TMSCA HIGH SCHOOL MATHEMATICS TEST# 13 (UIL D) ©

MARCH 4, 2017

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

1. **Evaluate:** $(5 - 4) + 3^2 \times 11 \div (2^3 + 4) - 5$

   (A) $7\frac{3}{5}$ **(B)** $4\frac{1}{4}$ **(C)** $4\frac{1}{6}$ **(D)** $3\frac{1}{3}$ **(E)** $2\frac{1}{2}$

2. **Imin Shock's electric company charges a monthly base fee of $9.95 plus 7.5¢ per kilowatt for the first 1,000 kilowatts used and 5.5¢ per kilowatt for any kilowatts used over 1,000. What would his monthly bill be if he used 1,500 kilowatts?**

   (A) $20.20 (B) $102.45 (C) $112.45 (D) $168.75 (E) $204.95

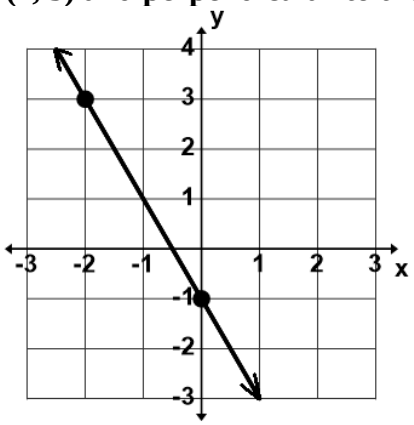
3. **If B= {b, o, a, t}, C = {c, a, n, o, e}, S = {s, c, o, w} and Y = {y, a, c, h, t} then $(Y \cap C) \cup (S \cap B)$ contains how many distinct elements?**

   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

4. **If P is 20% more than Q and Q is 30% less than R, then P is what percent of R?**

   (A) 36% (B) 50% (C) 60% (D) 84% (E) 90%

5. **An equation of a line through (4, 5) and perpendicular to the line shown is:**



   (A) $2x + y = 14$ **(B)** $2x - y = -8$ **(C)** $x + 2y = 13$ **(D)** $x - 2y = -14$ **(E)** $x - 2y = -6$

6. **Larry, Moe, and Curly were given $80.00 to share. Moe gets twice as much as Larry and Curly gets $5.00 less than Moe. How much did Curly get?**

   (A) $12.00 (B) $17.00 (C) $22.00 (D) $29.00 (E) $34.00

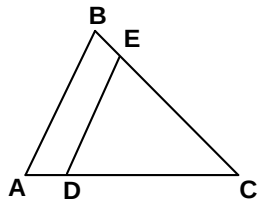
7. **If** $6x^2 + x - 35 = (2x + a)(3x + b)$ **then** $a + b =$ _____.

   (A) 36 (B) 12 (C) 2 (D) −1 (E) −2

8. **Simplify:** $14\sqrt[3]{3w^2} \div 7\sqrt[3]{24w^5}$

   (A) $\frac{1}{w}$ **(B)** $\frac{1}{2w}$ **(C)** $2w$ **(D)** $\frac{1}{4w}$ **(E)** w

9. $\triangle ABC$ and $\triangle CDE$ exist such that $\overline{AB} \parallel \overline{DE}$, AC = 5 cm, DC = 4 cm and BE = 2 cm. Find CE.



(A) 8 (B) 2.5 (C) 10 (D) 1.6 (E) not enough information given

10. The measure of an exterior angle of a regular heptagon is ___. (nearest degree)

(A) 129° (B) 120° (C) 60° (D) 51° (E) 7°

11. Which of the following mathematicians was known for their work in algebraic logic?

(A) Blaise Pascal (B) George Boole (C) Alicia Stott (D) John Napier (E) Leonard Euler

12. A box contains a dozen golf balls. A fourth of them are green, a third of them are yellow, and the rest of them are white. What are the odds of randomly selecting a white ball?

