - 2x^2 + 5$  on the interval  $[2, 7]$   
 A. 77                      B. 54                      C. 50                      D. 183                      E. 55
22. What is the arithmetic mean of the first six triangular numbers?  
 A.  $15\frac{1}{6}$                       B. 56                      C.  $9\frac{1}{3}$                       D.  $3\frac{1}{2}$                       E.  $13\frac{5}{6}$
23. How many distinguishable arrangements can be made from the letters in the word "SASSAFRASS"?  
 A. 181440                      B. 90720                      C. 5040                      D. 10080                      E. 720
24.  $876_9 = k_3$ . Find the sum of the digits in  $k$ .  
 A. 9                      B. 8                      C. 7                      D. 10                      E. 6
25. Which of the following quadrant(s) does not contain a solution to  $7x - 5y \leq -49$  ?  
 A. QIV                      B. QI & QII                      C. QIII & QIV                      D. QIII                      E. QI
26. Phil is filling up his empty conical water tank. The diameter of the base is 18 feet and the height of the tank is 6 ft. What is the least number of whole gallons of water he will need to fill the tank?  
 A. 11421 gal                      B. 3808 gal                      C. 4847 gal                      D. 952 gal                      E. 318 gal
27. Given  $a_{n+1} = a_{n-1} + (a_n)^2$ ,  $a_0 = 2$  and  $a_1 = 3$ , find  $a_4$ .  
 A. 2608                      B. 15387                      C. 124                      D. 6898                      E. 1611

28. Jill can paint a room alone in 8 hours. She painted for the first hour by herself then her friend Kylie joined her. They finished in a total of 6 hours instead of 8. How many hours would it have taken Kylie to do the whole job alone?  
 A. 20 hours      B. 18 hours      C. 15 hours      D. 12 hours      E. 10 hours
29.  $3\log a^2b^2 + \frac{1}{2}\log a^3b - 3\log ab =$   
 A.  $\log a^3b^4\sqrt{ab}$       B.  $-\log \frac{a^3b^3}{2}$       C.  $\log a^4b^3\sqrt{ab}$       D.  $\log \frac{1}{a^3b^3}$       E.  $\log a^8b^7\sqrt{ab}$
30. If  $\int_{-3}^8 f(x)dx = 42$ , then  $\int_{-3}^8 [2 + 5f(x)]dx =$   
 A. 210      B. 212      C. 139      D. 218      E. 232
31. The equation  $5x^2 - 6x + k = 0$  has two positive solutions when which of the following is true?  
 A.  $-1.2 < k < 1.2$       B.  $k > 1.2$       C.  $k < 1.8$       D.  $-1.2 < k < 1.8$       E.  $0 < k < 1.8$
32. There are 84 students in the senior class in Texas HS senior class. The ratio of boys to girls is 7:5. If the number of boys remains constant, how many new girls would have to enroll to change the ratio to 1:1?  
 A. 24      B. 35      C. 21      D. 7      E. 14
33. Let  $x$  and  $y$  exist such that  $8 < x < y$ . If 8,  $x$ , 24 form an arithmetic sequence and  $x$ ,  $y$ , 25 form a geometric sequence, then  $xy = ?$   
 A. 320      B. 400      C. 256      D. 160      E. 324
34. Find  $C$  if the remainder of  $2x^3 - 7x^2 + 3x + C$  divided by  $x - 4$  is 16.  
 A. -236      B. 35      C. -12      D. 7      E. 14
35.  $A = \begin{bmatrix} 3 & 5 \\ -2 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 7 \\ 3 & -4 \end{bmatrix}$ , so  $\det BA =$   
 A. -376      B. 186      C. -377      D. -324      E. 203
36. The chords  $\overline{AD}$  and  $\overline{BC}$  intersect inside circle  $O$  at point  $P$ . If  $AD = 23$ ,  $AP = 5$  and  $BP = 3$ , find  $CB$ .  
 A. 30      B. 27      C.  $\frac{124}{3}$       D. 33      E.  $\frac{115}{3}$
37. The altitudes of a triangle intersect at the  
 A. Orthocenter      B. Incenter      C. Circumcenter      D. Median      E. Origin
38. The graph of  $9x^2 - y^2 - 36x - 6y + 18 = 0$  is a  
 A. Circle      B. Hyperbola      C. Parabola      D. Ellipse      E. Cartoid
39. John's Ice Cream carries eight gourmet flavors of home-made ice cream every day. If they also carry four choices of cones, how many different 2-scoop cone choices are there every day?  
 A. 180      B. 45      C. 220      D. 210      E. 144
40. Two roots of  $f(x) = x^3 + bx^2 + cx + d$  are 7 and  $3 + i$ . Find  $b + c + d$ .  
 A. -34      B. -31      C. -1      D. -10      E. 17
41. The graph of the parametric equations  $x = 3\cos t$  and  $y = 5\sin t$  is a(n) \_\_\_\_\_.  
 A. Ellipse      B. Hyperbola      C. Circle      D. Parabola      E. Line

