# TMSCA HIGH SCHOOL

# MATHEMATICS

# TEST #12 ©

# MARCH 1, 2014

## GENERAL DIRECTIONS

1. About this test:
   A.  You will be given 40 minutes to take this test.
   B.  There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided.  If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**.  Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself.  You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer.  There is a penalty for all incorrect answers.
7. Calculators  used on this test must be conform to the UIL standards.  Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points.  **TWO** points will be deducted for all problems answered incorrectly.  No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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1. Evaluate the following to the nearest tenth:  $2 + 4! \div (8)^{\frac{2}{3}} - 16 \times 32^{(-1)}$

   (A) $-1.9$      (B) $-0.3$      (C) 0.6      (D) 4.0      (E) 7.5

2. Let D = {d,a,y}, M = {m,o,n,t,h},  Y = {y,e,a,r}, and C = {c,e,n,t,u,r,y}. The number of distinct elements  in  $(D \cup Y) \cap (M \cup C)$  is _____.

   (A) 12      (B) 13      (C) 5      (D) 3      (E) 2

3. The net shown is used to fold into a cube. What letter of the cube will be on the opposite side of side Q?

   |   |   |
   |---|---|
   | R | E |

   | Q | U | A |
   |---|---|---|
   | S |   |   |
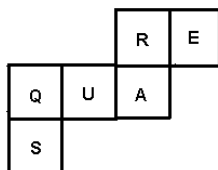
   (A) Q      (B) U      (C) A      (D) R      (E) E

4. Simplify:  $\dfrac{3x - 12}{x^2 + 8x + 16} \times \dfrac{4}{x^2 - x - 12} \div \dfrac{2x}{x^3 + 7x^2 + 12x}$

   (A) $\dfrac{6}{x+4}$      (B) $\dfrac{6(x-4)}{x+4}$      (C) $\dfrac{6}{x-4}$      (D) $\dfrac{x+3}{x-4}$      (E) $\dfrac{6x}{x+3}$

5. Which of the following equations has a graph of a line with a negative slope and does not contain points in quadrant I ?

   (A) $7x + 2y = 5$  (B) $5x + 7y = 2$  (C) $2x + 7y = 5$  (D) $2x - 7y = -5$  (E) $7x + 2y = -5$

6. Find k when $3 \div 7 \times 20 + 14 \times (3 - k) = 14$.

   (A) $2\frac{53}{60}$      (B) $11\frac{3}{10}$      (C) $10\frac{7}{10}$      (D) $2\frac{30}{49}$      (E) $19\frac{3}{7}$

7. The perimeter of a rectangle is 108 inches. The ratio of its width to length is 2:7. The area of the rectangle is:

   (A) $504 \text{ in}^2$      (B) $486 \text{ in}^2$      (C) $324 \text{ in}^2$      (D) $270 \text{ in}^2$      (E) $252 \text{ in}^2$

8. The measure of an interior angle of a regular pentagon is ___. (nearest degree)

   (A) $72°$      (B) $108°$      (C) $60°$      (D) $120°$      (E) $140°$

9. Points A, B, and C lie on a circle  with center O, and m$\angle$AOB $= 75°$. Find  m$\angle$ACB.

   (A) $105°$      (B) $37.5°$      (C) $75°$      (D) $150°$     (E)  not enough information

10. If you slice a cone with a plane that is parallel to the base of the cone, the intersection is a(n) __.

   (A) parabola      (B) ellipse      (C) circle      (D) hyperbola     (E) line

11. If $5^{(x+y)} = 125$ and $6^{(x-y)} = 36$ then xy equals _____.

    (A) 1.25        (B) 1.5        (C) 2.75        (D) 3        (E) 6

12. The graph best depicts Willie Makette's 6 hour hiking trip. During which of the following time intervals was Willie walking the slowest?