(A) $\frac{5}{12}$ (B) $\frac{7}{12}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$ (E) $\frac{5}{7}$

13. If $\frac{5x-3}{Ax+B} + \frac{2x-1}{x+2} = \frac{11x^2+14x-11}{(Ax+B)(x+2)}$, where A and B are constants, then A — B equals:

(A) 8 (B) 5 (C) 2 (D) — 1 (E) — 2

14. The graph of $x^2 + y^2 - 6x - 2y = 26$ is a(n):

(A) parabola (B) line (C) hyperbola (D) ellipse (E) circle

15. Saul Sage bought three dozen eggs and five loaves of bread that cost $6.10 on Monday. He bought four dozen eggs and two loaves of bread that cost $5.10 on Tuesday. What would Saul pay for two dozen eggs and three loaves of bread on Wednesday if the unit prices stayed the same during the week?

(A) $1.60 (B) $3.20 (C) $3.85 (D) $4.00 (E) $4.15

16. If $\sin x + \cos x = \frac{3}{4}$ then the $\tan x + \cot x$ = ? (nearest hundredth)

(A) — 4.57 (B) — 4.3 (C) — 4.11 (D) 0.75 (E) 2.81

17. Find a + b + c + d given the Fibonacci characteristic sequence: 3, a, b, 9, 15, c, d, ...

(A) 69 (B) 72 (C) 75 (D) 81 (E) 87

18. Using the following pattern of numbers, determine which of the following numbers will not be in row 10.

1	(row 0)
1 1	(row 1)
1 2 1	(row 2)
1 3 3 1	(row 3)
1 4 5 4 1	(row 4)
1 5 7 7 5 1	(row 5)
1 6 9 10 9 6 1	(row 6)
...	(row ...)

(A) 17 (B) 22 (C) 24 (D) 25 (E) 26

19. Which of the following expressions is not equal to 1?

(A) $\cot(\theta)\sin(\theta)\sec(\theta)$ **(B)** $\tan(\theta)\csc(\theta)\cos(\theta)$ **(C)** $\cos^2(\theta) + \sin^2(\theta)$

(D) $\sec^2(\theta) - \tan^2(\theta)$ **(E)** $\cot^2(\theta) - \sec^2$

20. Rusty Yaht sailed his boat on a bearing of 38° for 2 miles. Then he turned and sailed his boat on a bearing of 47° for 4 miles. What bearing would he have to sail his boat to go directly back to where he started?

(A) 230° (B) 224° (C) 220° (D) 217° (E) 183°

21. How many squares, various sizes, are there in the figure?

(A) 9 (B) 12 (C) 14 (D) 16 (E) 19

22. If $A = \begin{bmatrix} 2 & 1 \\ 0 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 0 & k \\ 1 & 7 \end{bmatrix}$, **and the determinant of AB is 3 then k = ?**

(A) $-\frac{3}{14}$ **(B)** $-\frac{3}{7}$ **(C)** $-\frac{1}{3}$ **(D)** $\frac{3}{7}$ **(E)** $\frac{1}{14}$

23. Let $f(x) = |x + 1| + |x - 2|$. **Find the minimum value of f(x).**

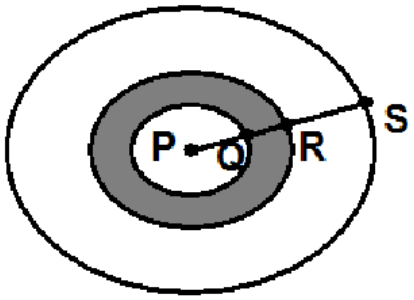
(A) -2 **(B)** -1 **(C)** -0.5 **(D) 1 (E) 3**

24. Let $f(x) = \begin{cases} (x-1)^{-1} & \text{if } x < 1 \\ x^3 - 2x + 5 & \text{if } x \geq 1 \end{cases}$. **Which of the following is/are true?**

1. $\lim_{x \to 1^+} f(x)$ **exists 2.** $\lim_{x \to 1^-} f(x)$ **exists 3.** $\lim_{x \to 1} f(x)$ **exists**