42. If  $4^x \cdot 16^{2y} = 1$  and  $7^{5x} \cdot 49^y = \frac{1}{49}$ , find the value of  $x + y$ .

- A.  $\frac{4}{9}$       B.  $\frac{1}{9}$       C.  $-\frac{4}{9}$       D.  $\frac{1}{3}$       E.  $-\frac{1}{3}$

43. Find the digit in the millionths place of the sum of the series  $1 - \frac{2}{1!} + \frac{4}{2!} - \frac{8}{3!} + \frac{16}{4!} - \dots$ .

- A. 6      B. 5      C. 7      D. 4      E. 3

44. Let  $a, b$  and  $c$  be real numbers such that  $c = a + b + 8$ ,  $c^2 = a^2 + b^2$  and  $ab = 8$ . Find the value of  $6c$ .

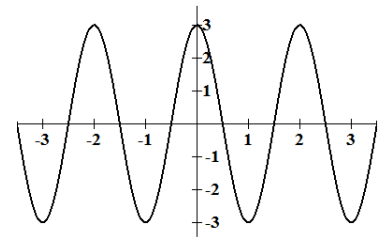
- A. 3      B. 18      C. 30      D. -24      E. -30

45. What is the angle between the hour and minute hands on a clock at 6:23 pm?

- A.  $53.5^\circ$       B.  $35.5^\circ$       C.  $42^\circ$       D.  $56.5^\circ$       E.  $47.5^\circ$

46. Which of the following functions yields the graph shown?

- A.  $y = 3\sin(\pi(x-2))$       C.  $y = 6\sin(\pi x - 2)$       E.  $y = 3\cos(\pi(x-2))$   
 B.  $y = 3\sin(\pi x - 2)$       D.  $y = 6\cos(\pi(x-2))$



47. The Real value solution for  $x^2 - 2x - 3 < 0$  is?

- A.  $\{x | -1 < x < 3\}$       C.  $\{x | \{x < -3\} \cup \{x > 1\}\}$       E.  $\{x | x > -1\}$   
 B.  $\{x | \{x < -1\} \cup \{x > 3\}\}$       D.  $\{x | -3 < x < 1\}$

48. Find the smallest positive angle between the vectors  $v_1 = \langle -16, 3 \rangle$  and  $v_2 = \langle 11, 9 \rangle$ . (nearest degree)

- A.  $29^\circ$       B.  $23^\circ$       C.  $130^\circ$       D.  $40^\circ$       E.  $140^\circ$

49. Find the slope of  $2x^2 + 3y^2 = 29$  at the point  $(-1, 3)$ .

- A.  $\frac{9}{2}$       B.  $-\frac{9}{2}$       C.  $-\frac{8}{3}$       D.  $-\frac{2}{9}$       E.  $\frac{2}{9}$

50. Given triangle  $ABC$  such that  $m\angle A = 60^\circ$ ,  $AC = 17$  cm and  $BC = 15$  cm there are two possible lengths for the side  $\overline{AB}$ . Find the sum of these two lengths.