Distance (miles) vs Time (hours)
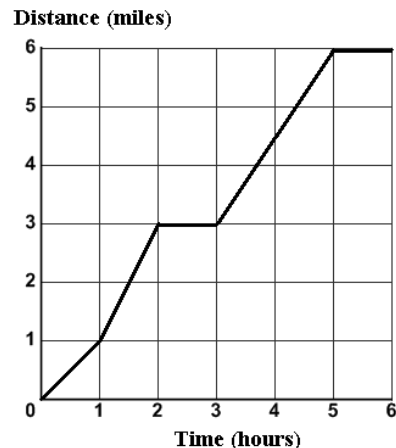
    (A) 0 - 1        (B) 1 - 2        (C) 2 - 3        (D) 3 - 5        (E) 5 - 6

13. Potter's McNuggets are sold in packs of 6, 9, 20, and in a happy meal pack of 4. What is the largest number of McNuggets that cannot be purchased using these packs?

    (A) 43        (B) 35        (C) 22        (D) 11        (E) 7

14. The end of a cable attached at ground level is 80 feet from the base of a cell phone tower. The other end of the cable is attached to a specific point on the tower. The angle of elevation of the cable is 60°. Another cable 40 feet from the base is attached at ground level and to the tower with an angle of elevation of 30°. How far apart are the two ends attached to the tower? (nearest foot).

    (A) 210 feet        (B) 143 feet        (C) 115 feet        (D) 89 feet        (E) 60 feet

15. Which of the following is equivalent to $\dfrac{\sin\theta}{\tan^2\theta} + \dfrac{\tan\theta}{\sec\theta}$ ?

    (A) $\sin^2\theta$        (B) $\csc\theta$        (C) $\cot^2\theta$        (D) $\cos\theta$        (E) 1

16. Willie Hittett throws a dart at the March calendar on the wall. Assuming the dart hit one of the dates on the calendar, what are the odds that the date was a Lucas number ( 2, 1, 3, ...)? Each date had an equal chance of being hit.

    (A) $\dfrac{4}{11}$        (B) $\dfrac{7}{24}$        (C) $\dfrac{8}{23}$        (D) $\dfrac{4}{15}$        (E) $\dfrac{8}{31}$

17. Which of the following is not a *polite number*?

    (A) 45        (B) 32        (C) 14        (D) 9        (E) 5

**18.** If $(3-i) \times (2+i) + (1-i) = a + bi$ then $a + b$ is:

    (A) 8     (B) 7     (C) 4     (D) $-1$     (E) $-9$

**19.** If the three numbers 112, 232, and 400 are each divided by the number D, each of their quotients will have the same remainder R. Find R where $R > 1$.

    (A) 2     (B) 3     (C) 5     (D) 7     (E) 16

**20.** Find C if the remainder of $2x^3 - 5x^2 + 7x + C$ divided by $x - 2$ is 3.

    (A) $-1$     (B) $-7$     (C) 1     (D) 3     (E) 11

**21.** Which of the following polar equations will produce the graph shown?
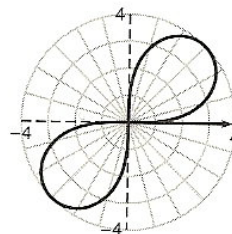


    (A) $r^2 = 16\cos 2\theta$   (B) $r = 4\sin 2\theta$   (C) $r^2 = 16\sin 4\theta$   (D) $r = 4\cos 2\theta$   (E) $r^2 = 16\sin 2\theta$

**22.** A water tank in the shape of an inverted cone has an altitude of 24 feet and a base radius of 12 feet. The cone is being emptied at the rate of 6 cubic feet per minute. How fast is the water level lowering when the water is 4 feet deep? (nearest tenth)

    (A) 1.3 in/min   (B) 2.8 in/min   (C) 4.0 in/min   (D) 5.7 in/min   (E) 7.6 in/min

**23.** The probability that statement P is true is 62.5%, and the probability that statement Q is true is 60%. Determine the probability that $P \rightarrow Q$ is true.

    (A) 75%     (B) 61.25%     (C) 50%     (D) 25%     (E) 38.5%

**24.** A sub-area of Algebra that led to the use of the digits 0 and 1 in the development of computer science was introduced by which of these mathematicians?

    (A) Boole     (B) Cantor     (C) Goldbach     (D) Leibniz     (E) Zeno

**25.** Two numbers exist such that their product is 24 and the sum of their squares is 409. Find the absolute value difference of the two numbers.

    (A) 21     (B) 20     (C) 19     (D) 18     (E) 17

**26.** A scalene triangle has side lengths of 5", 8", and k". How many integral values of k exist such that the scalene triangle is an obtuse triangle?

