# TMSCA HIGH SCHOOL

MATHEMATICS

TEST #2 ©

NOVEMBER 1, 2014

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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1. What is 56.25% of $2.\overline{3} \div 0.35$?
(A) $\dfrac{49}{30}$ (B) $\dfrac{50}{63}$ (C) $\dfrac{15}{14}$ (D) $\dfrac{5}{6}$ (E) $\dfrac{15}{4}$

2. Carol paid a total of $45.92 for dinner at a restaurant. If this sum included an 18% tip, what was the amount of the tip?
(A) $8.27   (B) $7.01   (C) $6.70   (D) $5.75   (E) $6.38

3. Carrie's drives to work every weekday on the highway. Her average daily speeds for the week are 62 mph, 68 mph, 57 mph, 65 mph, and 65 mph. What is her average speed for the week?
(A) 63.17 mph   (B) 63.60 mph   (C) 63.40 mph   (D) 63.72 mph   (E) 63.75 mph

4. An ice cream shop offers 8 flavors of ice cream and 2 types of cones. How many distinct 2-scoop cone orders are possible?
(A) 48   (B) 28   (C) 72   (D) 56   (E) 96

5. At Cup o' Joe's, Columbian coffee sells for $14.40 per pound while Guatemalan coffee sells for $17.40 per pound. How much Columbian coffee should be included in a blend that sells for $15.33 per pound?
(A) 4.96 oz.   (B) 5.24 oz.   (C) 3.92 oz.   (D) 10.76 oz.   (E) 11.04 oz.

6. Points P and Q have coordinates $(-5,10)$ and $(7,-6)$ respectively. Which of the following is an equation of the perpendicular bisector of $\overline{PQ}$?
(A) $4x-3y=2$ (B) $4x+3y=-2$ (C) $3x-4y=-10$ (D) $3x-4y=-5$ (E) $3x-4y=2$

7. Two dice are rolled. What are the odds that the sum of the dice is greater than 7?
(A) 5:11   (B) 7:5   (C) 5:7   (D) 1:1   (E) 6:11

8. A triangle is inscribed in a circle. The center of the circle is the intersection of the _____ of the triangle.
(A) Perpendicular Bisectors   (B) Altitudes   (C) Medians   (D) Angle Bisectors   (E) Sides

9. Regular hexagon ABCDEF is inscribed in a circle. Find $m\angle BFC$.

(A) 45°   (B) 90°   (C) 60°   (D) 120°   (E) 30°
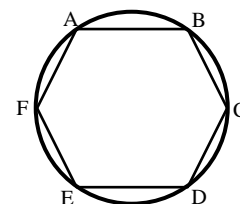
10. A parabola has roots at 0.4 and -3. The equation is $f(x)=6-ax-bx^2$. Find the value of b.
(A) -13   (B) 2.6   (C) 1.2   (D) 5   (E) -1.6

11. The cost of gasoline varies directly with the number of miles that a car drives. If it costs Carl $38.50 to drive 319 miles, how much should he budget for gas on a 782 mile trip? (nearest cent)
(A) $87.39   (B) $91.24   (C) $94.38   (D) $98.92   (E) $101.64

12. What is the lateral surface area of a right pyramid that has a square base with side length 8.24 in. and a height of 10.28 in.? (nearest square cm.)
(A) 464 cm^2 (B) 1178 cm^2 (C) 589 cm^2 (D) 182 cm^2 (E) 928 cm^2

13. All the edges of a cube are expanding at a rate of 4.25 cms^{-1}. How fast is the volume of the cube changing when the length of one edge is 10 cm?
(A) 541.875 cm^{3}s^{-1} (B) 1275 cm^{3}s^{-1} (C) 4250 cm^{3}s^{-1} (D) 114.75 cm^{3}s^{-1} (E) 767.656 cm^{3}s^{-1}

14. Jerry, Kyle and Larry can each tile a floor in 8 hours, 6.4 hours and 5.6 hours respectively. How long would it take them to tile a floor that is twice as long and three times as wide together? (nearest minute)
(A) 12 h. 52 min.   (B) 13 h.   (C) 13 h. 22 min.   (D) 13 h. 3 min.   (E) 11 h. 48 min.

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15. Use the Venn diagram to determine the set $A' \cap B'$.

