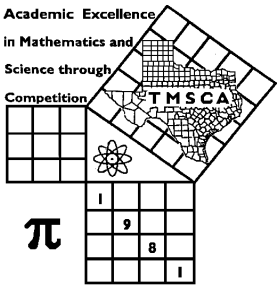


TMSCA HIGH SCHOOL MATHEMATICS TEST# 11©

FEBRUARY 18, 2017

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2016-2017 TMSCA Mathematics Test Eleven

1. **Evaluate $10.8 \div \left(\frac{5}{2}\right)^{-2} - 3! - 7.8$.**

   (A) -9.48 (B) 26.8 (C) 28.8 (D) 53.7 (E) 67.2

2. **Three billion, eighty-seven is subtracted from sixteen billion, four hundred sixty-one thousand, eight hundred seventy-two. What is the sum of the digits in the difference?**

   (A) 47 (B) 35 (C) 26 (D) 63 (E) 39

3. **If $\frac{3}{4}$ of A is 22.5% of B, and B is $\frac{2}{3}$ of C, then A is what percent of C?**

   (A) 20% (B) 60% (C) 2% (D) 40% (E) 4%

4. **Find an equation of the line with slope $-\frac{3}{2}$ that passes through the midpoint of the line segment with endpoints $A(-6,5)$ and $B(4,-3)$.**

   (A) $3x+2y=1$ (B) $3x-2y=4$ (C) $3x-2y=-3$ (D) $3x+2y=4$ (E) $3x+2y=-1$

5. **Given that $(2x+3)$ is a factor of $2x^3+ax^2-22x+3a$, find the value of a.**

   (A) 5 (B) 3 (C) -5 (D) 7 (E) -15

6. **Susan plans on buying 2 shirts for \$22.95 each, 3 skirts for \$17.99 each and a pair of shoes for \$42.95. If the local tax rate is 8.75%, how much money will Susan save by making her purchases on tax-free weekend.**

   (A) \$12.50 (B) \$11.78 (C) \$12.14 (D) \$12.93 (E) \$12.19

7. **What is the sum of the infinite geometric sequence $16.2+10.8+7.2+...$?**

   (A) 32 (B) 9.72 (C) 32.4 (D) 48.6 (E) 52.3

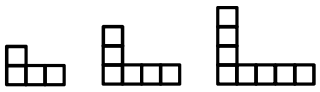
8. **A box contains 2 black marbles, 7 red marbles and 8 green marbles. If Leon draws out 3 marbles 1 at a time without replacement, what are the odds that he will draw out 3 red marbles?**

   (A) 7:136 (B) 7:129 (C) 35:816 (D) 35:781 (E) 7:965

9. **$\angle A$ and $\angle B$ are supplementary. If $m\angle A = 3x^2 - 10$ and $m\angle B = 6x + 1$, find the measure of the larger angle.**

   (A) 143° (B) 137° (C) 128° (D) 63° (E) 151°

10. **The three shapes below are made up of one unit squares. If the pattern continues, the area of the figure with a perimeter of 82 units is _____square units.**



   (A) 34 (B) 42 (C) 38 (D) 36 (E) 40

11. If $\frac{Ax+B}{2x-9}+\frac{x-3}{3x+1}=\frac{11x^2+9x+34}{6x^2-25x-9}$, where A and B are constants, then $A+B=$?

(A) 12 **(B)** 7 **(C)** 5 **(D)** 10 **(E)** 15

12. A tank in the form of a rectangular prism is completely empty. The length of the tank is three times the width and the height is double the width. How many gallons of water will fill the tank to 80% capacity if the height of the tank is 5 feet? (nearest gallon)

(A) 561 gal **(B)** 723 gal **(C)** 701 gal **(D)** 578 gal **(E)** 404 gal

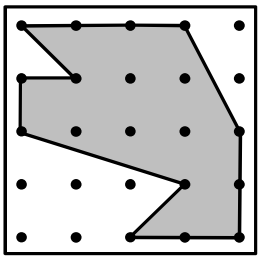
13. Let $A=\begin{bmatrix} 2 & 7 \\ -13 & -9 \end{bmatrix}$, $B=\begin{bmatrix} -7 & 0 \\ 1 & -11 \end{bmatrix}$ and $C=AB$. Find $C_{2,1}$.

