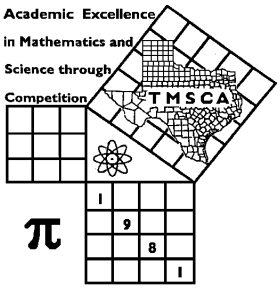


TMSCA HIGH SCHOOL MATHEMATICS TEST#11 ©

FEBRUARY 21, 2015

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

1. $117 \div 13 + (64)^{\frac{2}{3}} \times (8 - 3^2) =$

A) -25 B) $-\frac{4096}{3}$ C) $-\frac{4069}{3}$ D) 25 E) -7

2. At the Texas State Fair, Isaac discovers that he can buy 3 fried pies and 2 funnel cakes for $19.25 or he can buy 2 fried pies and 3 funnel cakes for $20.75. What would his total bill be if he bought six of each?

A) $45.00 B) $51.00 C) $48.00 D) $40.00 E) $42.00

3. Bob wants to sell packages of 6 lollipops. If he has lemon, cherry, blueberry and strawberry how many distinct packages can he make.

A) 210 B) 126 C) 252 D) 30 E) 84

4. Which of the following is an equation of the perpendicular bisector of $\overline{AB}$ when $A(2,-10)$ and $B(-9,11)$.

A) $21x+11y=-68$ B) $11x+21y=-28$ C) $21x+11y=79$ D) $21x+11y=76$ E) $11x-21y=-49$

5. The median $\overline{AD}$ in triangle ABC has a length of 8.52 cm. If E is the centroid of in triangle ABC, then $AE =$

A) 2.84 cm B) 4.26 cm C) 5.68 cm D) 2.13 cm E) 0.71 cm

6. Tanya's little motorboat can make the 18 mile trip from Town A to Town B in 3 hours downstream, but it would take her 15 hours upstream at the same rate. How fast could Tanya's little boat go in still water?

A) 4.8 mph B) 2.4 mph C) 5.4 mph D) 3.6 mph E) 3.0 mph

7. A red die and a green die are both rolled and the top numbers on each are recorded. What are the odds that the number recorded for the red die is greater than the number recorded for the green die?

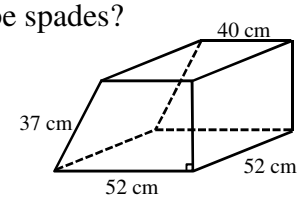
A) 1:1 B) 5:7 C) 1:2 D) 5:12 E) 7:15

8. $\sin^4\theta + 2\sin^2\theta\cos\theta + \cos^4\theta =$

A) $\sin\theta\cos\theta$ B) $\tan^2\theta$ C) $\cos\theta\csc\theta$ D) $\sin\theta\csc\theta$ E) $\sin\theta\sec\theta$

9. Three cards are dealt face up. What is the probability that the three cards will all be spades?

A) $\frac{33}{850}$ B) $\frac{2197}{140608}$ C) $\frac{33}{2704}$ D) $\frac{11}{850}$ E) $\frac{2197}{132600}$



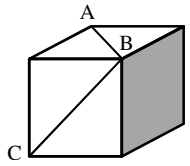
10. The surface area of the right trapezoidal prism shown is _____cm^2.

A) 11932 B) 8528 C) 83720 D) 11748 E) 88504

11. The distance around a track is 300 m. Two boys begin running at the same place and time, but in opposite directions. One travels at a rate of 6.25 meters per second and the other travels at a rate of 5.75 meters per second. How far will the slower runner have travelled the second time the boys meet?

A) 287.5 m B) 143.75 m C) 312.5 m D) 300 m E) 156.25 m

12. A, B and C are each vertices of the cube shown. Find $m\angle ABC$.



A) 90° B) 67.5° C) 60° D) 45° E) 75°

13. How many distinct seating arrangements can be made for eleven people sitting at a round table?

A) 39916800 B) 362880 C) 831600 D) 3628800 E) 2494800

14. A number is defined as *highly composite* if it has more positive divisors than all smaller whole numbers. Forty-eight is a highly composite number. What is the largest highly composite number less than 48?

