

**TMSCA HIGH SCHOOL
MATHEMATICS
TEST # 12 ©
FEBRUARY 23, 2013**

GENERAL DIRECTIONS

1. About this test:
 - A. You will be given 40 minutes to take this test.
 - B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

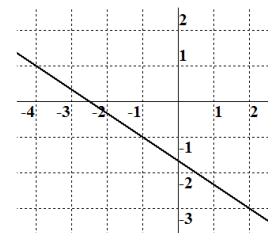
2012-2013 TMSCA High School Mathematics

1. What is $58.\overline{3}\% \div \frac{7}{9} + 0.375$?

- (A) $\frac{9}{8}$ (B) $\frac{13}{8}$ (C) $\frac{297}{280}$ (D) $\frac{2899}{7560}$ (E) $\frac{303}{4}$

2. The equation of the line on the graph shown on the right is

- (A) $2x - 3y = 5$ (B) $2x - 3y = 13$ (C) $2x - 3y = 8$
 (D) $2x + 3y = 13$ (E) $2x + 3y = -5$



3. The intersection of the medians of a triangle is the _____.

- (A) median (B) centroid (C) orthocenter (D) circumcenter (E) incenter

4. Chicago, IL and Houston, TX are approximately 1080 miles apart. An airplane flying with the wind can fly from Chicago to Houston in 2 hours and 30 minutes. An airplane flying against the same wind can make the return trip in 3 hours and 12 minutes. If the speed of the plane and the speed of the wind remain constant through both trips, the speed of the wind is

- (A) 47.25 mph (B) 37.50 mph (C) 384.75 mph (D) 61.71 mph (E) 407.86 mph

5. A cylindrical tank has a diameter of 2.5 yards and a height of 85 inches? If it is filling with water at a rate of 11 gal/min, how long will it take to fill?

- (A) 7 hr 6 min (B) 3 hr 33 min (C) 4 hr 24 min (D) 2 hr 12 min (E) 5 hr 34 min

6. The polynomial $x^2 + 4x - 21$ is a factor of $x^3 + ax^2 + (2a - 7)x - 231$. Find the value of the constant a .

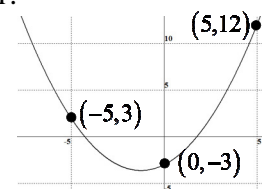
- (A) 7 (B) -11 (C) 18 (D) 3 (E) 15

7. Find the value of k for which the system of equations $3x - ky = 11$ and $-9x + (2k - 4)y = 13$ has no solutions.

- (A) 4 (B) 3 (C) -4 (D) -3 (E) 2

8. Gary is playing a game where he will draw a card from a standard deck then roll a die. In order to win, he must draw a face card and roll a prime number. What is the probability that he will win?

- (A) $\frac{3}{13}$ (B) $\frac{3}{26}$ (C) $\frac{1}{8}$ (D) $\frac{3}{52}$ (E) $\frac{1}{8}$



9. What is the sum of the roots of the parabola shown on the right?

- (A) $-\frac{5}{2}$ (B) $-\frac{2}{5}$ (C) $\frac{15}{7}$ (D) $\frac{2}{5}$ (E) $-\frac{15}{7}$

10. Two teams are going to play a series of four games. The probability that team A will win an individual game is 0.25 and the probability that team B will win an individual game is 0.65. What is the probability that the series will be a tie?