(A) –77 **(B)** –12 **(C)** –13 **(D)** 82 **(E)** 7

14. An operation "Ω" is defined by $a\Omega b=b^a-ab$. What is the value of $(4\Omega 2)\Omega(2\Omega 3)$?

(A) 488 **(B)** 4048 **(C)** 681 **(D)** 6537 **(E)** 5288

15. A rubber band was stretched on the geoboard to form this 10-sided figure. What is the area?



(A) 11 units^2 **(B)** 10 units^2 **(C)** 9 units^2

(D) 11.5 units^2 **(E)** 10.5 units^2

16. Determine the frequency of $f(x)=7+4\cos[3\pi(x-3)]$.

(A) $\frac{3}{2}$ **(B)** $\frac{3}{4}$ **(C)** 3 **(D)** $\frac{2}{3}$ **(E)** $\frac{1}{3}$

17. Given the arithmetic sequence -12, a, b, 3, c, …, find $a+b+c$.

(A) -9 **(B)** –1 **(C)** 2 **(D)** 1 **(E)** 3

18. Find the remainder when x^4-3x^3+8x-5 is divided by $x-2$.

(A) 19 **(B)** 4 **(C)** 3 **(D)** 15 **(E)** –17

19. Which of the following functions is neither even nor odd? $f(x)=$ _____.

(A) $\sin(2x)$ **(B)** $|x-2|$ **(C)** $|x|-2$ **(D)** $3x^2+1$ **(E)** $\cos(x)$

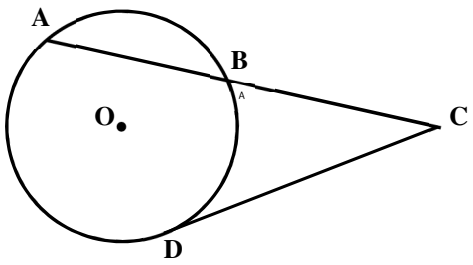
20. Let $f'(x)=9x^2+2x+9$ and $f(2)=39$. Find $f(-2)$.

(A) 49 **(B)** 41 **(C)** 39 **(D)** –18 **(E)** –45

21. The graph of $y=7+5\sin(3x-150^\circ)$ reaches a maximum value at:

(A) $(20^\circ,2)$ **(B)** $(75^\circ,12)$ **(C)** $(0^\circ,12)$ **(D)** $(320^\circ,12)$ **(E)** $(-45^\circ,12)$

22. Given the circle with center O shown, find CD if AB = 11' and BC = 15'. (nearest inch)



(A) 16 **(B) 13** **(C) 19**

(D) 20 **(E) 22**

23. Find the eccentricity of the ellipse $49x^2 + 81y^2 = 3969$. (nearest hundredth)

(A) 0.63 **(B) 0.47** **(C) 0.67** **(D) 0.72** **(E) 0.49**

24. Let $f(x) = x^2 + 1$ and $g(x) = x^4$. Calculate $f(g(3))$.

(A) 6561 **(B) 6560** **(C) 6562** **(D) 6564** **(E) 6566**

25. Simplify: $\dfrac{(n+3)!(n-2!)}{(n+5)!(n-3)!}$.

(A) $\dfrac{1}{n^3+9n^2+20n}$ **(B)** $\dfrac{n}{n+5}$ **(C)** $\dfrac{n-2}{n^2+20}$ **(D)** $\dfrac{n}{n^2+20}$ **(E)** $\dfrac{n-2}{n^2+9n+20}$

26. Caroline's Ice Creams offers 12 flavors of ice cream, two toppings and three container options. How many different 2-scoop, one topping desserts can be ordered?

(A) 468 **(B) 396** **(C) 165** **(D) 198** **(E) 144**

27. Let the "1" at the top of Pascal's triangle be row 0. Determine the third number in row 42.

(A) 903 **(B) 820** **(C) 780** **(D) 861** **(E) 800**

28. Coach Euclid's UIL team consists of 7 boys and 9 girls. How many different 4-member teams consisting of at least two girls could he make up?