A) 24 B) 32 C) 36 D) 40 E) 42

15. The harmonic mean of two integers is one less than their arithmetic mean. The larger number is twice the smaller. What is the larger number?

A) 6 B) 8 C) 12 D) 16 E) 18

16. Find the value of c for which the roots of $3x^2 - 28x + c = 0$ are in a ratio of 1:3.

A) 21 B) 49 C) 147 D) 7 E) 14

17. If $a\log_a\left(a^a\right) = 16$, find 16^a.

A) 65536 B) 4096 C) 256 D) 16777216 E) 32768

18. If a fair coin is tossed 8 times, what is the probability of getting exactly 3 heads?

A) $\frac{1}{256}$ B) $\frac{7}{64}$ C) $\frac{3}{256}$ D) $\frac{1}{32}$ E) $\frac{7}{32}$

19. If $\int_{13}^{k} \frac{1}{2x-1}\,dx = \ln(1.4)$, calculate the value of k.

A) 18 B) 26 C) 25 D) 50 E) 49

20. Each of the eighteen people in a conference room shakes hands with everyone else exactly once. How many handshakes take place?

A) 171 B) 162 C) 153 D) 324 E) 243

21. Larry had a bag of candy. He gave Meredith one-third of his candy plus two pieces. Then he gave Neill one-third of the remaining plus 2 pieces. Finally he gave away one-third of what he had left plus two pieces to Owen. If Larry had two pieces left for himself, how many pieces did he originally have?

A) 18 B) 27 C) 21 D) 36 E) 24

22. What is the area enclosed by the graph of the relation: $|x| + |y| = 12$?

A) 144 B) 72 C) 576 D) 288 E) $144\sqrt{2}$

23. Angle A is complementary to angle B and supplementary to angle C. If $m\angle B = 4x - 1$ and $m\angle C = 12x + 1$. Find the measure of angle A.

A) 43° B) 52° C) 38° D) 47° E) 55°

24. A 15-foot piece of wire is cut into two pieces. One piece is bent into an equilateral triangle and the other forms a circle inscribed in the triangle. What is the area of the circle? (nearest square inch)

A) 366 in^2 B) 304 in^2 C) 813 in^2 D) 120 in^3 E) 160 in^2

25. Evaluate: $\lim_{h\to 0} \frac{\sin\left[2\left(\frac{\pi}{3}+h\right)\right] - \sin\left[2\left(\frac{\pi}{3}-h\right)\right]}{2h}$

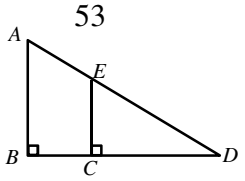
A) $\sqrt{3}$ B) -1 C) $-\sqrt{3}$ D) 1 E) Does not exist

26. What is the shortest distance between the point $(3,-1)$ and the line with the equation: $28y = 45x - 9$?

A) $\frac{172}{53}$ B) $\frac{11}{2}$ C) $\frac{154}{53}$ D) $\frac{98}{53}$ E) $\frac{116}{53}$

27. $\overline{EC}$ divides the triangle ABD into to polygons with equal area. $AB : EC =$

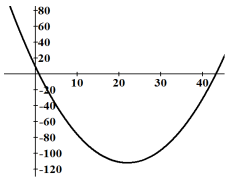
A) 2:1 B) $\sqrt{3}:1$ C) 4:3 D) 4:1 E) $\sqrt{2}:1$



28. The functions $f(x)$ and $g(x)$ are defined as $f(x)=x^2$ and $g(x)=2x+11$. Calculate $f(g(3))-g(f(3))$.

A) 167 B) 318 C) 225 D) 260 E) 153

29. The graph shown is a parabola that includes the points $(0,9)$, $(6,-48)$ and $(10,-76)$. Find $f(20)$.



A) -109 B) -110 C) -111 D) -112 E) -113

30. If *A* represents a digit 0-9 in the equation $2A8_9+3A1_6=111010_3$ find the value of *A*.

A) 1 B) 2 C) 3 D) 4 E) 5

31. If $f(x)=\ln(3x-5)$, for what value(s) of *x* does $\frac{dy}{dx}=\frac{dx}{dy}$?

A) $\frac{2}{3}$ B) $\frac{8}{3}$ C) $\frac{2}{3}$ and $\frac{8}{3}$ D) 2 E) $\frac{4}{3}$ and 2