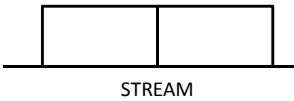
- (A) 0.178 (B) 0.169 (C) 0.125 (D) 0.158 (E) 0.163

11. If $f(x) = x^3$ and $g(x) = x - 2$, then $g(f(2x - 1)) =$

- (A) $8x^3 - 4x^2 + 3x - 3$ (B) $8x^3 - 12x^2 + 6x - 3$ (C) $8x^3 - 12x^2 + 6x - 1$
 (D) $8x^3 - 4x^2 + 3x - 1$ (E) $8x^3 - 4x^2 + 3x - 2$

12. A belt joins two pulleys. The larger pulley has a radius of 92 cm and revolves at a rate of 15 rpm. The smaller has a diameter of 9 cm. How fast is the smaller pulley revolving?
- (A) 75 rpm (B) $\frac{230}{3}$ rpm (C) 153 rpm (D) 150 rpm (E) $\frac{920}{3}$ rpm
13. $(\sin x - \cos x)^2$
- (A) $1 - \sin 2x$ (B) $1 + \sin 2x$ (C) $\sin 2x - 1$ (D) $1 + 2 \sin x$ (E) $2 \sin x - 1$
14. Two lines have vector equations $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} + t \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ -7 \end{pmatrix} + s \begin{pmatrix} 2 \\ -5 \end{pmatrix}$. What is the position vector of the intersection of the two lines?
- (A) $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ (B) $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ (C) $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ (D) $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ (E) $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$
15. Evaluate: $\lim_{\theta \rightarrow 0} \frac{\theta}{\sin(4\theta)}$
- (A) 4 (B) $\frac{1}{4}$ (C) $-\frac{1}{4}$ (D) 1 (E) does not exist
16. The numbers $a, 6, b, 23$ are arranged in order. The set has a range of 45 and a mean of 4. Find the value of b .
- (A) -3 (B) 3 (C) -22 (D) 6 (E) 9
17. $-8 - 9 - 4 + 7 + 24 + \dots + 376 =$
- (A) 1166 (B) 771 (C) 855 (D) 1055 (E) 1534
18. There were 1230 tickets sold for a concert. The full price of the tickets was \$24.50. Some of the tickets were sold at a 30% discount for students and some were sold at a 20% discount at the last minute. If there were twice as many tickets sold at full price than at a discount and the total sales totaled \$27513.50, how many student tickets were sold?
- (A) 820 (B) 180 (C) 160 (D) 250 (E) 220
19. A quadrilateral has vertices $(-3, -4)$, $(-2, 7)$, $(4, 10)$ and $(6, 1)$ respectively. Find the area of the quadrilateral.
- (A) 38 (B) 69 (C) 76 (D) 19 (E) 77
20. Triangle ABC has vertices $A(9, 9)$, $B(3, 2)$, and $C(9, 4)$. Find the equation of the perpendicular bisector of \overline{BC} .
- (A) $x + 3y = -3$ (B) $3x + y = 11$ (C) $x - 3y = -3$ (D) $3x + y = 21$ (E) $3x - y = 15$
21. Solve $\sin \theta = -\sin^2 \theta$, for $0 < x \leq \pi$.
- (A) $\frac{3\pi}{2}$ (B) $\frac{\pi}{2}$ (C) 0 (D) π (E) no solution
22. A box contains 35 red discs and 5 black discs. A disc is selected at random and its color noted. The disc is then replaced in the box. If this process is repeated eight times. What is the probability that red disc will be drawn exactly five times?
- (A) 0.625 (B) 0.513 (C) 0.001 (D) 0.002 (E) 0.056