    (A) 2     (B) 4     (C) 5     (D) 7     (E) 9

27. Shee Noh Soh had 7 yards of material to make a matching blouse and skirt. Shee cut the material such that the ratio of the material for the blouse to the skirt was 5:4 with 9 inches left over. How much material did she use for the blouse?

   (A) 3 yds     (B) 3 yds 4 in     (C) 3 yds 2 ft 3 in     (D) 3 yds 2 ft 8 in     (E) 4 yds

28. The two lines shown intersect at point P(x,y). Find x + y.
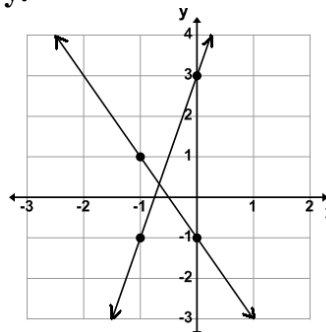


   (A) $-\frac{5}{12}$     (B) $-\frac{1}{6}$     (C) $-\frac{1}{2}$     (D) $-\frac{5}{9}$     (E) $-\frac{1}{3}$

29. An elongated pentagonal cupola has 25 vertices and 45 edges. How many faces does it have?

   (A) 35     (B) 32     (C) 28     (D) 22     (E) 18

30. Let vector $u$ = (3, 2) and vector $v$ = ($-2, 5$). Find the measure of the angle with initial side $u$ and terminal side $v$. (nearest minute)

   (A) 33° 41'     (B) 68° 12'     (C) 78° 7'     (D) 101° 53'     (E) 111° 48'

31. $(2\text{cis}\,(\frac{7\pi}{4})\,)^2$ equals:

   (A) $4 - 4i$     (B) $-4$     (C) $2i$     (D) $-4i$     (E) $2 - 4i$

32. Ten students from Take-A-Chance high school write their names on slips of paper, one name per slip, to enter a contest. Two names are drawn at random without replacement. If the probability that both winners are girls is $\frac{1}{15}$, how many of the ten students are girls?

   (A) 1     (B) 2     (C) 3     (D) 4     (E) 5
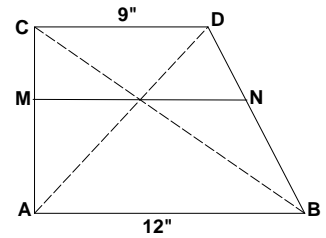
33. How many ordered pairs (a, b) exist such that the four-digit number, a04b, is divisible by both 8 and 9?

   (A) 1     (B) 2     (C) 4     (D) 6     (E) 17

34. Find the sum:  $\log_2 8 + \log_4 8 + \log_8 8 + \log_{16} 8 + \log_{32} 8$

   (A) 6.85     (B) 7.35     (C) 7.666...     (D) 8     (E) 8.85

**35.** Given the trapezoid shown where AB‖MN‖CD, find MN.   (nearest tenth)



(A) 10.3"          (B) 10.4"          (C) 10.5"          (D) 10.6"          (E) 10.7"

**36.** If $f''(x) = 80x^3 + 36x$ and $f'(1) = 40$ and $f(-1) = -11,$ then $f(1) = $ _____.

(A) 22          (B) 13          (C) 4          (D) 0          (E) $-1$

**37.** Find $\lim_{x \to 1} f(x)$, if it exists. Let $f(x) = \begin{cases} 2x + 3 & \text{if } x < 1, \\ 2 & \text{if } x = 1, \\ 7 - 2x & \text{if } x > 1. \end{cases}$

(A) $-1$          (B) 1          (C) 2          (D) 5          (E) does not exist

**38.** Wynn Agin lost 5 of the first 12 matches he played. How many of the remaining 8 matches must he win to have at least a 75% winning percentage for the season?

(A) 8          (B) 7          (C) 4          (D) 3          (E) 1

**39.** Uncle Donald gave his nephews, Huey, Dewey, and Louie, $\frac{2}{3}$ of a pizza for their lunch. Huey ate $\frac{1}{4}$ of what they were given and Dewey ate $\frac{1}{5}$ of what Louie ate. What fractional part of the pizza did Louie eat?