(A) $\{\theta\}$ (C) $\{2,4,5,9,17\}$ (E) $\{5,17\}$

(B) $\{2,4,9\}$ (D) $\{\theta,2,4,5,9,17\}$
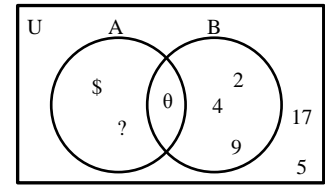
16. If $h(x) = x - 3$, $g(x) = x^2$, and $f(x) = 0.5x$, then $f(g(h(4))) =$

(A) 0.5   (B) 2   (C) 1   (D) -0.5   (E) -1

17. For all θ, which of the following is always equivalent to $f(\theta) = \frac{1}{2}\cos\left(3\theta - \frac{3\pi}{2}\right)$?

(A) $f(\theta) = \frac{1}{2}\cos\left(3\theta - \frac{\pi}{2}\right)$ (C) $f(\theta) = -\frac{1}{2}\sin\left(3\theta + \frac{\pi}{12}\right)$ (E) $f(\theta) = -\frac{1}{2}\cos\left(3\theta - \frac{\pi}{6}\right)$

(B) $f(\theta) = \frac{1}{2}\sin\left(3\theta + \frac{\pi}{12}\right)$ (D) $f(\theta) = \frac{1}{2}\sin\left(3\theta - \frac{3\pi}{2}\right)$

18. How many 3-digit numbers exist such that the sum of their digits equals 4?

(A) 13   (B) 9   (C) 10   (D) 4   (E) 7

19. Let $f(x) = 3x^2 - 2x + 4$ and $g(x) = 3x + 1$. Find $g(f'(x-2))$.

(A) $18x - 28$ (B) $18x - 41$ (C) $9x^2 - 42x + 61$ (D) $18x - 5$ (E) $9x^2 - 6x + 12$

20. Calculate the probability that when nine books are arranged on a shelf at random that three particular books are together.

(A) $\frac{1}{12}$ (B) $\frac{1}{72}$ (C) $\frac{1}{84}$ (D) $\frac{1}{60480}$ (E) $\frac{1}{720}$

21. If $f'(x) = 12x^3 + 6x^2 - 10x + 2$ and $f(-1) = 3$ then $f(1) =$

(A) 10   (B) 9   (C) 11   (D) 2   (E) 8

22. What is the circumference of the circle with the equation $x^2 + y^2 + 8x - 6y = 11$?

(A) 36π (B) 16π (C) 12π (D) 24π (E) 8π

23. What is the slope of the line normal to $2x^2 + 3y^2 + xy = 16$ at the point $(2, -2)$?

(A) -1   (B) $-\frac{3}{5}$ (C) $\frac{4}{5}$ (D) $-\frac{5}{3}$ (E) $-\frac{4}{5}$

24. The repeating decimal $2.242424..._6$ can be written as which of the following fractions in base 6?
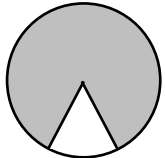
(A) $\frac{222}{30}_6$ (B) $\frac{222}{55}_6$ (C) $\frac{222}{35}_6$ (D) $\frac{220}{35}_6$ (E) $\frac{220}{30}_6$

25. The product of two positive integers is 8, and the reciprocal of one is 18 times the reciprocal of the other. Find the larger number.

(A) $\frac{3}{2}$ (B) 12   (C) $\frac{2}{9}$ (D) $\frac{2}{3}$ (E) 9

26. Which of the following are sides of an isosceles, obtuse triangle?

(A) 11, 11, 15   (B) 48, 55, 73   (C) $14, 14\sqrt{2}, 14$ (D) 8, 14, 8   (E) 19, 11, 17

27. The real solution set to $|2x-7|\le 9$ is

(A) $\{x|\{x\le -1\}\cup\{x\ge 8\}\}$ (B) $\{x|\{x\le -8\}\cup\{x\ge 1\}\}$ (C) $\{x|-1\le x\le 8\}$ (D) $\{x|-8\le x\le 1\}$ (E) $\{x|\{x\le -8\}\cup\{x\ge -1\}\}$

28. Triangle PQR is such that $m\angle R=60°$, $PR=16$ and $PQ=14$. There are two possible values for QR. Find the sum of the two values.

