T M S C A H I G H S C O O L

M A T H E M A T I C S

T E S T # 1 2 ©

F E B R U A R Y 2 8, 2 0 1 5

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

1. Evaluate: $4! \div (16)^{\frac{1}{2}} - 4 \times (16)^{-1} + 4 \times 16^{0}$

(A) 4.125 **(B)** 5.75 **(C)** 9.75 **(D)** 10.25 **(E)** 16

2. The *I. C. Ewe* glass store sells glasses and contacts. The store averages 25 customers per day. On average per day, 15 customers buy glasses, 8 buy glasses and contacts, and 2 don't buy anything. How many customers buy contacts only?

(A) 0 **(B)** 2 **(C)** 8 **(D)** 10 **(E)** 16

3. Find the greatest common divisor of 234, 456, and 789.

(A) 3 **(B)** 6 **(C)** 9 **(D)** 72 **(E)** 108

4. Simplify: $\left(a^{-2} \div b^{3}\right)^{-1} \times a^{4} \div b^{-5}$

(A) a^2b^{-2} **(B)** a^2b^2 **(C)** a^6b^{-8} **(D)** ab **(E)** a^6b^8

5. *Evil* and *Odious* numbers are based on _____ .

(A) the number of their factors **(B)** their prime factorization **(C)** the binary system
(D) their reciprocals **(E)** their politeness

6. Let Y vary inversely to X, and Y = $\frac{3}{4}$ when X = 2. Find Y when X = 5.

(A) $\frac{3}{10}$ **(B)** $\frac{2}{15}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{2}{5}$ **(E)** $\frac{3}{40}$

7. Willis Cheep needs to buy four tires for his ATV Four Wheeler. Which of the following is the cheapest deal before taxes?

(A) \$40.75 each **(B)** \$44.95 each and he has a 10% off coupon
(C) buy 3 at \$54.50 each and get 1 free **(D)** buy 2 at \$55.75 each and get 2 at half off
(E) buy 3 at \$48.00 and get 40% off of the 4th one

8. Which of the following properties, if any, is not used in this example ?

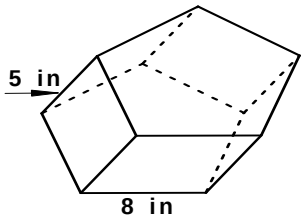
$3(k + 2) = 5$
$3k + 6 = 5$
$3k + 6 - 6 = 5 - 6$
$3k + 0 = -1$
$3k = -1$
$3k \times \frac{1}{3} = -1 \times \frac{1}{3}$
$k = -\frac{1}{3}$

(A) distributive **(B)** additive inverse **(C)** additive identity
(D) multiplicative identity **(E)** all of them are used

9. $F(x) = \frac{2x+1}{3x-2}$ **has an inflection point at x = ?**

(A) $\frac{2}{3}$ **(B) 0** **(C)** $-\frac{1}{2}$ **(D)** -1 **(E) does not exist**

10. Find the lateral surface area of the regular pentagonal prism shown. (nearest sq. in). Drawing is not to scale.



(A) 200 sq. in **(B) 184 sq. in** **(C) 368 sq. in** **(D) 280 sq. in** **(E) 292 sq. in**

11. The point (— 2, 5) lies on a circle whose center is (1, 3). Where does the point (2, — 1) lie in reference to the circle?

(A) on the circle **(B) outside the circle** **(C) inside the circle**
(D) in quadrant III **(E) cannot be determined**

12. A transversal intersects two parallel lines such that the measure of one of the interior angles is 38°. The measure of the opposite interior angle is _____.

(A) 19° **(B) 38°** **(C) 52°** **(D) 57°** **(E) 142°**

13. Let E be an even integer greater than 3, such that $P_m + P_n = E$, where P_m and P_n are prime numbers (note: $n \geq m$). The integer E is considered to be a _____ number.

(A) Euclidian **(B) Boolean** **(C) Archimedian** **(D) Euler** **(E) Goldbach**

14. If $\frac{5x+2}{4x+3} - \frac{3x-4}{2x-5} = \frac{Ax^2+Bx+C}{Px^2+Qx+R}$, **then** $\frac{A+B+C}{P+Q+R}$ **equals:**

(A) $\frac{2}{3}$ **(B)** $\frac{6}{7}$ **(C)** $1\frac{1}{2}$ **(D)** $2\frac{10}{21}$ **(E)** $3\frac{3}{7}$

15. Lynn Kahn has 40 coins consisting of nickels, dimes, and quarters. He has two times as many quarters as dimes and 4 more dimes than nickels. How much money does he have?