(A) $\frac{1}{30}$          (B) $\frac{5}{12}$          (C) $\frac{2}{5}$          (D) $\frac{7}{60}$          (E) $\frac{5}{8}$

**40.** Let p and q be the roots of $x^2 + 5x + 4 = 0$.  Find $p^3 + 3p^2q + 3pq^2 + q^3$.

(A) $-125$          (B) $-27$          (C) $-1$          (D) 9          (E) 25
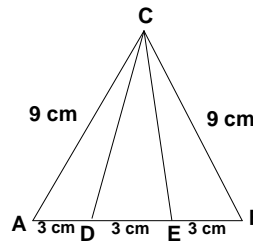
**41.** Determine the sum of all of the numbers in the first 10 rows of Pascal's triangle.

(A) 2,049          (B) 1,728          (C) 1,034          (D) 1,023          (E) 999

**42.** The function $f(x) = x^2 - 5x - 14$ crosses the x-axis at two points. Find the midpoint of the segment joining the two points.
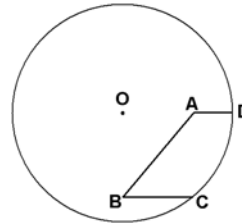
(A) (4.5, 0)          (B) (3, 0)          (C) (2.5, 0)          (D) (2.8, 0)          (E) (4.75, 0)

**43.** Find the perimeter of △CDE. (nearest tenth)



(A) 17.1 cm (B) 18.9 cm (C) 21.0 cm (D) 27.0 cm (E) 35.1 cm

**44.** Given the circle with center O. Points A, D, and O are collinear; $\overline{AD}\|\overline{BC}$; $\overline{AC} \perp \overline{OD}$; $\overline{OB} \perp \overline{OD}$. Find the diameter of the circle if AD = 5" and AB = 12 "



(A) 26" (B) 24" (C) 17" (D) 15" (E) 13"

**45.** Let $A = \begin{bmatrix} 1 & 3 \\ -5 & -7 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & -6 \\ 4 & -2 \end{bmatrix}$. Find $\left|(A + B)^T\right|$.

(A) 48 (B) 16 (C) $-64$ (D) $-78$ (E) $-84$

**46.** If (1, 1), (2, 3), and (k, 5) are members of the function $\{(x, y) \mid y = 4ax - b\}$, then k = ?

(A) 8 (B) 6 (C) 5 (D) 3 (E) 1.5

**47.** Sir Vayer is plotting three points on his map. The bearing of point B from A is 80°. The bearing of point C from point A is 160°. And, the bearing of C from B is 200°. He knows the distance from point A to point B is 2000 yards. How far is it from point A to point C? (nearest yard)

(A) 3,064 yds (B) 2,695 yds (C) 2,261 yds (D) 1,759 yds (E) 1,484 yds

**48.** The graph of the polar equation $r = 2 + 3\cos(\theta)$ intersects the x-axis on the Cartesian coordinate system. What is the sum of all of the x-values where the graph intersects?

(A) 6 (B) 5 (C) 3 (D) 2 (E) 1

**49.** In the expansion of $(3x - 2)^4$, the sum of the coefficients of the $2^{nd}$, $3^{rd}$ and $4^{th}$ term is:

(A) 192 (B) 81 (C) 14 (D) $-84$ (E) $-96$

**50.** The line tangent to the curve $f(x) = x^3 - 2x^2 + 3$ at the point (1,2) intersects the x-axis at x = ?

(A) $-1\frac{1}{3}$ (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3

51. If the set of odd numbers continue in the triangular array shown below, the median of the 14$^{th}$ row would be?

$$
\begin{array}{ccccc}
1 & & & & \\
3 & 5 & & & \\
7 & 9 & 11 & & \\
13 & 15 & 17 & 19 & \\
21 & 23 & 25 & 27 & 29 \\
\end{array}
$$
...

   (A) 169     (B) 182     (C) 188     (D) 196     (E) 225

52. Find the area (in square units) of the region bounded by the curve $y = x^3 - 2x^2 - 5x + 6$, the x-axis, and the lines $x = -1$ and $x = 2$.