(A) none of these (B) 1 & 2 but not 3 (C) 1 only (D) 2 only (E) 1, 2, & 3

25. Arch Uhr shot an arrow at the target. The target consisted of three concentric circles with center P, PQ = 5", PR = 8", and PS = 19". What is the probability that the arrow hit in the shaded area shown? (nearest whole percent)



(A) 39 (B) 33% (C) 18% (D) 14% (E) 11%

26. Let $\frac{dy}{dx} = 3x^2 - x + 4$, and y = 5 when x = 0. Find y when x = — 1.

(A) — 5.5 (B) — 1 (C) — 0.5 (D) 5 (E) 8

27. Given the function $f(x) = x^3 - x^2 + x - 1$, the slope of the secant line between x = 0 and x = 2 is?

(A) 3 (B) 5 (C) 9 (D) — 1 (E) — 15

28. Let $f_1 = 1$, $f_2 = 3$, $f_3 = 4$, $f_4 = 7$, ..., $f_{20} = 15{,}127$, ... Find f_{23}.

(A) 64,079 (B) 103,682 (C) 73,063 (D) 73,428 (E) 39,603

29. $11101011_2 + 1001_4 + 11_8 =$ _____ 16

(A) 15D (B) 153 (C) ED (D) D5 (E) 135

30. Find k if GCF(k, 80) = 16 and LCM(k, 80) = 160.

(A) 16 (B) 32 (C) 48 (D) 64 (E) 96

31. If the roots of $x^3 + bx^2 + cx + d = 0$ are — 5, — 3, and 2, then $b + c + d$ equals:

(A) 37 (B) 35 (C) — 1 (D) — 6 (E) — 25

32. Tim Burr cut 4 branches from a tree. The lengths of the branches are 2 feet, 3 feet, 5 feet and 7 feet. How many different triangles can he make using three branches as a time?

(A) 4 (B) 3 (C) 2 (D) 1 (E) 0

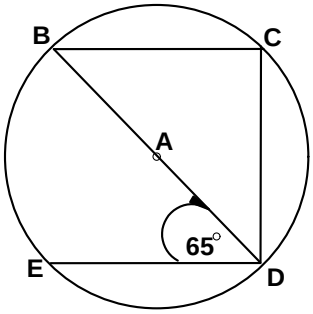
33. Let f(x) = 4 — x, g(x) = x — 3, h(x) = 2x + 1, and $f\big(g(\,h(x)\,)\big) = 6$. Find x.

(A) 0 (B) 1.5 (C) — 6 (D) 6 (E) — 3

34. The *Slow Snail* train leaves at 1:00 pm. The average speed of the *Snail* is 30 mph. The *Speed Burner* train leaves the same station on a parallel track at 2:00 pm. The average speed of the *Burner* is 75 mph. How long will it take the *Burner* to catch the *Snail*?

(A) 1 hr 40 min (B) 1 hr 23 min (C) 1 hr 15 min (D) 45 min (E) 36 min

35. Given the circle with center A shown. Find $m\overset{\frown}{ED}$.



(A) 25° (B) 50° (C) 65° (D) 90° (E) 130°

36. If $a_1 = -1$, $a_2 = 1$, $a_3 = 2$ and $a_n = (a_{n-1})(a_{n-3}) + (a_{n-2})$ where $n > 3$ then a_5 = ?

(A) -1 (B) 0 (C) 1 (D) 2 (E) 3

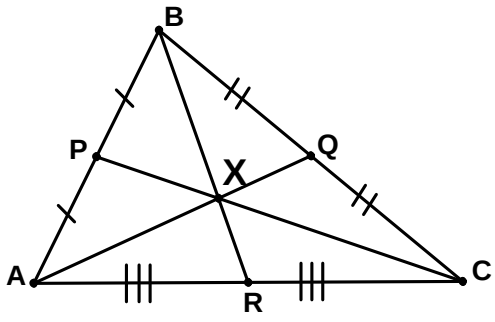
37. Given: $f(x) = 4\sin[3\pi(x + 2)] - 1$. What quadrant(s) would the graph of f(x) be in if the vertical displacement was decreased by 5 and the phase shift was increased by 5?