(A) 14   (B) 10   (C) 28   (D) 16   (E) 24

29. Given $a_{n+2}=a_n(a_{n+1})$, $a_1=-2$ and $a_2=3$ find a_6.

(A) -209952   (B) 108   (C) -1944   (D) -108   (E) 27648

30. If $xy-\dfrac{1}{3}=y-x=6-x-y$, what is the value of $x+y$?

(A) 1.2   (B) $0.41\overline{6}$ (C) $1.1\overline{6}$ (D) $1.\overline{83}$ (E) $3.\overline{83}$

31. Which of the following is in the 15th row of Pascal's triangle?

(A) 1820   (B) 1287   (C) 24310   (D) 6435   (E) 11440

32. Evaluate $\int_{-n}^{n}\left(3x^5-x^3+5\right)dx$.

(A) $n^6-\dfrac{n^4}{2}$ (B) $10n$ (C) $\dfrac{n^6}{2}-\dfrac{n^4}{2}+10n$ (D) $\dfrac{n^6}{2}-\dfrac{n^4}{4}+10n$ (E) $n^6-\dfrac{n^4}{4}+10n$

33. A line crosses the y-axis at $y=8$ and passes through the point $(1,4)$. Another line crosses the x-axis at $x=1$ and passes through the point $(-3,-3)$. The two lines intersect at (x,y). What is the value of x?

(A) $\dfrac{29}{19}$ (B) $\dfrac{29}{13}$ (C) $\dfrac{35}{19}$ (D) $\dfrac{29}{17}$ (E) $\dfrac{35}{13}$

34. The central angle of the sector illustrated is 56° and the radius of the circle is 7.54 cm. The area of the shaded region is _____ cm^2. (nearest hundredth)

(A) 178.60   (B) 27.78   (C) 48.01   (D) 56.86   (E) 150.82

35. Ned left his front porch and travelled 548 yards on a bearing of 52° then turned and travelled 372 yards on a bearing of 184°. How far will Ned have to travel to go directly back to his porch? (nearest foot)

(A) 2760 ft.   (B) 1222 ft.   (C) 920 ft.   (D) 1380 ft.   (E) 407 ft.

36. In the binomial expansion of $\left(2x^2-\dfrac{3}{x}\right)^6$ what is the coefficient of the constant term?

(A) 324   (B) 4860   (C) 81   (D) -1215   (E) -4320

37. Allen plays a prize game where he rolls a single dice. If he rolls a prime number he wins $3.00. If he rolls a composite number, he wins $10.00. Otherwise, he wins $40.00. What are Allen's expected winnings on a single throw?

(A) $11.50   (B) $8.83   (C) $6.50   (D) $9.00   (E) $8.75

38. The graph of the polar equation $r=4\sin(5\theta)$ is a

(A) Circle   (B) Lemniscate   (C) Rose curve   (D) Limacon   (E) Cardioid

39. $\det\begin{pmatrix} \cos A & \sin A \\ \cos B & \sin B \end{pmatrix} =$

(A) $\sin(A+B)$ (B) $\cos(A+B)$ (C) $\sin(B-A)$ (D) $\sin(A-B)$ (E) $\cos(B-A)$

40. Solve $\log_2(5x^2 - x - 2) = 2 + 2\log_2 x$.

(A) -1   (B) 3   (C) -2   (D) 2   (E) 4

41. Simplify $a^2 \times b^3 \times a^{-2} \div b^{-3} \div a^3 \times b^{-5}$.

(A) $\dfrac{b}{a^3}$ (B) $a^3 b$ (C) $\dfrac{a^3}{b^5}$ (D) $a^3 b^5$ (E) $\dfrac{1}{a^3 b}$

42. Find the cosine of the angle between the two vectors $u = 3i + 4j$ and $v = -2i + j$.

(A) $-\dfrac{2}{5}$ (B) $-\dfrac{2\sqrt{5}}{25}$ (C) $-\dfrac{3}{8}$ (D) $\dfrac{3\sqrt{2}}{4}$ (E) $\dfrac{\sqrt{5}}{5}$

43. Which "trapezoidal mean" can be used to find the volume of a frustum of a cone?
(A) Harmonic   (B) Geometric   (C) Centroidal   (D) Algebraic   (E) Heronian

44. An electronics store reduced the price of a television by 25% resulting in a profit of 5% over the cost of the television. What percentage profit would the store have made if the television had been sold for the original price?
(A) 55%   (B) 40%   (C) 20%   (D) 35%   (E) 50%