(A) $5.30 **(B) $5.70** **(C) $6.60** **(D) $6.95** **(E) $7.50**

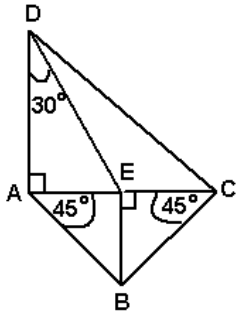
16. The number of integers between 1 and 60 that are relatively prime to 60 is ?

(A) 12 **(B) 15** **(C) 16** **(D) 20** **(E) 30**

17. If $0 < x < 1\frac{1}{2}$ **and** $1 - 2\cos^2(\frac{\pi}{2} - x) = \frac{1}{4}$ **then x is _____. (nearest hundredth)**

(A) 1.14 **(B) 0.89** **(C) 0.66** **(D) 0.52** **(E) 0.45**

18. Find m∠DCE, nearest degree, if AB = $\sqrt{8}$ cm.



(A) 22° **(B) 25°** **(C) 38°** **(D) 41°** **(E) 52°**

19. $(1+i)^7$ equals:

(A) $8-8i$ **(B) $7+7i$** **(C) $7\sqrt{2}+7\sqrt{2}\,i$** **(D) $7\sqrt{2}-7i$** **(E) $8+8i$**

20. Willie Finnish can paint a car in 6 hours. He painted the first hour by himself, then a friend, Betty Wont joins Willie to finish the job. They finish it in 4 hours instead of 6. How many hours would it have taken Betty do the whole job alone?

(A) 24 hrs **(B) 18 hrs** **(C) 12 hrs** **(D) 9 hrs** **(E) 6 hrs**

21. The sum of the coefficients of the 2nd term in the expansion of $(x+2)^2$, the 3rd term of $(x+2)^3$, the 4th term of $(x+2)^4$, and 5th term of $(x+2)^5$ is:

(A) 36 **(B) 128** **(C) 58** **(D) 192** **(E) 14**

22. Given that the set of natural numbers continue in the triangular pattern shown below, find the sum of the fourth number in row five and the seventh number in row eight.

			1				(row 1)
		2	3	4			(row 2)
	5	6	7	8	9		(row 3)
10	11	12	13	14	15	16	(row 4)
			...				(...)

(A) 68 **(B) 76** **(C) 92** **(D) 123** **(E) 135**

23. A particle is moving along the straight line with a function of $f(t) = 7t^2 - 11t - 15$, where f(t) is the distance in meters per second. Find the instantaneous rate of change at a time of 4 seconds.

(A) 32 m/s **(B) 37 m/s** **(C) 45 m/s** **(D) 53 m/s** **(E) 56 m/s**

24. U. R. Kahnfuzed needs help solving this problem. Let ⊕ k ⊕ be the largest prime number less than k and let ⊖ k ⊖ be the smallest prime number greater than k. What does ⊕ 3 ⊕ − ⊖ 14 ⊖ + ⊕ 20 ⊕ − ⊖ ⊕ 15 ⊕ ⊖ equal?

(A) −32 **(B) −13** **(C) −7** **(D) −6** **(E) −4**

25. If $f''(x) = 18x - 4$ and $f'(1) = 9$ and $f(1) = 4$, then $f(-1) =$ _____.

(A) 14 **(B)** 13 **(C)** -5 **(D)** -10 **(E)** -22

26. Find the odds of choosing a vowel from a letter box containing the letters from the word MATHEMATICIAN?

(A) $\frac{6}{13}$ **(B)** $\frac{3}{7}$ **(C)** $\frac{10}{13}$ **(D)** $\frac{3}{13}$ **(E)** $\frac{6}{7}$

27. The United States Supreme Court consists of 3 women and 6 men. In how many ways can the a four-member committee be formed if each committee must have at least one man?

(A) 243,000 **(B)** 3,024 **(C)** 936 **(D)** 270 **(E)** 126

28. If P, Q, and R represent digits then $RPQ_4 - QRP_3 + PQR_2$ has a numeric value in base 10 of:

(A) $7P - 3Q + 6R$ **(B)** $-3P - 10Q + 11R$ **(C)** $3P + 3Q + 3R$
(D) $7P - 6Q + 14R$ **(E)** $11P - 12Q + 21R$

29. Golf balls were numbered 1 to 20, put into a bag, and mixed up. Tye Gurr drew a golf ball at random from the bag. What is the probability that the ball Tye drew has a number which is a multiple of 3 or 5?

(A) 50% **(B)** 45% **(C)** 60% **(D)** $35\frac{3}{5}\%$ **(E)** $53\frac{2}{5}\%$

30. Line *m* intersects the x-axis at $x = -2$ and the y-axis at $y = 3$. Which of the following equations of line *n* exists such that $m \perp n$.