(A) I & II (B) I & IV (C) II & III (D) III & IV (E) I, II, III, & IV

38. Let $\triangle PQR$ exists such that $m\angle QPR = 45°$. Point A lies on $\overline{PR}$ and $\overline{QA}$ is the altitude $\triangle PQR$. Find $m\angle QRP$ if the area of $\triangle AQR$ is one and a half times the area of $\triangle AQP$. (nearest degree)

(A) 28° (B) 34° (C) 45° (D) 56° (E) 62°

39. The point of concurrency X of the triangle shown is called the:



(A) incenter (B) centroid (C) orthocenter (D) circumcenter (E) line of Euler

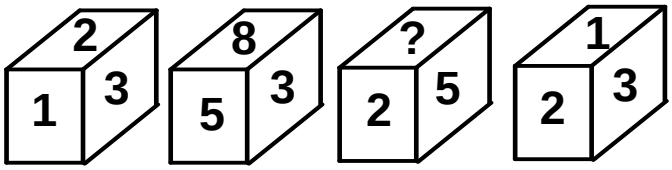
40. If $\log_2(x^2 - 6x) = 3 + \log_2(1 - x)$ then x is:

(A) 2 (B) 2.5 (C) — 3 (D) — 4 (E) — 4.5

41. The vertical asymptote and the oblique asymptote of $f(x) = \frac{x^2 + 3x + 2}{x - 2}$ intersect at point (x , y). Find the value of y.

(A) — 5 (B) — 3 (C) 2 (D) 7 (E) 9

42. Each face of a rectangular prism is labeled with the digits, 1, 1, 2, 3, 5, 8. Four views of the prism are shown. Which of the digits should appear on the face where the "?" is? (Ignore orientation of the digits.)



(A) 1 (B) 2 (C) 3 (D) 5 (E) 8

43. Find the digit in the thousandth place of the series $\frac{2^1}{1!} - \frac{2^3}{3!} + \frac{2^5}{5!} - \frac{2^7}{7!} + \ldots$.

(A) 0 (B) 2 (C) 4 (D) 7 (E) 9

44. The function $f(x) = x^4 - 4x^3$ is concave down on which of the intervals?
I. (0.5, 1.5) II. (— 1.5, — 0.5) III. (— 1.5, 0.5) IV. (— 0.5, 1.5)

(A) I only (B) II only (C) II & III (D) I & IV (E) II, III, & IV

45. Which of the following words has only 30 unique permutations of its letters?

(A) letter (B) green (C) tartar (D) level (E) odious

46. How many five letter distinguishable code words can be created from the letters in MULTIPLY such that the first letter is an M, the third letter is a vowel and the last letter is a Y?

(A) 8 (B) 9 (C) 12 (D) 18 (E) 24

47. Let $f_1 = 3$, $f_2 = 6$, $f_3 = 9$, $f_4 = 15$, ... be the terms of a Fibonacci characteristic sequence. Find f_{10}.

(A) 432 (B) 341 (C) 297 (D) 267 (E) 165

48. Simplify: If the pattern of the sequence, 1, 2, 5, 10, 17, 26, 37, ... , continues find the difference between the 19^{th} term and the 20^{th} term.

(A) 37 (B) 39 (C) 41 (D) 43 (E) 45

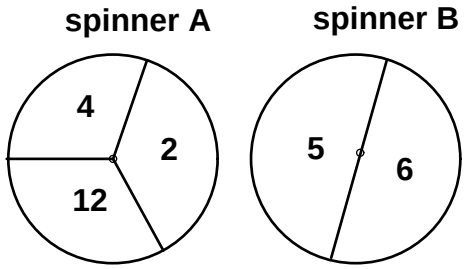
49. R. U. Shur was given the following problem for homework. Let $\oplus$ # be the prime number closest to # and let $\ominus$ # be the composite number closest to #. What should $\oplus 11 - \ominus 3 + \oplus 20 - \ominus 28$ equal?

