



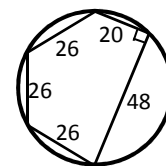
**TMSCA HIGH SCHOOL
MATHEMATICS
TEST # 11 ©
FEBRUARY 16, 2013**

GENERAL DIRECTIONS

1. About this test:
 - A. You will be given 40 minutes to take this test.
 - B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2012-2013 TMSCA High School Mathematics

1. What is $41.\bar{6}\%$ of the quotient of $\frac{1}{3}$ and $\frac{5}{2}$?
- (A) $\frac{7}{120}$ (B) $\frac{1}{18}$ (C) $\frac{25}{72}$ (D) $\frac{35}{96}$ (E) $\frac{2}{9}$
2. Mr. Green's statistics class passed out a survey to 102 seniors. Forty-two students did not return the survey. Of the students who returned the survey, 32 agreed with proposition one and 39 agreed with proposition two. How many students agreed with both propositions?
- (A) 28 (B) 60 (C) 22 (D) 11 (E) 49
3. A small motor boat travelling from point A to point B along the river can make the 14 mile trip in 6 hours. The return trip takes 8 hours and 24 minutes. If the speed of the boat and the current remained constant throughout both trips, what is the speed of the current?
- (A) 1760 ft/hr (B) 12320 ft/hr (C) 10560 ft/hr (D) 4928 ft/hr (E) 8800 ft/hr
4. Find the x -intercept of the line containing $(7,3)$ and perpendicular to $5x - 4y + 11 = 0$.
- (A) $(8.6,0)$ (B) $(10.75,0)$ (C) $(0,10.75)$ (D) $(0,8.6)$ (E) $(0,-10.75)$
5. Given the geometric sequence : 1440, 960, 640, find the value of the 13th term.
- (A) $\frac{176534}{23859}$ (B) $\frac{765275040}{256}$ (C) $\frac{655360}{59049}$ (D) $\frac{765275040}{512}$ (E) $\frac{88267}{7953}$
6. If $\int_2^k \frac{1}{x+7} dx = \ln 2$, find the value of k .
- (A) 18 (B) 25 (C) 11 (D) 9 (E) 4
7. Poppy's Pet Store sells assorted tropical fish at a price of \$18.75 each. The weekly special is buy three, get one free, with an additional 15% discount for purchases exceeding 10 fish. How much would it cost to stock a tank with a dozen fish?
- (A) \$162.56 (B) \$117.19 (C) \$191.25 (D) \$143.44 (E) \$154.22
8. $\angle A$ and $\angle B$ are supplementary. If $m\angle A = (4x)^\circ$ and $m\angle B = (5x+18)^\circ$, find $m\angle B$.
- (A) 18° (B) 72° (C) 36° (D) 90° (E) 108°
9. A large cylindrical tank holds 6000 gallons of water. If the height of the tank is 3 yards, what is the radius of the tank to the nearest inch?
- (A) 60 in (B) 156 in (C) 111 in (D) 64 in (E) 75 in
10. Find the area of the pentagon inscribed in the circle on the right.
- (A) $989\sqrt{3}$ (B) $480+507\sqrt{3}$ (C) $989\sqrt{2}$ (D) $480+169\sqrt{3}$ (E) $240+507\sqrt{2}$



11. Find the total surface area of a right hexagonal prism with an apothem of 12 cm and a height of 22 cm.
- (A) $1344\sqrt{3} \text{ cm}^2$ (B) $1056\sqrt{3} \text{ cm}^2$ (C) $288+1056\sqrt{3} \text{ cm}^2$ (D) $1632\sqrt{3} \text{ cm}^2$ (E) $1056+576\sqrt{3} \text{ cm}^2$
12. Given that $(a+7i)(b-2i) = 17+i$, where $a, b \in \mathbb{Z}$, find the value of $a+b$.
- (A) 3 (B) 4 (C) -7 (D) 7 (E) -10

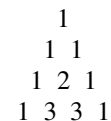
13. If F_n is the n^{th} Fibonacci number then $\lim_{n \rightarrow \infty} \left(\frac{F_{n+1}}{F_n} \right) =$