(A) $3x + 2y = 5$ **(B)** $2x - 3y = -5$ **(C)** $5x + 3y = -2$ **(D)** $3x - 2y = -5$ **(E)** $2x + 3y = 5$

31. The equation $5x^2 - 4x + k = 0$ always has two positive roots when which of the following is true?

(A) $-1.25 < k < 1.25$ **(B)** $0.8 > k > 0$ **(C)** $k > 1.25$
(D) $k < 0.8$ **(E)** $-1.25 < k < 0.8$

32. How many acute triangles with integral sides exist if the side lengths are 7", 11" and x"?

(A) 4 **(B)** 5 **(C)** 8 **(D)** 10 **(E)** 12

33. Find the area in square units of a scalene triangle whose vertices are $(-1, -2)$, $(4, 1)$, and $(1, 5)$. (nearest tenth)

(A) 12.8 **(B)** 13.5 **(C)** 14.0 **(D)** 14.5 **(E)** 14.8

34. Let $A = \begin{bmatrix} -1 & 1 \\ 2 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$. Find $\left|(A - B)^T\right|$.

(A) 9 **(B)** 4 **(C)** 0 **(D)** 11 **(E)** -1

35. Bill Spender has one-dollar bills, five-dollar bills, ten-dollar bills, and twenty-dollar bills. Bill puts seven bills in each envelope for his daily spending. How many different envelopes of money can he put together?

(A) 840 (B) 330 (C) 210 (D) 120 (E) 35

36. Let x and y exist such that $4 < y < x$. If 4, 8, x form an arithmetic sequence and y, x, 18 form a geometric sequence then xy = ?

(A) 144 (B) 216 (C) 96 (D) 48 (E) 384

37. How many elements are in $\{x \mid \cos(2x) = \sin(x)\ x \in [0, 2\pi)\}$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

38. The *U.S.S. Poly Gawn* sailed 120 miles from Port Trye on a bearing of 325° to Port Qwad, then sailed 100 miles from Port Qwad on a bearing of 75° to Port Penta. What bearing would the *Poly Gawn* have to sail to return directly to Port Trye? (nearest degree)

(A) 250 ° (B) 242 ° (C) 230 ° (D) 193 ° (E) 167 °

39. Find C if the remainder of $2x^3 - 5x^2 + 6x + C$ divided by $x + 3$ is 4.

(A) -23 (B) -18 (C) 30 (D) 121 (E) 72

40. Find the digit in the millionths place of the sum of the series $1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots$

(A) 9 (B) 7 (C) 6 (D) 4 (E) 3

41. Which of the following surfaces is generated by $x^2 - y^2 + z^2 = 0$?

(A) elliptic hyperboloid of one sheet (B) ellipsoid (C) elliptic cone
(D) elliptic paraboloid (E) elliptic hyperboloid of two sheets

42. Let a, b, and c be real numbers such that $c = a + b + 6$, $c^2 = a^2 + b^2$, and $ab = 6$. Find the numerical value of 6c.

(A) 2 (B) 6 (C) 12 (D) 26 (E) 3

43. If $a_1 = -3$, $a_2 = 1$, $a_3 = 2$, and $a_n = [(a_{n-3})(a_{n-1})] \div (a_{n-2})$, where $n \geq 4$, then a_8 = ?

(A) -6 (B) -3 (C) -1 (D) 2 (E) 1

44. If you connect the centers of all the faces of a dodecahedron you will form a(n):

(A) octahedron (B) icosahedron (C) rectangular prism
(D) square pyramid (E) decahedron

45. If $16^{(x+y)} = 256$ and $8^{(x-y)} = 64$ then $x^2 - y^2$ equals _____.

(A) 0 (B) 1 (C) 4 (D) 9 (E) 16

46. How many 3-digit numbers exist such that the sum of their digits equals 6?

(A) 15 (B) 18 (C) 21 (D) 24 (E) 27

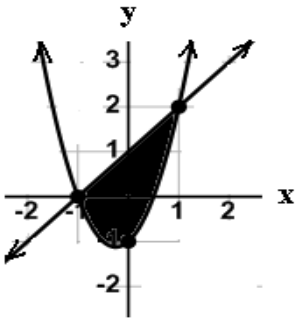
47. Let $4x^5 - 5x^4 + 2x^3 - 6x^2 + x + 15 = 0$. According to Descartes' Rule of Signs how many possible negative roots are there?