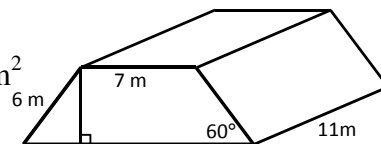
23. What is the slope of the line normal to $3x^2 - 2y^2 + 2x - y = 15$ at the point $(2, -1)$?
- (A) $\frac{14}{3}$ (B) $-\frac{8}{3}$ (C) $\frac{8}{3}$ (D) $\frac{3}{8}$ (E) $\frac{3}{14}$
24. On triangle ABC , $AB = 80$ cm, $BC = 70$ cm, and $m\angle A = 60^\circ$. What are the two possible lengths of AC ?
- (A) 50 cm, 40 cm (B) 30 cm, 40 cm (C) 40 cm, 70 cm (D) 30 cm, 50 cm (E) 50 cm, 70 cm
25. The fourth number in the 32nd row of the triangle shown on the right is the sum of the first ____ triangular numbers.
- (A) 28 (B) 29 (C) 30 (D) 31 (E) 32
- $$\begin{array}{ccccccc} & & & & & & 1 \\ & & & & & & 1 & 1 \\ & & & & & & 1 & 2 & 1 \\ & & & & & & 1 & 3 & 3 & 1 \end{array}$$
26. The ratio of sides of triangle is 8:14:19. What is the measure of the largest angle to the nearest hundredth of a degree?
- (A) 72.54° (B) 76.80° (C) 46.78° (D) 116.80° (E) 41.12°
27. The graph of $f(x) = \frac{1}{2} \sin\left(\frac{1}{3}x - \frac{\pi}{2}\right)$ has a phase shift of
- (A) $\frac{2\pi}{3}$ (B) 3π (C) π (D) $\frac{3\pi}{2}$ (E) $\frac{\pi}{2}$
28. Brigit wants to find three-digit numbers such that the hundreds digit is three times the units digit and the tens digit is the square of the units digit. What is the sum of all such numbers?
- (A) 862 (B) 993 (C) 1820 (D) 953 (E) 1946
29. What is the solution set for $|2x + 13| > 9$?
- (A) $\{x | -11 < x < 2\}$ (B) $\{x | \{x < -11\} \cup \{x > -2\}\}$ (C) $\{x | -11 < x < -2\}$
 (D) $\{x | \{x < -2\} \cup \{x > 11\}\}$ (E) none of these
30. What is the area of the ellipse with the equation $7x^2 + 3y^2 - 42x + 6y = -45$?
- (A) $\frac{\pi\sqrt{21}}{4}$ (B) $\frac{21\pi}{2}$ (C) $\pi\sqrt{21}$ (D) $\frac{\pi\sqrt{21}}{2}$ (E) 21π
31. A ship is initially at a port A and travels 63 km on a bearing 55° followed by 87 km on a bearing of 150° to reach port B, then finally 212 km on a bearing of 110° to port C. After this journey, the captain wants to return directly to position A? To the nearest kilometer, how far will she have to travel?
- (A) 315 km (B) 107 km (C) 135 km (D) 201 km (E) 362 km
32. Express the complex number $(1 - i\sqrt{3})^{10}$ in standard form.
- (A) 512 (B) $512 + 512i\sqrt{3}$ (C) $512 - 512i\sqrt{3}$ (D) $-512 + 512i\sqrt{3}$ (E) $-512 - 512i\sqrt{3}$
33. The point (x, y) is a point of inflection on $f(x) = \frac{\cos x}{1 + \sin x}$, where $0 < x \leq 2\pi$. Find the value of x .
- (A) 0 (B) π (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$ (E) 2π
34. The length of one edge of a regular dodecahedron is 4 in. What is the volume to the nearest cubic inch?
- (A) 140in^3 (B) 490in^3 (C) 315in^3 (D) 330in^3 (E) 139in^3