- A.  $17\sqrt{3}$       B. 17      C.  $15\sqrt{3}$       D. 15      E.  $\frac{17\sqrt{3}}{2}$

51. Let  $f(x) = \frac{2x^2 - 7x + 5}{x - 8}$  and  $s(x)$  be the slant asymptote of  $f$ . Find the value of  $s(-6)$ .

- A. 12      B. -3      C. 77      D. -35      E. -12

52. Find the value of  $A + B + C$ , where  $A, B$  and  $C$  are non-negative integers such that:  $\frac{41}{8} = A + \left( \frac{1}{B + \frac{1}{C+1}} \right)$

- A. 12      B. 7      C. 15      D. 18      E. 9

53. Carmen has white marbles and yellow marbles in a bag. The probability that she will select a white marble is  $\frac{1}{4}$ . If 30 white marbles are added to the bag, the probability of selecting a white marble becomes  $\frac{1}{3}$ . How many yellow marbles are in the bag?
- A. 180                      B. 60                      C. 90                      D. 240                      E. 270
54. Let  $f_0 = 0, f_1 = 1, f_2 = 1, f_3 = 2, f_4 = 3, \dots$  be the terms of the Fibonacci sequence. Find  $(f_7)^2 + (f_8)^2$ .
- A. 233                      B. 1597                      C. 34                      D. 610                      E. 55
55. If  $f(x) = ax^5 + bx^3 - 10$  and  $f(3) = -40$ , then  $f(-3) =$
- A. 20                      B. -30                      C. -20                      D. -50                      E. 30
56. Find the area of the convex quadrilateral with vertices  $(2,7), (4,1), (1,-5)$  and  $(-6,2)$ .
- A. 24                      B. 27.5                      C. 60.5                      D. 59.5                      E. 30.5
57. How many solutions  $(x, y)$  are there to  $5x + 3y = 900$  where  $x$  and  $y$  are both positive integers?
- A. 61                      B. 58                      C. 60                      D. 59                      E. 62
58. If  $y = 9 - x$  and  $xy = 16$  then  $|x - y| =$
- A.  $3\sqrt{7}$                       B.  $\sqrt{17}$                       C. 4                      D. 7                      E.  $\sqrt{33}$
59.  $\tan \theta > 0$  and  $\sin \theta < 0$ . Where will  $\theta$  terminate?
- A. QI                      B. QII                      C. QIII                      D. QIV                      E.  $y$  - axis
60.  $444_5 + 777_8 - 222_3 = \text{_____}_{10}$ .
- A. 400                      B. 609                      C. 630                      D. 398                      E. 611

## 2015-2016 TMSCA Mathematics Test Two Answers

1. C	21. D	41. A
2. C	22. C	42. E
3. B	23. C	43. B
4. A	24. A	44. B
5. E	25. A	45. A
6. B	26. B	46. E
7. A	27. B	47. A
8. E	28. A	48. C
9. B	29. C	49. E
10. B	30. E	50. B
11. D	31. E	51. B
12. B	32. E	52. A
13. C	33. A	53. A
14. B	34. C	54. D
15. D	35. C	55. A
16. B	36. D	56. C
17. C	37. A	57. D
18. C	38. B	58. B
19. A	39. E	59. C
20. A	40. B	60. B



2015-2016 TMSCA Mathematics Test Two Selected Solutions

4.  $\frac{d}{6} + \frac{d}{26} = \frac{32}{60}$  because the two fractions on the right side of the equation represent the times for the trip and return trip. The solution to the equation is 2.6 miles.