   (A) $8\frac{1}{4}$     (B) $12\frac{3}{4}$     (C) $13\frac{1}{12}$     (D) 12     (E) $6\frac{11}{12}$

53. A special code is made up of five characters in a group. How many groups of five characters can be made from the letters of the word **THURSDAY** if four letters of the group must be consonants and one must be a vowel (a, e, i, o, u, y)?

   (A) 56     (B) 15     (C) 30     (D) 160     (E) 560

54. A bag of cash contains ten $5 bills, ten $10 bills, and ten $20 bills. Les Dough is blindfolded and told to draw out one bill at a time from the bag. He continues to draw one bill at a time until he has three of the same type of bills. What is the greatest amount of money will Les be able to draw?

   (A) $140.00     (B) $60.00     (C) $105.00     (D) $75.00     (E) $90.00

55. The repeating decimal 0.3111... in base 4 can be written in simplest form as which of the following fractions in base 4?

   (A) $\frac{30}{130}_4$     (B) $\frac{10}{12}_4$     (C) $\frac{31}{100}_4$     (D) $\frac{22}{30}_4$     (E) $\frac{11}{12}_4$

56. Willie Fennish participated in the 40 mile "Iron Man" challenge. He spent a fourth of the time swimming at 5 mph, a third of the time on a bicycle at 15 mph, and the rest of the time running at 10 mph. How long did it take Willie to finish the race? (nearest minute)

   (A) 1 hr 20 min  (B) 1 hr 36 min  (C) 3 hrs 30 min  (D) 3 hrs 50 min  (E) 4 hrs 15 min

57. The coordinates of the vertices of $\triangle$PQR are (6, 1), (5, 3) and (7, 8). The coordinates of the centroid of $\triangle$PQR is:

   (A) (6, 3.5)     (B) (4, 6)     (C) (6, 5.5)     (D) (4, 6.5 )     (E) ( 6, 4)

**58.** Let $f(x) = x - 3$ and $g(x) = x^2 - 9$ and $h(x) = x + 3$. Find $g\left(h\left(f(-3)\right)\right)$.

   (A) $-12$     (B) $-9$     (C) 0     (D) 3     (E) 30

**59.** The Mayan number system consists of three symbols,   ●   ▬   ◯ . The symbols have a value of 1, 5, and 0 respectively. They use base 20 instead of base 10 and write their numbers vertically instead of horizontally. What base 10 number would the following Mayan number be?
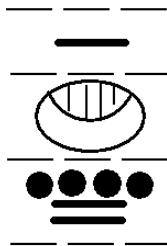


   (A) 5,014     (B) 422     (C) 50,455     (D) 524     (E) 2,014

**60.** The ratio of the radii of the three concentric circles is 1:2:4. What is the probability that an arrow shot at the target will hit the shaded section? The arrow does hit the interior of one or more of the circles. (nearest percent)
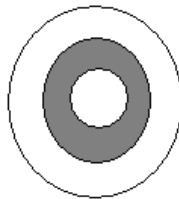


   (A) 6%     (B) 19%     (C) 25%     (D) 31%     (E) 33%

# 2013-14 TMSCA HS Math Test #12
## Answer Key

| # | Ans | # | Ans | # | Ans |
|---|-----|---|-----|---|-----|
| 1. | E | 21. | E | 41. | D |
| 2. | D | 22. | D | 42. | C |
| 3. | C | 23. | A | 43. | B |
| 4. | A | 24. | A | 44. | B |
| 5. | E | 25. | C | 45. | E |
| 6. | D | 26. | C | 46. | D |
| 7. | A | 27. | C | 47. | B |
| 8. | B | 28. | E | 48. | A |
| 9. | E | 29. | D | 49. | E |
| 10. | C | 30. | C | 50. | E |
| 11. | A | 31. | D | 51. | D |
| 12. | A | 32. | C | 52. | C |
| 13. | D | 33. | B | 53. | B |
| 14. | C | 34. | A | 54. | E |
| 15. | B | 35. | A | 55. | E |
| 16. | C | 36. | B | 56. | D |
| 17. | B | 37. | D | 57. | E |
| 18. | A | 38. | A | 58. | C |
| 19. | E | 39. | B | 59. | E |
| 20. | B | 40. | A | 60. | B |