32. What is the sum of the first 8 terms of the sequence 1225, 245, 49, 9.8…?

A) $\frac{957019}{625}$ B) $\frac{6125}{4}$ C) $\frac{23925769}{15625}$ D) $\frac{4785144}{3125}$ E) 1531

33. If $f(x)=Ax^4+Bx^2+x-9$ and $f(4)=235$, then $f(-4)=$

A) 244 B) 226 C) 248 D) 227 E) 231

34. A right circular conical tank has a vertex angle of 37° and a height of 18 ft. How many gallons of liquid can the tank hold? (nearest gallon)

A) 2161 gal B) 5115 gal C) 2691 gal D) 4317 gal E) 3467 gal

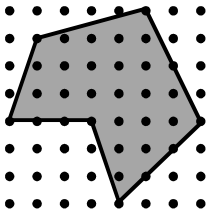
35. Which of the following will produce the same graph in the Cartesian plane as the polar equation $r=5\cos\theta$ does on the polar coordinate system?

A) $x^2+(y-2.5)^2=5$ B) $(x-2.5)^2+y^2=5$ C) $x^2+y^2=5$ D) $(x-2.5)^2+y^2=6.25$ E) $x^2+y^2=6.25$

36. Joann would like to put $10,000 in a CD rather than risk the stock market. She would like to have $16,000 after ten years and all of the CD's she researched compound monthly. What is the minimum interest rate that Joann should consider? (nearest tenth of a percent)

A) 4.8% B) 4.5% C) 5.1% D) 4.9% E) 4.7%

37. The horizontal and vertical distance between the dots is 3 cm. What is the area of the hexagon?



A) 76.5 cm^2 B) 153 cm^2 C) 459 cm^2 D) 229.5 cm^2 E) 306 cm^2

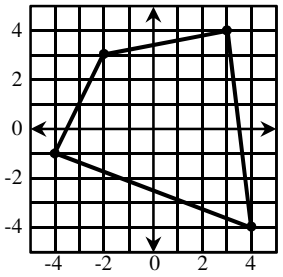
38. If *w* is 10% larger than *x*, *x* is 40% larger than *y*, and *y* is 50% smaller than *z*, by what percentage is *w* smaller than *z*?

A) 70% B) 27% C) 73% D) 35% E) 23%

39. If *f* is continuous on the closed interval $[a,b]$, then there exists a number *c* in the closed interval $[a,b]$ such that $\int_a^b f(x)\,dx=f(c)(b-a)$.

A) Rolle's Theorem
B) Intermediate Value Theorem
C) Mean Value Theorem
D) Definition of Derivative
E) Fundamental Theorem of Algebra

40. What is the acute angle formed by the diagonals of the quadrilateral shown? (nearest degree)



A) 89° B) 86° C) 85° D) 88° E) 87°

41. Classify the graph of with the equation: $9x^2+6xy+2y^2+2x-3y+5=0$

A) Ellipse B) Centroid C) Hyperbola D) Circle E) Parabola

42. What are the coordinates of the *x*-intercept of the line tangent to $f(x)=\sqrt{3x^2-2}$ at $(3,5)$?

A) $\left(-\frac{1}{5},0\right)$ B) $\left(\frac{1}{3},0\right)$ C) $\left(-\frac{2}{5},0\right)$ D) $\left(\frac{2}{9},0\right)$ E) $\left(-\frac{1}{9},0\right)$

43. The Washington Monument in Washington, D.C. casts a shadow that is 142 ft. long at the same time that a person who is 156 cm tall cast a shadow that is 40 cm long. What is the height of the Washington Monument? (nearest foot)

A) 554 ft. B) 535 ft. C) 387 ft. D) 36 ft. E) 169 ft.

44. A continuous random variable *X* has probability density function given by: $f(x)=k\left(2x-x^2\right)$ for $0\le x\le 2$ and $f(x)=0$ for all other values of *x*. What is the value of *k*?