35. The point $M(k, -6)$ is the midpoint of the line segment with endpoints $P(-22, 17)$, $Q(16, -29)$. Find the value of k .
- (A) -6 (B) 6 (C) 3 (D) -3 (E) 9
36. John is at the candy store buying a box of chocolates. There are six types of caramels, twelve types of soft-centers and five types of toffees. A box comes with three different caramels, eight different soft-centers and one toffee. How many different combinations of chocolate are available for one box?
- (A) 24750 (B) 9900 (C) 3060 (D) 360 (E) 49500
37. Solve $15e^x - e^{2x} = 56$
- (A) $\log 56, 0$ (B) $\log 8, \log 7$ (C) $\ln 8, \ln 7$ (D) $\ln 56, 0$ (E) 8, 7
38. A regular octagon has vertices A, B, C, D, E, F, G and H respectively. Find the measure of $\angle BFD$.
- (A) 60° (B) 36° (C) 45° (D) 135° (E) 90°
39. What is the length of side a on the diagram shown right?
- (A) $3\sqrt{15}$ (B) $\frac{9\sqrt{30}}{2}$ (C) $9\sqrt{15}$ (D) $\frac{9\sqrt{15}}{2}$ (E) $3\sqrt{30}$
40. If $\frac{8x^2 - 4x + 17}{x^3 - 3x^2 + 2x - 6} = \frac{A}{x-3} + \frac{Bx+C}{x^2+2}$, then $A+B+C =$
- (A) 0 (B) 2 (C) 7 (D) 9 (E) 5
41. Seventy-five miles per hour equals _____ inches per second.
- (A) 79200 (B) 110 (C) 7920 (D) 1320 (E) 6600
42. $\frac{x^3 - 7x^2 - 4x + 28}{x^3 - 21x^2 + 147x - 343} \div \frac{x+2}{x^2 - 49} =$
- (A) $x^2 - 5x - 14$ (B) $\frac{x^2 + 5x - 14}{x-7}$ (C) $\frac{x^2 - 5x - 14}{x-7}$
- (D) $\frac{x^2 + 9x + 14}{x-7}$ (E) $\frac{x^2 + 9x + 14}{x+7}$
43. The probability that Joy will get a multiple choice question right is 0.25. If she guesses on every question on a 15 question quiz, what is the probability that she will make above a 30%?
- (A) 0.686 (B) 0.836 (C) 0.164 (D) 0.096 (E) 0.314
44. The point $(7, 12)$ is thirty units away from the point $(31, y)$. What are the two possible values of y ?
- (A) 6, 24 (B) -6, 30 (C) -6, -24 (D) 6, 30 (E) -6, -30
45. A farmer has 1000 m of fencing to fence a rectangular field with one division down the middle. One side of the field doesn't need fencing because it is bounded by a stream as shown in the diagram. What is the area of the largest field he can fence?
- (A) 8000 m^2 (B) 41667 m^2 (C) 83333 m^2 (D) 80000 m^2 (E) 75000 ft^2
- 
46. The sum of the first n terms of a sequence is given by $S_n = 2n^2 - 3n$, where $n \in \mathbb{Z}^+$. Find the sixth term of the sequence.
- (A) 27 (B) 19 (C) 44 (D) 13 (E) 17
47. Three ditch diggers can dig a hole 5 ft by 8 ft by 3 ft in four hours. How long will it take four diggers digging at the same individual rate to dig a hole three times as long, wide and deep?
- (A) 15 hours (B) 108 hours (C) 48 hours (D) 27 hours (E) 81 hours

48. Mr. Data gives a ten question quiz to his class. When he is done grading, he gives the following frequency table to his class and offers extra credit to the first student to find the mode. What is the mode of number of questions the students got right on the quiz?