(A) $\frac{\pi}{2}$ (B) e (C) φ (D) π (E) ε

14. Given $y = -2 \cos x$, find the value of x for which $\frac{dy}{dx} = \frac{dx}{dy}$, where $0 \leq x \leq \frac{\pi}{2}$

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$ (E) π

15. Given the pyramid at right, the sum of all the numbers in the first 10 rows is _____



(A) 512 (B) 1023 (C) 511 (D) 1024 (E) 255

16. The measure of one central angle of a regular dodecagon is

(A) 36° (B) 45° (C) 60° (D) 72° (E) 30°

17. If $\frac{x^2 + 3x - 40}{x^3 + 24x^2 + 192x + 512} \div \frac{x^2 - 25}{x^2 + 7x + 10} = \frac{x + a}{x^2 + bx + c}$, find $a + b + c$.

(A) 78 (B) 82 (C) 64 (D) 80 (E) 60

18. A fair tetrahedral die with sides labeled 2, 3, 3, 6 is thrown. What is the expected value of a single roll?

(A) $\frac{7}{2}$ (B) $\frac{8}{3}$ (C) 3 (D) $\frac{11}{6}$ (E) $\frac{5}{2}$

19. If the region enclosed by the curve $y = 6x^2 + x - 35$ is rotated 360° around the x -axis, the volume of the solid generated is

(A) 352.19 (B) 9944.51 (C) 354.72 (D) 9944.09 (E) 3165.44

20. $\sin x + \cos x \cot x =$

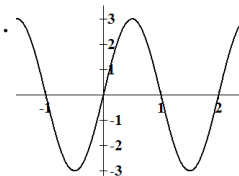
(A) $\csc x$ (B) $\cos x$ (C) $\cot x$ (D) $\sec x$ (E) $\sin x$

21. A circle circumscribed about a triangle. The center of the circle is the _____ of the triangle.

(A) centroid (B) circumcenter (C) incenter (D) orthocenter (E) foci

22. If the equation of the function graphed below is $y = a \sin bx + c$, find the value of $a \cdot b \cdot c$.

(A) $-\frac{\pi}{3}$ (B) -3π (C) 0 (D) $\frac{\pi}{3}$ (E) 3π



23. 80 miles per hour equals _____ inches per second.

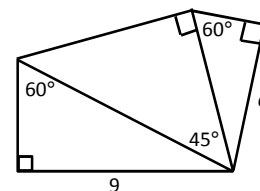
(A) 9.77 in/sec (B) 5068800 in/sec (C) 84480 in/sec (D) 16896 in/sec (E) 1408 in/sec

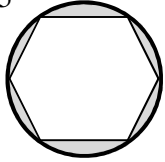
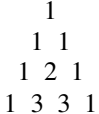
24. Carries Ice Cream Shop has nine flavors of ice cream and three types of cones. How many distinct 3-scoop cones can be ordered?

(A) 660 (B) 165 (C) 45 (D) 495 (E) 135

25. Find the value of a on the picture to the right.

(A) $6\sqrt{6}$ (B) $\frac{27\sqrt{2}}{2}$ (C) $6\sqrt{3}$ (D) $3\sqrt{6}$ (E) $\frac{9\sqrt{2}}{2}$