45. Solve $\dfrac{4}{3y} + \dfrac{2x}{7} = \dfrac{5}{9}$ for x.

(A) $\dfrac{7}{12y}$ (B) $-\dfrac{49}{18y}$ (C) $\dfrac{35y - 84}{18}$ (D) $\dfrac{35y - 84}{18y}$ (E) $-\dfrac{49}{18}$

46. The apothem of a regular hexagon has a length of 15.3. What is the area of the hexagon?
(A) 76.51   (B) 810.91   (C) 153.02   (D) 4213.62   (E) 1053.41

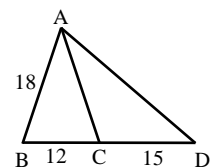47. The second term of an arithmetic sequence is 7 and the sum of the first four terms is 12. Find the first term.
(A) 15   (B) -8   (C) -8.5   (D) 15.5   (E) $\dfrac{40}{3}$

48. Find $f(2) - f(4) + f(-2)$ if $f(x) = \begin{cases} x+3, & x<0 \\ x^2, & 0 \le x \le 3 \\ \dfrac{x}{2}, & x>3 \end{cases}$

(A) 5   (B) 7   (C) 6   (D) 4   (E) 3

49. Given that $\overline{AB} \cong \overline{AC}$, find the area of triangle ABD.

(A) $162\sqrt{2}$ (B) 243   (C) $540\sqrt{5}$ (D) $81\sqrt{3}$ (E) $243\sqrt{2}$

50. The roots of $f(x) = x^3 + ax^2 + bx + c$ are -2, 4 and 11. $a + b + c =$
(A) 82   (B) 115   (C) 89   (D) 101   (E) -88

51. The chord $\overline{AB}$ has a length of 14 cm and the circle has a diameter of 50 cm. How far is $\overline{AB}$ from the center of the circle?
(A) 48 cm   (B) 21 cm   (C) 24 cm   (D) 45 cm   (E) 37 cm

52. Calculate $0.21 - \dfrac{0.21^3}{6} + \dfrac{0.21^5}{120} - \dfrac{0.21^7}{5040} \ldots$ to the nearest ten-thousandth.
(A) 0.9780   (B) 0.2131   (C) 0.9990   (D) 0.0037   (E) 0.2085

53. Quadrilateral ABCD has vertices $(-7,3)$, $(-4,6)$, $(5,5)$ and $(9,-2)$ respectively. What is the area of ABCD?
(A) 77   (B) 71   (C) 45   (D) 67   (E) 61

54. Solve $e^{2x} - 10e^x + 21 = 0$.
(A) $\ln 3, \ln 7$ (B) $0, \log 21$ (C) $\log 3, \log 7$ (D) $0, \ln 21$ (E) 3, 7

55. Let $f(x) = ax^7 - bx^3 - cx + 6$. If $f(3) = 15$ then $f(-3) =$
(A) -3   (B) -1   (C) 9   (D) -20   (E) -15

56. If $y = x^{x^2}$ find $\dfrac{dy}{dx}$.
(A) $2x^{x^2}(1 + \ln x)$ (B) $2x^{2x}$ (C) $4x^{2x}$ (D) $x^{x^2+1}(1 + \ln x^2)$ (E) $x^{2x}(2 + \ln x)$

57. How many solutions are there for $2x + 5y = 1342$ where x and y are both positive integers?
(A) 134   (B) 137   (C) 136   (D) 135   (E) 133

58. $2345_6 - 1234_5 = \underline{\qquad}_{10}$.
(A) -123   (B) 891   (C) 375   (D) 2029   (E) 1641

59. Ms. Angle must send 3 girls and 4 boys to the counselor for a survey. If her class has 8 girls and 9 boys, how many distinct groups could she send?
(A) 182   (B) 7056   (C) 1016064   (D) 19448   (E) 3360

60. On the diagram of circle F below, $2AE = EF$. If the area of triangle ABC is 78 cm^2, find the area of the shaded region.