A) $\frac{3}{2}$ B) $\frac{1}{2}$ C) $-\frac{1}{2}$ D) $\frac{3}{4}$ E) $-\frac{3}{4}$

45. A large cylinder has a volume of 600 cm^3. What is the total surface area of the cylinder in terms of *r*, the length of the radius?

A) $2\pi r^2+600r$ B) $2\pi r^2+\frac{1200}{r}$ C) $\pi r^2+\frac{1200}{r}$ D) $2\pi r^2+\frac{600}{r}$ E) $\pi r^2+\frac{600}{r}$

46. $\frac{2x^2-9x-35}{x^2-8x+16}\div\frac{4x^2+20x+25}{x^2-11x+28}=$

A) $\frac{4x^2+20x+25}{x^2-8x+16}$ B) $\frac{x^2-14x+49}{2x^2-3x-20}$ C) $\frac{x-7}{2x^2-3x-20}$ D) $\frac{x^2-14x+49}{2x^2-3x+5}$ E) $\frac{x^2-14x+49}{2x+5}$

47. Right before her final, Amanda's semester math average is 77%. If the final counts as 20% of her semester grade, what is the lowest grade she can make on the final to have an 80% or above for the semester?

A) 83% B) 88% C) 92% D) 94% E) 97%

48. The seven trapezoidal means are constructed as segments in a trapezoid. Which one is the shortest?

A) Heronian B) Contraharmonic C) Geometric D) Harmonic E) Root-mean square

49. $\prod_{k=2}^{5}(2k+1)=$

A) 315 B) 3465 C) 32 D) 945 E) 9009

50. Solve: $2e^{2x}-11e^{x}=-5$.

A) $\log 2$ and $\log 5$ B) $\ln 2$ and $\ln 5$ C) $-\ln 2$ and $\ln 5$ D) $\ln 5$ and $\ln 2$ E) $-\log 5$ and $\log 2$

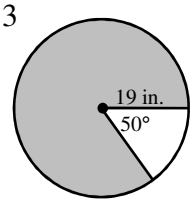
51. If a hiker travels 6 miles on a bearing of 18°, then another 7 miles on a bearing of 334°, what is the shortest distance back to his starting point? (nearest tenth of a mile)

A) 13 mi B) 1.2 mi C) 12.1 mi D) 9.2 mi E) 3.9 mi

52. $2.\overline{7}_8 =$ _____$_{10}$.

A) $\frac{25}{7}$ B) $\frac{23}{8}$ C) $\frac{23}{7}$ D) $\frac{25}{8}$ E) 3

53. What is the area of the shaded region on the picture shown right? (nearest square inch)



A) 1134 in^2 B) 315 in^2 C) 119 in^2 D) 158 in^2 E) 977 in^2

54. What is the 10^{-9} digit in the expansion of $x^3 - \frac{x^9}{3!} + \frac{x^{15}}{5!} - \frac{x^{21}}{7!}$.....when $x = 7$?

A) 5 B) 3 C) 8 D) 6 E) 2

55. A belt joins two pulleys. The larger pulley has a radius of 105 cm and revolves at a rate of 15 rpm. The smaller has a radius of 7 cm. How fast is the smaller pulley revolving?

A) 112.5 rpm B) 450 rpm C) 225 rpm D) 337.5 rpm E) 675 rpm

56. If $\int_0^k a\cos\theta d\theta = 16$, then $\int_{-k}^k a\cos\theta d\theta =$.

A) 8 B) 0 C) 256 D) 4 E) 32

57. Simplify $\frac{(x+2)!}{(x-3)!} \div \frac{(x+1)!}{(x-1)!}$.

A) $\frac{x+2}{x^2-3x+2}$ B) $x^4 - x^3 + 2x^2 + 4x$ C) $\frac{x^4 - 5x^2 + 4}{x^2}$ D) $x^3 - x^2 - 4x + 4$ E) $x^4 - 5x^2 + 4$

58. What is the area of a triangle with side lengths 48 cm, 37 cm and 50 cm? (nearest cm^2)

A) 838 cm^2 B) 380 cm^2 C) 391 cm^2 D) 805 cm^2 E) 782 cm^2

59. The probability that Joy will get a multiple choice question right is 0.20 by randomly guessing. If she guesses on every question on a 15 question quiz, what is the probability that she will make above a 30%?