(A) 1, 3, or 5 (B) 1 or 3 (C) 0, 2, or 4 (D) 2 or 0 (E) 1

48. Which of the following equations in rectangular form can be written as $r + 8 \sin\theta = 0$ in polar form?

(A) $x^2 + (y+4)^2 = 16$ (B) $x^2 + y^2 = 64$ (C) $x^2 + y^2 = 2\sqrt{2}$
(D) $(x+4)^2 - y^2 = 16$ (E) $x^2 + (y+2)^2 = 8$

49. Find the area of the shaded region between the line with the two points shown and the parabola whose x-intercepts are $(-1, 0)$ and $(\frac{1}{2}, 0)$. in square units.



(A) $4\frac{1}{3}$ (B) $3\frac{1}{2}$ (C) $3\frac{5}{6}$ (D) $2\frac{3}{4}$ (E) $2\frac{2}{3}$

50. Willie Pikette puts 3 red chips, 4 blue chips, and 5 white chips in a bag. He randomly chooses 2 chips, without replacement. What is the probability that the 2 chips Willie chose were the same color? (nearest per cent)

(A) 17% (B) 29% (C) 34% (D) 58% (E) 73%

51. $2015_7 - 314_7 - 413_7 =$ _____ $_7$

(A) 265 (B) 321 (C) 462 (D) 561 (E) 655

52. Millie Whatt has white bulbs and yellow bulbs in a bin. The probability that she will select a white bulb is 40%. If 20 white bulbs are added to the bin, the probability of selecting a white bulb becomes 60%. How many yellow bulbs are in the bin?

(A) 24 (B) 30 (C) 32 (D) 36 (E) 40

53. Let $f(x) = \frac{(1-x)^3}{x^2}$ and s(x) be the slant asymptote of f. Find the value of $s(-1)$.

(A) 8 **(B)** 6 **(C)** 4 **(D)** -1 **(E)** -2

54. $\triangle DEF$ exists such that $m\angle DEF = 90°$ and point M is the midpoint of segment DF. If EM = 20.5 cm, and DE = 40 cm, find $\sin(\angle FME)$. (nearest hundredth)

(A) 0.43 **(B)** 0.34 **(C)** 0.17 **(D)** 0.08 **(E)** 0.04

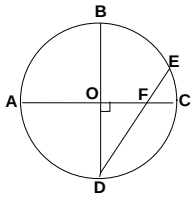
55. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. Find $(f_6)^2 + (f_7)^2$.

(A) 377 **(B)** 233 **(C)** 169 **(D)** 144 **(E)** 85

56. If $x + \frac{1}{x} = 5$, then $x^3 + \frac{1}{x^3} = ?$

(A) 125 **(B)** 120 **(C)** 115 **(D)** 110 **(E)** 105

57. Given the circle O with perpendicular diameters and a chord, find the shortest distance from the origin to the segment DF if EF = 6" and DE = 14" inches. (nearest tenth)



(A) 2.0" **(B)** 2.2" **(C)** 2.6" **(D)** 3.0" **(E)** 3.4"

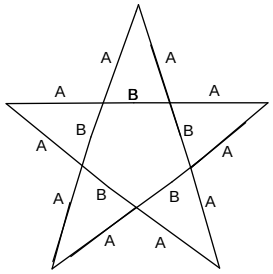
58. $\left\{(x, y) \middle| x,y \in \{\text{Integers}\}, -6 \le x \le 9, \text{ and } -9 \le y \le 6\right\}$ is the solution set of $3x + 2y = 15$. How many such ordered pairs exist?

(A) 6 **(B)** 5 **(C)** 4 **(D)** 3 **(E)** 2

59. The number 321 in base 4 is equivalent to the number k in base 2. Find the sum of the digits in the number k.

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

60. Given the pentagram shown, find B if A = 4". (nearest tenth)



(A) 2.8" **(B)** 2.2" **(C)** 2.6" **(D)** 2.0" **(E)** 2.5"

2014-15 TMSCA HS Math Test #12
Answer Key

1. C	**21. B**	**41. C**
2. C	**22. B**	**42. C**
3. A	**23. C**	**43. A**
4. E	**24. B**	**44. B**
5. C	**25. D**	**45. C**
6. A	**26. E**	**46. C**
7. B	**27. E**	**47. E**
8. E	**28. D**	**48. A**
9. E	**29. B**	**49. E**
10. A	**30. E**	**50. B**
11. B	**31. B**	**51. E**
12. B	**32. B**	**52. A**
13. E	**33. D**	**53. C**
14. A	**34. D**	**54. A**
15. D	**35. D**	**55. B**
16. B	**36. C**	**56. D**
17. C	**37. D**	**57. C**
18. D	**38. D**	**58. B**
19. A	**39. D**	**59. C**
20. A	**40. A**	**60. E**