(A) 26 cm^2 (B) 13 cm^2 (C) 39 cm^2 (D) 24 cm^2 (E) 52 cm^2
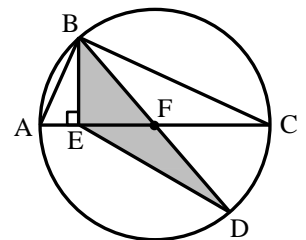
2014-2015 TMSCA Mathematics Test Two Answers

1. E	21. C	41. A
2. B	22. C	42. B
3. A	23. D	43. E
4. C	24. B	44. B
5. E	25. B	45. D
6. D	26. D	46. B
7. C	27. C	47. A
8. C	28. D	48. E
9. E	29. C	49. A
10. D	30. E	50. C
11. C	31. D	51. C
12. B	32. B	52. E
13. B	33. C	53. E
14. D	34. E	54. A
15. E	35. E	55. A
16. A	36. B	56. D
17. C	37. A	57. A
18. C	38. C	58. C
19. B	39. C	59. B
20. A	40. D	60. E

2013-2014 TMSCA Mathematics Test Two Select Solutions

3. Let her distance be 1 mile. Her weekly time will be $t = \left(\dfrac{1}{62} + \dfrac{1}{68} + \dfrac{1}{57} + \dfrac{2}{65}\right)$. Her average weekly speed will be $\dfrac{5}{t} \approx 63.17$

4. $_{(8+2-1)}C_2 \cdot 2 = 72$

9. $m\angle BFC = \dfrac{1}{2} mBC = 30\,°$

14. The combined rate is $\left(\dfrac{1}{8} + \dfrac{1}{6.4} + \dfrac{1}{5.6}\right)$ and the total job is 6 times the size of the original floor. Solve $\left(\dfrac{1}{8} + \dfrac{1}{6.4} + \dfrac{1}{5.6}\right)t = 6$
$t = 13$ h. 3 min.

20. Treat the 3 books as a block that can appear in 6 arrangements. Then do
$\dfrac{(7!)(6)}{9!} = \dfrac{1}{2}$

21. Complete the squares:
$(x+4)^2 + (y-3)^2 = 11+16+9 = 36$ so the radius is 6 and the circumference is 12π

24. $0.2424..._6 = \dfrac{16}{35}_{10}$ and
$2\dfrac{16}{35} = \dfrac{86}{35} = \dfrac{222}{55}_{\,6}$

28. Use the law of cosines
$14^2 = 16^2 + x^2 - 2(16)x\cos 60$
$0 = x^2 - 8x + 60$ the sum of the roots is 8.

30. $y - x = 6 - x - y$, so $y = 3$,
$3x - \dfrac{1}{3} = 3 - x$ and $x = \dfrac{5}{6}$. $x + y = 3.8\overline{3}$.

32. $\displaystyle\int_{-n}^{n} 5dx = [5x]_{-n}^{n} = 10n$ The other terms don't matter, because individually they are odd functions.

36. $_6C_4\left(2x^2\right)\left(-\dfrac{3}{x}\right)^4 = 4860$

39. $\cos A \sin B - \cos B \sin A = \sin(B - A)$

40. $\log_2\left(\dfrac{5x^2 - x - x}{x^2}\right) = 2$ so
$\left(\dfrac{5x^2 - x - x}{x^2}\right) = 4$ and $4x^2 = 5x^2 - x - 2$
which has two solutions 2 and -1, but -1 is not a solution to the original equation.

42. $\dfrac{(3)(-2) + (4)(1)}{\sqrt{3^2 + 4^2}\,\sqrt{2^2 + 1^2}} = \cos\theta = -\dfrac{2\sqrt{5}}{25}$

46. The area of a regular hexagon given the apothem is $2a^2\sqrt{3}$, so $A \approx 810.91$

49. $h^2 = 18^2 - 6^2 = 12\sqrt{2}$ so
$A = \dfrac{1}{2}(27)\left(12\sqrt{2}\right) = 162\sqrt{2}$

51. $\sqrt{25^2 - 7^2} = 24$

52. McClaurin series for $f(x) = \sin x$.
$f(0.21) \approx 0.2084598998$

54. $\left(e^x - 7\right)\left(e^x - 3\right) = 0$ so $e^x = 7$ or $e^x = 3$ and $x = \ln 7$ or $x = \ln 3$

55. Because of the odd nature of the first three terms let
$X + 6 = 15$ so $X = 9$ and
$-X + 6 = -9 + 6 = -3$

56. $\ln y = x^2 \ln x$, $\dfrac{1}{y}\dfrac{dy}{dx} = x^2\left(\dfrac{1}{x}\right) + \ln x \cdot 2x$
$\dfrac{dy}{dx} = y(x + 2x\ln x) = x^{x^2}(x + 2x\ln x)$

60 $EF = \dfrac{1}{3} AC$ so and the heights of triangles AEF and DEF are the same as the height of triangle ABC, so the area of triangle the shaded region is $\dfrac{2}{3}(78) = 52$