A) 0.164 B) 0.939 C) 0.836 D) 0.061 E) 0.434

60. Given $(f \circ g)(x) = 2x + 6$ and $g(x) = 2x + 1$, calculate $f(7)$.

A) 12 B) 36 C) 41 D) 14 E) 35

2014-2015 TMSCA Mathematics Test Eleven Answers

1. E	21. C	41. A
2. C	22. D	42. D
3. E	23. D	43. A
4. E	24. A	44. D
5. C	25. B	45. B
6. D	26. C	46. B
7. B	27. E	47. C
8. D	28. D	48. D
9. D	29. C	49. B
10. D	30. E	50. C
11. A	31. B	51. C
12. C	32. D	52. E
13. D	33. D	53. E
14. C	34. B	54. A
15. C	35. D	55. C
16. B	36. E	56. E
17. A	37. D	57. D
18. E	38. E	58. A
19. C	39. C	59. A
20. C	40. C	60. A

2014-2015 TMSCA Mathematics Test Eleven Select Solutions

3. ${}_{6+4-1}C_6 = 84$

8. $\sin^4\theta + 2\sin^2\theta\cos\theta + \cos^4\theta =$
$\left(\sin^2\theta + \cos^2\theta\right)^2 = 1 = \sin\theta\csc\theta$

12. Each side of the triangle is a diagonal of a face, so they are all the same and $m\angle ABC = 60^\circ$.

13. $10! = 3628800$

15. $\frac{2ab}{a+b} + 1 = \frac{a+b}{2}$ and $a = 2b$. Solve for $a = 6$

17. $a^2 = 16$, so $a = 4$ and $16^4 = 65536$.

19. $\frac{1}{2}\ln\left|2x-1\right|\Big]_{13}^{k} = \ln\left(\frac{7}{5}\right)$ yields
$\ln\frac{2k-1}{25} = \ln\frac{49}{25}$, so $k = 25$.

26. $45x - 28y - 9 = 0$ so the distance from the line to $(3,-1)$ is equal to
$\frac{45(3) - 28(-1) - 9}{\sqrt{45^2 + 28^2}} = \frac{154}{53}$

30. $2\left(9^2\right) + 9A + 8 + 3\left(6^2\right) + 6A + 1 =$
$3^5 + 3^4 + 3^3 + 3$, so $15A + 279 = 354$ and $A = 5$.

33. Let $g(x) = Ax^4 + Bx^2$, so $g(4) = 240$ and $f(-4) = 240 - 4 - 9 = 227$.

37. Let *I* be interior points and *P* be perimeter points.
$\frac{2I + P - 2}{2} = \frac{42 + 11 - 2}{2} = \frac{51}{2}$
And each unit of area on the grid represents 9 square units.
$\left(\frac{51}{2}\right)(9) = 229.5$

40. Let the vectors $\begin{pmatrix}7\\5\end{pmatrix}$ and $\begin{pmatrix}6\\-7\end{pmatrix}$ represent the diagonals. The angle between the vectors can be found using:
$\frac{42 - 35}{\sqrt{(49+25)(36+49)}} = \cos\theta$, and
$\theta \approx 84.94^\circ$.

44. $\int_0^2 k\left(2x - x^2\right)dx = 1$, $1 = k\left(4 - \frac{8}{3}\right)$ so
$k = \frac{3}{4}$.

52.
$$\begin{array}{r} 1\ 0\ n = 2\ 7.\overline{7} \\ -\ 1\ n = \quad 2.\overline{7} \\ \hline 7\ n = 2\ 5 \end{array}$$, and
$\frac{25}{7}_{8} = 3_{10}$

54. McClaurin series for $\sin\left(x^3\right)$, so use
$\sin 343 \approx -0.5365983552$

56. $f(x) = a\cos\theta$ is an even function, so
$\int_{-k}^{k} f(x)\,dx = 2\int_0^k f(x)\,dx$.

58.
$A = \sqrt{67.5(67.5-48)(67.5-37)(67.5-50)}$