21. The average rate of change will just be the slope of the secant line connecting the endpoints of the function on the given interval.

$$m = \frac{f(7) - f(2)}{7 - 2} = 183.$$

23. There are a total of 10 letters with "S" repeating 5 times and "A" repeating thrice, so the number of distinguishable

$$\text{arrangements will be: } \frac{10!}{(5!)(3!)} = 5040.$$

28. Jill's rate is  $\frac{1}{8}$  of a room per hour. Let Kylie's rate be  $r$ .

$$\frac{1}{8} + 5\left(\frac{1}{8} + r\right) = 1. \text{ Solving for } r \text{ gives a}$$

rate for Kylie of  $\frac{1}{20}$  of a room per hour, so it will take her 20 hours to do the job alone.

30.

$$\int_{-3}^8 [2 + 5f(x)] dx = \int_{-3}^8 2 dx + 5 \int_{-3}^8 f(x) dx = 22 + 5(42) = 232.$$

33. Because of the definition of an arithmetic sequence,  $x = \frac{8 + 24}{2} = 16$ . We

can find the ratio of the terms in the geometric sequence using  $16r^2 = 25$ , so

$$r = \frac{5}{4} \text{ and } y = 16\left(\frac{5}{4}\right) = 20. \quad xy = 320.$$

39. There are 8 flavors and we are going to order 2 scoops, so the number of choices for ice cream is  ${}_{8+2-1}C_2 = 36$ . There are also 4 choices for cones, so the total number of options is  $4(36) = 144$ .

43. This is the power series for  $f(x) = e^x$  when  $x = -2$ .  $e^{-2} \approx 0.1353352832$  with the 5 being the digit in the millionths place.

44. A little bit of algebraic manipulation of the first equation yields:

$$(c - 8)^2 = (a + b)^2 \text{ then}$$

$$c^2 - 16c + 64 = a^2 + 2ab + b^2 \text{ using}$$

$$\text{substitution from the equation } c^2 = a^2 + b^2$$

$$c^2 - 16c + 64 = c^2 + 2ab \text{ then}$$

$$-16c + 64 = 2ab \rightarrow -16c + 64 = 16, \text{ so}$$

$$c = 3 \text{ and } 6c = 18.$$

$$48. \cos \theta = \frac{(-16)(11) + (3)(9)}{\sqrt{(256 + 9)(121 + 81)}}, \text{ so}$$

$$\theta \approx 130.09^\circ.$$

$$49. 4x + 6y \frac{dy}{dx} = 0, \text{ so } -4 + 18 \frac{dy}{dx} = 0.$$

$$\text{Solving for } \frac{dy}{dx} = \frac{2}{9}.$$

50. Use the law of cosines to set up the quadratic equation:

$$225 = 289 + x^2 - 34x \cos 60, \text{ where}$$

$$x = AB. \text{ Arranging the equation to have a}$$

$$0 \text{ on one side: } 0 = x^2 - 17x + 64, \text{ so the}$$

$$\text{sum of the roots (or two possible lengths)}$$

$$\text{is } 17.$$

$$52. \frac{41}{8} = 5 + \frac{1}{8} = 5 + \frac{1}{7 + \frac{1}{0+1}}, \text{ so}$$

$$5 + 7 + 0 = 12.$$

$$55. f(3) = ax^5 + bx^3 - 10 = -40, \text{ so for}$$

$$x = 3, ax^5 + bx^3 = -30. \text{ Because both}$$

$$\text{terms have odd exponents, for } x = -3,$$

$$ax^5 + bx^3 = 30, \text{ and } f(-3) = 30 - 10 = 20.$$

$$58. (x + y)^2 = x^2 + 2xy + y^2 = 81, \text{ and}$$

$$(x - y)^2 = x^2 - 2xy + y^2, \text{ or } (x + y)^2 - 4xy$$

$$\text{so that } (x - y)^2 = 81 - 4(16) = 17 \text{ and}$$

$$|x - y| = \sqrt{17}.$$