(A) -1 (B) 0 (C) 1 (D) 2 (E) 3

50. Lotta Phence built a pen 12 yards by 8 yards for her sheep. Her brother, Les Phence, built a pen for his goats that is 3 yards longer in length and 2 yards shorter in width than Lotta's pen. How much more fence will Les need than Lotta?

(A) 21 yds (B) 14 yds (C) 6 yds (D) 5 yds (E) 2 yds

51. Spinner A is divided into three equal sectors and spinner B into two halves. Willie Wynn spins each spinner once. If A is bigger than B then Willie receives A — B dollars. If A is smaller than B then Willie loses B — A dollars. What is the mathematical expectation of playing the game many times?



(A) \$.50 (B) \$1.50 (C) \$2.00 (D) \$2.50 (E) \$3.00

52. The equation $x^2 + y^2 = 81$ written in polar equation form is:

(A) $\cos\theta + \sin\theta = 9$ (B) $3\cos\theta + 3\sin\theta = 1$ (C) $r = 9$ (D) $\theta = 9$ (E) $r = 9\theta$

53. If $y^2 = 2 - i$ and $y^3 = 3 + i$ where $y = a + bi$ then $a + b$ equals:

(A) -3 (B) -1 (C) 0 (D) 2 (E) 4

54. The fraction $\frac{11}{16}$ base 8 can be written as which of the following decimals in base 8?

(A) $0.555..._8$ (B) $0.5444..._8$ (C) $0.5111..._8$ (D) $0.1555..._8$ (E) $0.515151..._8$

55. Which of the following numbers is a deficient, odious number?

(A) 60 (B) 72 (C) 76 (D) 80 (E) 84

56. $\frac{1^2 + 2^2 + 3^2 + 4^2 + ... + 11^2 + 12^2}{364} =$ _____.

(A) 1 (B) $1\frac{11}{14}$ (C) $1\frac{3}{7}$ (D) $2\frac{1}{14}$ (E) $3\frac{1}{7}$

57. Let $g(x) = -(\sqrt{x})^{-1}$. Which of the following statements are true about g(x)?

I. domain is $(0, \infty)$ **II. asymptote at x = 0**

III. increasing on $(0, \infty)$ **IV. concave down on $(0, \infty)$**

(A) I, II & III (B) I, II & IV (C) I & II (D) all of them (E) none of them

58. Roland Bones tossed a fair 6-sided die 6 times. What is the probability that he rolled at least one 6? (nearest whole percent)

(A) 79% (B) 67% (C) 60% (D) 36% (E) 33%

59. Simplify: Let T_n be the nth triangular number, P_n be the nth pentagonal number, and H_n be the nth hexagonal number. Then $T_n + P_{(n+1)}$ has the same value as:

(A) $T_{(n+2)}$ (B) $H_{(n+1)}$ (C) $P_{(n+2)}$ (D) $T_{(n+1)}$ (E) $H_{(n+2)}$

60. Let $x = 2 + \cfrac{3}{2 + \cfrac{3}{2 + \cfrac{3}{2 + \cfrac{3}{2 + \ldots}}}}$. Find x. (nearest tenth)

(A) 2.666... (B) 3 (C) 3.232323... (D) 3.5 (E) 4

2016-17 TMSCA HS Math Test #13
Answer Key

1. B	**21. C**	**41. D**
2. C	**22. A**	**42. C**
3. D	**23. E**	**43. E**
4. D	**24. C**	**44. A**
5. E	**25. E**	**45. D**
6. D	**26. C**	**46. E**
7. E	**27. A**	**47. D**
8. A	**28. A**	**48. A**
9. A	**29. E**	**49. C**
10. D	**30. B**	**50. E**
11. B	**31. E**	**51. A**
12. E	**32. D**	**52. C**
13. E	**33. A**	**53. D**
14. E	**34. A**	**54. C**
15. C	**35. B**	**55. C**
16. A	**36. C**	**56. B**
17. B	**37. D**	**57. D**
18. C	**38. B**	**58. B**
19. E	**39. B**	**59. B**
20. B	**40. D**	**60. B**