26. If two coplanar lines are parallel to the same line then the two lines are:
 (A) intersecting (B) perpendicular (C) transversal (D) skew (E) parallel
27. A belt joins two pulleys. The pulley with a diameter of 55 cm rotates at a rate of 90 rpm. The smaller pulley has a radius of 11 cm. How fast is the smaller pulley rotating?
 (A) 225 rpm (B) 550 (C) 450 (D) 375 (E) 275
28. A regular hexagon is inscribed in a circle with a radius of 8 cm. Find the area of the shaded region.
 (A) 159.49 cm^2 (B) cm^2 (C) 34.79 cm^2 (D) 48.89 cm^2 (E) 75.40 cm^2
- 
29. If $\tan \theta < 0$ and $\csc \theta > 0$, then where will θ terminate?
 (A) QI (B) QII (C) QIII (D) QIV (E) x - axis
30. Helicopter leaves the pad and travels 10 miles on a bearing of 210° , then turns and travels 13 miles at a bearing of 55° . What bearing would the helicopter need to travel in order to go straight back to the pad?
 (A) 95° (B) 78° (C) 282° (D) 12° (E) 155°
31. If $f(x) = 2\cos^2 x$, $g(x) = x^2 + 9$, and $h(x) = g(f'(x))$, find $h\left(\frac{\pi}{3}\right)$.
 (A) 21.00 (B) 9.25 (C) 18.00 (D) 13.91 (E) 12.00
32. The total surface area of a regular icosahedron is $60\sqrt{3} \text{ cm}^2$. Find the length of one edge?
 (A) $8\sqrt{2}$ in (B) $4\sqrt{3}$ in (C) $4\sqrt{2}$ in (D) $2\sqrt{3}$ in (E) $8\sqrt{3}$ in
33. Uncle Jon bought seven boxes of chocolate bars from a school fundraiser. He gave half of his order to his niece, then two-thirds of the remaining bars to his nephew. He ate six bars then put the remaining box away to use for holidays. How many bars came in each box?
 (A) 48 (B) 12 (C) 72 (D) 24 (E) 36
34. The Real value solution set for $|2x+10|-3 < 4$ is
 (A) $\{x | -8.5 < x < -1.5\}$ (B) $\{x | -8.5 < x < 1.5\}$ (C) $\{x | \{x < -8.5\} \cup \{x > -1.5\}\}$
 (D) $\{x | \{x < -1.5\} \cup \{x > 8.5\}\}$ (E) $\{x | 1.5 < x < 8.5\}$
35. Shady Lawn Service charges \$45 for a fertilizer treatment on a lawn with 1000 square feet. How much should they charge to fertilize a lawn with 180 square yards?
 (A) \$72.90 (B) \$24.30 (C) \$97.20 (D) \$48.60 (E) \$121.50
36. Given the pyramid at right, the fourth number in the 18th row is the sum of the first _____ triangular numbers.
 (A) 16 (B) 18 (C) 17 (D) 15 (E) 14
- 
37. The chord \overline{AB} is 9 cm from the center of circle O. If $AB = 80$ cm. The area of circle O to the nearest cm^2 is
 (A) 5281 (B) 251 (C) 5027 (D) 258 (E) 20360
38. If $m\angle A + m\angle B = 180^\circ$ and $m\angle A + m\angle C + m\angle D = 180^\circ$ then $m\angle A + m\angle B = m\angle A + m\angle C + m\angle D$ is an example of the _____ property.
 (A) distributive (B) reflexive (C) symmetric (D) transitive (E) commutative
39. What is the equation of a line tangent to $y^2 - x = 191$ at the point $(5, 14)$?
 (A) $28x + y = 154$ (B) $x - 28y = -387$ (C) $28x + y = 397$ (D) $28x + y = 126$ (E) $x - 28y = 397$