Questions Right	2	3	4	5	6	7	8	9	10
Number of Students	1	4	2	5	4	8	4	2	1

- (A) 6 (B) 6.5 (C) 3.44 (D) 6.03 (E) 7
49. The line $y = 20x - 7$ is tangent to the curve $y = 5x^3 + ax^2 + bx + 11$ at the point $(1, 13)$. Find the value $a + b$.
 (A) -3 (B) -7 (C) 7 (D) -11 (E) 3
50. The vectors u and v are given by $u = 3i - 5j$, $v = i - 2j$. Scalars a and b are such that $a(u + v) = 12i + (b - 2)j$. Find the value of b .
 (A) 3 (B) -7 (C) 4 (D) -19 (E) 2
51. Find the units digit of 107^{2012} .
 (A) 3 (B) 1 (C) 7 (D) 0 (E) 9
52. If $\frac{x-9}{x+11} + \frac{x+11}{x-9} = A + \frac{B}{(x+11)(x-9)}$, then $B =$
 (A) 4 (B) 81 (C) 400 (D) 121 (E) 324
53. What is the 10^{-8} digit in the expansion of $1 + x - \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} \dots$ when $x = 4$
 (A) 9 (B) 2 (C) 2 (D) 0 (E) 1
54. If $y = x^{-x}$, then $\frac{dy}{dx} =$
 (A) $-x \cdot x^{-x-1}$ (B) $-x^{-x}(1 + \ln x)$ (C) $-1 - \ln x$ (D) $-x^{-x}$ (E) $-x^{-x} \ln x$
55. If $\begin{pmatrix} a & -3 & 5 \\ 2 & 0 & a \end{pmatrix} \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 9 \\ 33 \end{pmatrix}$, find the value of a .
 (A) -5 (B) -13 (C) 7 (D) 9 (E) -7
56. The operation ϵ is defined as $A\epsilon B = (AB) \bmod 5$. Compute $3\epsilon(4\epsilon 7)$.
 (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
57. Find the volume of the right, isosceles trapezoidal prism shown.
 (A) $352 + 30\sqrt{3} \text{ m}^2$ (B) 165 m^2 (C) $165\sqrt{3} \text{ m}^2$ (D) $330\sqrt{3} \text{ m}^2$ (E) 352 m^2
58. If $f(x) = ax^6 + bx^4 + cx^2 + x$ and $f(2) = 48$ then $f(-2) =$
 (A) 46 (B) -48 (C) 24 (D) 44 (E) 50
59. The mean number of faces on the Platonic solids is
 (A) 6 (B) 7 (C) 10 (D) 11 (E) 12
60. The function f is such that $\int_{-1}^3 f(x) dx = 7$. What is the value of $\int_3^{-1} (2f(x)) dx$?
 (A) 14 (B) -7 (C) 21 (D) 7 (E) -14



2012-2013 TMSCA High School Mathematics Test 12 Key

1. A	21. D	41. D
2. E	22. E	42. B
3. B	23. E	43. E
4. A	24. D	44. B
5. B	25. B	45. C
6. E	26. D	46. B
7. C	27. D	47. E
8. B	28. E	48. E
9. E	29. B	49. A
10. A	30. C	50. D
11. B	31. A	51. B
12. E	32. D	52. C
13. A	33. C	53. E
14. C	34. B	54. B
15. B	35. D	55. C
16. E	36. E	56. E
17. E	37. C	57. D
18. D	38. C	58. D
19. E	39. B	59. C
20. D	40. C	60. E

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6. 3 is a root so $27 + 9a + 3(2a - 7) - 231 = 0$, $a = 15$

7. $\det \begin{pmatrix} 3 & -k \\ -9 & 2k - 4 \end{pmatrix} = 0$, $6k - 12 - 9k = 0$, $k = 4$

10. 4 ties, 12 arrangements of 2 ties and 1 win each, 6 arrangements of 2 wins each

$p(\text{tie}) = (0.1)^4 + 12(0.1)^2(0.25)(0.65) + 6(0.25)^2(0.65)^2$
 $p(\text{tie}) \approx 0.178$

11. use binomial theorem

$(2x - 1)^3 - 2 = 8x^3 - 3 \cdot 4x^2 + 3 \cdot 2x - 1 - 2$
 $= 8x^3 - 12x^2 + 6x - 3$

13.