(A) 246 **(B) 501** **(C) 1470** **(D) 756** **(E) 966**

29. Which of the following mathematicians is one of the few American Indians who received a Ph.D. in mathematics.

(A) Freda Porter **(B) Sophie Germain** **(C) Emmy Noether**

(D) Grace Williams **(E) Marin Mersenne**

30. Given that $x - y = -8$ and $xy = 28$, find $x^3 - y^3$.

(A) −160 **(B) −960** **(C) −736** **(D) −1184** **(E) −440**

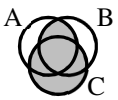
31. The mathematical statement $(2x-3)(4x-7)=8x^2-26x+21$ is an example of _____property.

(A) Associative **(B) Commutative** **(C) Identity**

(D) Transitive **(E) Distributive**

32. In which of the following Venn diagrams does the shaded region represent the set $(B \cup C) \cap (A \cup C)$?

(A) 

(B) 

(C) 

(D) 

(E) 

33. Use the table of values to create a function and find K.

X	1	2	3	4	5	…	18
Y	–1	0	9	32	75		K

(A) 4335 **(B) 6137** **(C) 5236** **(D) 5661** **(E) 5184**

34. Find $\lim_{x \to -\infty} \dfrac{2x^2-x-3}{2x^3-9x+9}$

(A) 0 **(B) 1** **(C) $-\infty$** **(D) ∞** **(E) –1**

35. Let $f_0=0, f_1=1, f_2=1, f_3=2, f_4=3, \ldots$ be the terms of the Fibonacci sequence. How many digits are in f_{32}?

(A) 4 **(B) 5** **(C) 6** **(D) 7** **(E) 8**

36. If $\left[(5+3i)(3-2i)\right] \div (2-i) = a+bi$, then $a+b=$?

(A) 9.6 **(B) 12.4** **(C) 7.8** **(D) 8.6** **(E) 9.2**

37. The chart shows the losses and gains in an investment over the course of five years. What was the average growth rate over the course of the five years? (nearest hundredth)

Year	1	2	3	4	5
Percent Growth	+ 10%	-3%	-8%	+4%	+6%

(A) +8.22% **(B) +1.80%** **(C) +1.59%** **(D) +1.64%** **(E) +5.01%**

38. Which of the following relations describes a 1-1 function?

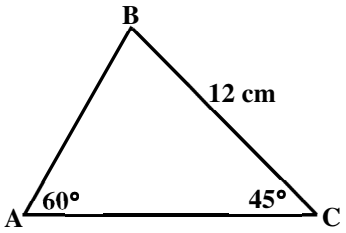
(A) $\{(2,4),(-2,-4),(3,5),(-3,-5)\}$ **(B)** $\{(2,4),(-2,4),(3,5),(-3,5)\}$ **(C)** $\{(2,4),(2,-4),(3,5),(3,-5)\}$

(D) $\{(-2,4),(-2,-4),(-3,5),(-3,-5)\}$ **(E) none of these**

39. Which of the following are the side lengths of a scalene acute triangle?

(A) 5, 6, 6 (B) 4, 5, 6 (C) 5, 6, 8 (D) 6, 8, 10 (E) 8, 10, 13

40. Find the perimeter of triangle ABC. (nearest tenth)



(A) 30.3 (B) 35.2 (C) 37.6 (D) 36.4 (E) 64.6

41. If $\frac{7}{(x-2)(x+4)}-\frac{1}{(x-5)(x+4)}=\frac{k}{(x-2)(x-5)(x+4)}$, then k equals:

(A) $6x-37$ (B) $8x-37$ (C) $8x-35$ (D) $8x-33$ (E) $6x-33$

42. The intersection of the perpendicular bisectors of the three sides of a triangle is called the_____.

(A) Incenter (B) Orthocenter (C) Center (D) Centroid (E) Circumcenter

43. Find the mean value of the function $f(x)=2x^3-8x$ over the interval $[-1,9]$.

(A) 138 (B) 217 (C) 478 (D) 199 (E) 296

44. Given $y=\ln(2x-3)$, find the positive value of x for which $\frac{dy}{dx}=\frac{dx}{dy}$.

(A) 1.5 (B) 1 (C) 2.5 (D) 0 (E) 0.5

45. A polyhedron has 20 faces and 30 edges, how many vertices does it have?

(A) 12 (B) 52 (C) 14 (D) 48 (E) 10

46. If $h(x)\le f(x)\le g(x)$ for all x in an open interval containing c, except possibly at c itself, and if $\lim_{x\to c}h(x)=L=\lim_{x\to c}g(x)$ then $\lim_{x\to c}f(x)$ exists and is equal to L. This theorem is known as:

(A) Rolle's Theorem (B) Sandwich Theorem (C) Fundamental Theorem of Calculus

(D) Intermediate Value Theorem (E) Fundamental Theorem of Algebra

47. What is the constant term in the expansion of $\left(2x^2-\frac{3}{x}\right)^9$?

(A) 19683 (B) 734832 (C) 326592 (D) 5832 (E) 489888

48. $(212121_3 + 121212_3) \times 2_3 =$ _____$_3$.

(A) 200,000 **(B)** 1,000,000 **(C)** 100,000 **(D)** 2,000,000 **(E)** 2,222,220

49. The length of one edge of a regular tetrahedron is $7\sqrt{3}$ **cm. The surface area of the tetrahedron is** _____**cm^2. (nearest square centimeter)**

(A) 508 **(B)** 191 **(C)** 303 **(D)** 255 **(E)** 333

50. If $a_0 = -2$, $a_1 = 3$, $a_2 = 5$ **and** $a_n = (a_{n-3})(a_{n-2}) + a_{n-1}$ **for** $n \geq 3$ **, then** $a_6 =$ **:**

(A) 9 **(B)** 23 **(C)** −23 **(D)** −5 **(E)** 121

51. There are two values of k **for which** $\det\begin{bmatrix} 6 & 2 & 3 \\ 4 & -k & 7 \\ k & 0 & 5 \end{bmatrix} = -60$. **What is the smallest value of** k**?**

(A) 3 **(B)** $\frac{10}{3}$ **(C)** 2 **(D)** $\frac{5}{2}$ **(E)** −3

52. The function $f(x) = 2x^4 - 3x^2$ **is decreasing over which of the following intervals?**

(A) $(-\infty, 0]$ **(B)** $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right]$ **(C)** $[0, \infty)$ **(D)** $\left[0, \frac{\sqrt{6}}{2}\right]$ **(E)** $\left[0, \frac{\sqrt{3}}{2}\right]$

53. If $27^{(x+1)} = 3^{-6x}$ **, then** $x =$ **?**

(A) $\frac{2}{3}$ **(B)** −2 **(C)** −1 **(D)** $-\frac{1}{3}$ **(E)** 1

54. The lengths of the sides of ΔPQR are the roots of $f(x) = x^3 - 15x^2 + 68x - 84$. **Find the area of ΔPQR. (nearest tenth)**

(A) 5.6 **(B)** 11.1 **(C)** 1.9 **(D)** 1.6 **(E)** 3.2

55. Find the area of a convex quadrilateral with vertices at $(-1,9)$, $(3,11)$, $(7,2)$ **and** $(2,-5)$.

(A) 71.5 **(B)** 69.5 **(C)** 56.5 **(D)** 67.5 **(E)** 73

56. Find the shortest distance between the x**-intercept of the line** $2x + 7y = 14$ **to the line** $5x - 6y = -48$. **(nearest tenth)**

(A) 4.6 **(B)** 10.6 **(C)** 7.7 **(D)** 4.5 **(E)** 8.1

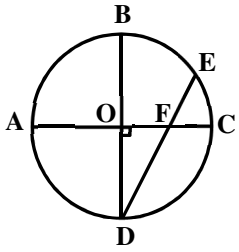
57. Caroline has 11 non-fiction books to shelve. She wants to keep her 6 math books together, but otherwise any order is fine. In how many different ways can Lily organize her shelf?