40. The school day at Austin Elementary School begins at 8:25 am and ends at 3:20 pm. What is the acute angle between the hour and minute hand at the end of the school day??
 (A) 20° (B) 30° (C) 25° (D) 10° (E) 15°
41. $\int \ln x dx = \underline{\hspace{2cm}} + C$, where C is some arbitrary constant.
 (A) $x \ln x + x$ (B) $x \ln x$ (C) $-x \ln x$ (D) $-x \ln x + x$ (E) $-x + x \ln x$
42. A particular model of car has an advertised gas mileage of 32 mpg for in-town driving. With further investigation, a consumer discovers that the gas mileage is normally distributed with a standard deviation of 3 mpg. What is the probability that the driver will get over 33 mpg for in-town driving?
 (A) 16% (B) 63% (C) 84% (D) 37% (E) 27%
43. There are seven boys and eleven girls on the student council. Their sponsor is asked to select three boys and three girls to attend an elementary school pep rally. How many different groups could the sponsor choose?
 (A) 200 (B) 5775 (C) 207900 (D) 693 (E) 6300
44. If $f(x) = \frac{x^3 - 3x^2 - 8}{x^2 + 8}$ and $s(x)$ be the slant asymptote of f , find the value of $s(2)$?
 (A) -0.75 (B) -1 (C) 1 (D) 0.75 (E) 2
45. $f(x) = 1 + x - \frac{x^2}{2} - \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} - \frac{x^6}{6!} \dots$. Find the 10^{-8} place of $f(4)$.
 (A) 1 (B) 6 (C) 9 (D) 5 (E) 2
46. The odds of drawing a pink raffle ticket at random from a bucket 306 tickets are 6:11. How many pink tickets would have to be removed from the bucket to reduce the odds to 1:3?
 (A) 66 (B) 42 (C) 99 (D) 9 (E) 54
47. Evaluate $\lim_{h \rightarrow 0} \frac{\tan\left(\frac{\pi}{3} + h\right) - \tan\left(\frac{\pi}{3}\right)}{h}$.
 (A) 2 (B) 4 (C) 3 (D) 0.75 (E) 0.25
48. What is the area of the quadrilateral with the vertices $(4,10)$, $(9,7)$, $(11,2)$ and $(2,2)$.
 (A) 83 (B) 182 (C) 9 (D) 45.5 (E) 20.25
49. If $f(x) = 3^x$, find $f'(x)$.
 (A) $3x$ (B) $x \cdot 3^{x-1}$ (C) 3^x (D) $3^x \ln 3$ (E) $\frac{3^x}{\ln 3}$
50. If the length of one edge of a regular dodecahedron is 26 cm. Then the total surface area is
 (A) 13957 cm^2 (B) 9135 cm^2 (C) 761 cm^2 (D) 15224 cm^2 (E) 3512 cm^2
51. Solve $\log_9 81 + \log_9 \left(\frac{1}{9}\right) - \log_9 3 = \log_9 x$ for x .
 (A) 0 (B) 1 (C) 3 (D) 9 (E) 27
52. The fourth term of a geometric sequence is 17280 and the sixth term is $\frac{31104}{5}$. The sum of the sequence is

- (A) 194401 (B) 80000 (C) 200000 (D) 2000000 (E) 140000

53. At which of the following x -values is the graph of $f(x) = x^4 - 15x^3 + 75x^2 - 125x$ concave down?

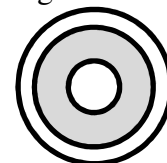
- (A) -1 (B) 1 (C) 0 (D) 4 (E) 6

54. If $\cos \theta = -\frac{7}{25}$ and $0 \leq \theta \leq \pi$, then $\tan \theta =$

- (A) $-\frac{24}{25}$ (B) $\frac{24}{7}$ (C) $-\frac{24}{7}$ (D) $\frac{7}{24}$ (E) $-\frac{7}{24}$

55. The diameters of the concentric circles on the right are 10 cm, 8 cm and 4 cm. If a dart hits the figure at random, what is the probability that it will **not** land in the shaded area?

- (A) 0.64 (B) 0.48 (C) 0.36 (D) 0.52 (E) 0.56



56. If $f(x) = ax^5 + bx^3 + cx + 9$ and $f(8) = 15$, find $f(-8)$.

- (A) -3 (B) 3 (C) 7 (D) -7 (E) 6

57. Find the constant term in the expansion of $\left(3x^2 - \frac{2}{x}\right)^9$.

- (A) 19683 (B) 145152 (C) -979776 (D) 489888 (E) -326592

58. How many solutions to $11x + 3y = 156$ where x and y are both positive integers.

- (A) 2 (B) 6 (C) 5 (D) 3 (E) 4

59. How many distinct arrangements can be made from the letters of the words MATH TEAM?

- (A) 5040 (B) 20160 (C) 720 (D) 40320 (E) 6720

60. If $\frac{x-7}{x+4} + \frac{x+4}{x-7}$ is equal to the mixed number $A + \frac{B}{(x+4)(x-7)}$, then $B =$

- (A) 64 (B) 8 (C) 121 (D) 24 (E) 9

2012-2013 TMSCA High School Mathematics Test 11 Key

1. B	21. B	41. E
2. D	22. C	42. D
3. A	23. E	43. B
4. B	24. D	44. B
5. C	25. E	45. A
6. C	26. E	46. B
7. D	27. A	47. B
8. E	28. C	48. D
9. D	29. B	49. D
10. B	30. C	50. A
11. D	31. E	51. C
12. B	32. D	52. C
13. C	33. E	53. D
14. D	34. A	54. C
15. B	35. A	55. D
16. E	36. D	56. B
17. B	37. A	57. B
18. A	38. D	58. E
19. B	39. B	59. A
20. A	40. A	60. C