$\sin^2 x - 2 \sin x \cos x + \cos^2 x = 1 - 2 \sin x \cos x = 1 - \sin 2x$

14. $5 + 3t = 6 + 2s$, and $8 + 5t = -7 - 5s$, solve system

if $t = -1$, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} - 1 \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

15. $\lim_{\theta \rightarrow 0} \frac{\theta}{\sin(4\theta)} = \lim_{\theta \rightarrow 0} \frac{4\theta}{4\sin(4\theta)} = \frac{1}{4}$

16. $23 - a = 45$, $a = -22$, $4 = \frac{-22 + 6 + b + 23}{4}$, $b = 9$

17. quadratic regression $y = 3x^2 - 10x - 1$, 13 terms

21. $\sin \theta (\sin \theta + 1) = 0$, $\sin \theta = 0$ or $\sin \theta = -1$, $\theta = \pi$

22. binomial probability: $\binom{8}{5} \left(\frac{7}{8}\right)^5 \left(\frac{1}{8}\right)^3 \approx 0.056$

24. $70^2 = 80^2 + x^2 - 2 \cdot 8x \cos 80$ graph or factor,
 $x = 30, 50$

25. the fourth number in the n th row is the sum of the first $(n - 3)$ triangular numbers.

28. $99 < 300A + 10A^2 + A < 1000$, use table,
 $311 + 642 + 993 = 1945$

$x = 63 \cos 55 + 87 \cos 150 + 212 \cos 110 \approx -111.717$

29. $y = 63 \sin 55 + 87 \sin 150 + 212 \sin 110 \approx 294.321$

$d = \sqrt{x^2 + y^2} \approx 314.811$

33. $f'(x) = \frac{(1 + \sin x)(-\sin x) - \cos x \cos x}{(1 + \sin x)^2} = \frac{-(1 + \sin x)}{(1 + \sin x)^2}$

$= \frac{-1}{1 + \sin x}$

$f''(x) = \frac{(1 + \sin x)(0) - (-1)(-\cos x)}{(1 + \sin x)^2} = \frac{\cos x}{(1 + \sin x)^2}$

$f''(x)$ changes signs at $x = \frac{\pi}{2}$

34. $V = \frac{s^3(15 + 7\sqrt{5})}{4} = \frac{64(15 + 7\sqrt{5})}{4} \approx 490$

38. If the octagon is inscribed in a circle, the intercepted arc of the inscribed angle is 90° , and the inscribed angle is 45°

40. $8x^2 - 4x + 17 = A(x^2 + 2) + (Bx + C)(x - 3)$

$= Ax^2 + 2A + Bx^2 - 3Bx + Cx - 3C$

$8 = A + B + 0C$

$-4 = 0A - 3B + C$

$17 = 2A + 0B - 3C$

$A = 7$, $B = 1$, $C = -1$, so, $A + B + C = 7$

42. $\frac{(x-7)(x-2)(x+2)}{(x-7)^3} \cdot \frac{(x-7)(x+7)}{x+2} = \frac{(x^2+5x-14)}{x-7}$

44. $900 = (31-7)^2 + (y-12)^2$, $(y-30)(y+6) = 0$,
 $y = -6, 30$

45. let x be the width of the field

$A = x(1000 - 3x)$, $A' = 1000 - 6x$, max at $x = \frac{500}{3}$

$A \approx 83333$

46. $u_6 = S_6 - S_5 = (72 - 18) - (50 - 15) = 19$

49. $13 = 5 + a + b + 11$, so $a + b = -3$

50. $u + v = 4i - 7j$, $4ai - 7aj = 12i(b - 2)j$, $a = 3$

52. $\frac{a}{b} = \frac{b}{a} = 2 + \frac{(a-b)^2}{ab}$, $(x-9-x-11)^2 = 400$

53. expansion = $\sin x + \cos x$,
 $\sin 4 + \cos 4 \approx -1.410446116$

54.

$\ln y = -x \ln x$

$\frac{1}{y} \frac{dy}{dx} = -x \cdot \frac{1}{x} + \ln x \cdot (-1)$

$\frac{dy}{dx} = (-1 - \ln x)y$

$\frac{dy}{dx} = (-1 - \ln x)x^{-x} = -x^{-x}(1 + \ln x)$

58. let $z = ax^6 + bx^4 + cx^2$, $48 = z + 2$, $z = 46$

$f(-2) = 46 - 2 = 44$