(A) 86400 **(B)** 518400 **(C)** 720 **(D)** 7920 **(E)** 55440

58. If p and q are the zeros of the function $f(x) = 35x^2 + 4x - 247$ then $pq^2 + p^2q =$

(A) $-\frac{988}{1225}$ **(B)** $\frac{494}{1225}$ **(C)** $\frac{988}{1225}$ **(D)** $-\frac{494}{1225}$ **(E)** $-\frac{247}{1335}$

59. Given the circle O with perpendicular diameters and a chord, find the area of triangle DFO if EF = 10" and DE = 22". (nearest tenth)



(A) $20.8\ in^2$ **(B)** $30.0\ in^2$ **(C)** $19.9\ in^2$ **(D)** $17.3\ in^2$ **(E)** $39.8\ in^2$

60. If $PQR_5 + RPQ_4 - QRP_3$ has a numeric value in base 10 of:

(A) $30P - 3Q + 20R$ **(B)** $30P - 3Q + 14R$ **(C)** $28P - 3Q + 20R$

(D) $28P - 3Q + 14R$ **(E)** $30P + 15Q + 14R$

Test Eleven Answer Key

1. D	21. D	41. E
2. B	22. D	42. E
3. A	23. A	43. E
4. E	24. C	44. C
5. C	25. E	45. A
6. A	26. A	46. B
7. D	27. D	47. E
8. B	28. C	48. E
9. B	29. A	49. D
10. E	30. D	50. D
11. D	31. E	51. C
12. A	32. B	52. E
13. D	33. E	53. D
14. D	34. A	54. A
15. E	35. D	55. D
16. A	36. B	56. B
17. B	37. C	57. B
18. C	38. A	58. C
19. B	39. B	59. C
20. E	40. B	60. D

Test Eleven Select Solutions

7. $\dfrac{16.2}{1-\dfrac{10.8}{16.2}} = 24.3$

8. To find the probability: $\dfrac{7}{17}\cdot\dfrac{6}{16}\cdot\dfrac{5}{15} = \dfrac{7}{136}$ **for odds:** $\dfrac{7}{136-7} = \dfrac{7}{129}$.

15. $A = \dfrac{2I+P}{2} - 1 = \dfrac{2(5)+13}{2} - 1 = 10.5$

16. The frequency is $\dfrac{b}{2\pi}$ **where** b **is the coefficient of the variable for** $\dfrac{3\pi}{2\pi} = \dfrac{3}{2}$.

20. $f(x) = 3x^3 + x^2 + 9x + C$ **for** $x = -7$ **and** $f(-2) = -45$.

22. $(15)(26) = (DC)^2 \approx 20$

26. $\left({}_{12+2-1}C_2\right)(3)(2) = 468$

37. $\sqrt[5]{(1.1)(0.97)(0.92)(1.04)(1.06)} \approx 1.0159$ **and average growth of 1.59%**

39. For an acute triangle, $a^2 + b^2 > c^2$ **and** $16 + 25 > 36$. **This is the only scalene choice with this characteristic.**

43. $\dfrac{1}{9-(-1)}\left(\int_{-1}^{9}\left(2x^3 - 8x\right)dx\right) = 296$

44. Graph the derivative of the function and find the points at which the derivative is equal to either 1 or -1, which is only true at $x = 2.5$.

45. Use Euler's formula relating the vertices, faces and edges of a polyhedron, $V + F - E = 2$ **for** $V + 20 - 30 = 2$ **and** $V = 12$.

47. Use the binomial theorem for $\left({}_9C_3\right)\left(2x^2\right)^3\left(-\dfrac{3}{x}\right)^6 = 489888$.

53. $3^{3(x+1)} = 3^{-6x}$ **so solve** $3x + 3 = -6x$ **for** $x = -\dfrac{1}{3}$.

54. The sum of the roots is 15, for a semi-perimeter of 7.5, so the area is $\sqrt{7.5f(7.5)} \approx 5.6$.

56. To find the x**-intercept,** $(7,0)$, **and the line** $5x - 6y + 48 = 0$ **by evaluating** $d = \dfrac{|7(5)+0(-6)+48|}{\sqrt{5^2+(-6)^2}} \approx 10.6$

57. Count the math books as if they were one group, then multiply by the number of arrangements of the math books themselves: $(6!)(6!) = 518400$