3. $6(b+c)=14$ and $8.4(b-c)=14$ solve system:

$$c = \frac{1 \text{ mi}}{3 \text{ hr}} = 1720 \frac{\text{ft}}{\text{hr}}$$

4. $0-3 = -\frac{4}{5}(x-7)$, $x=10.75$

5. $1440 \cdot \left(\frac{2}{3}\right)^{12} = 160 \left(\frac{2^{12}}{3^{10}}\right) = \frac{655360}{59049}$

6. $[\ln(x+7)]_2^k = \ln(k+7) - \ln 9 = \ln 2$, so $\frac{k+7}{9} = 2$, $k=11$

10. right triangle and 3 equilateral:

$$A = \frac{1}{2}(20 \cdot 48) + (3) \frac{26^2 \sqrt{3}}{4} = 480 + 507\sqrt{3}$$

11. $ab - 2ai + 7bi - 14i^2 = ab + 14 - 2ai + 7bi = 17 + i$

separate: $ab + 14 = 17$ and $-2a + 7b = 1$, $a\left(\frac{1+2a}{7}\right) = 3$

$a = 3$, $b = 1$, so $a + b = 4$

14. $\frac{dy}{dx} = 2\sin x = \frac{dx}{dy}$, when $2\sin x = \pm 1$, $\sin x = \frac{1}{2}$ at $\frac{\pi}{6}$

15. The sum of the first n rows is $2^n - 1 = 2^{10} - 1 = 1023$

17. $\frac{(x+8)(x-5)}{(x+8)^3} \cdot \frac{(x+5)(x+2)}{(x+5)(x-5)} = \frac{x+2}{x^2+16x+64}$,

$a+b+c=82$

18. $\frac{1}{4}(2+3+3+6) = \frac{7}{2}$

19. $V = \pi \int_{\frac{3}{2}}^{\frac{7}{2}} (6x^2 + x - 35)^2 dx \approx 9944.51$

20. $\sin x + \frac{\cos x \cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x} = \frac{1}{\sin x} = \csc x$

22. $a = 3$, $2 = \frac{2\pi}{b}$, so $b = \pi$, $c = 0$, so product is 0

24. $\binom{9+3-1}{3} \cdot 3 = 495$

25. $\sum_{n=1}^{10} n^3 + \sum_{n=1}^{10} 3 = \left[\frac{10(10+1)}{2}\right]^2 + 3 \cdot 10 = 3055$

30. 210° bearing is 240° on standard plane, 55° bearing is 35° destination is $(10\cos 240 + 13\cos 35, 10\sin 240 + 13\sin 35)$

about $(5.649, -1.204)$, $\arctan\left(\frac{-1.204}{5.649}\right) \approx -12.03^\circ$

to go back, $-12.03 + 180 \approx 154.97^\circ$, bearing of 282°

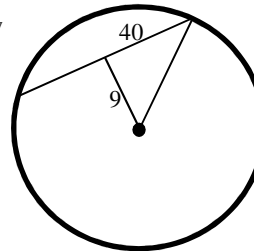
32. $SA = 20\left(\frac{s^2\sqrt{3}}{4}\right) = 60\sqrt{3}$, so $s^2 = 12$, $s = 2\sqrt{3}$

34. $|2x+10| < 7$

$2x+10 > -7$ $x > -8.5$	$2x+10 < 7$ $x < -\frac{3}{2}$
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35. $\frac{\$45}{1000 \text{ ft}^2} \cdot \frac{9 \text{ ft}^2}{1 \text{ yd}^2} \cdot 180 \text{ yd}^2 = \72.90

36. The fourth number in the n th row is the sum of the first $(n-3)$ triangular numbers



37. $40^2 + 9^2 = r^2$, $r = 41$, $A \approx 5281$

39. $2y \frac{dy}{dx} - 1 = 0$, $28 \frac{dy}{dx} = 1$, $m = \frac{1}{28}$

$y - 14 = \frac{1}{28}(x - 5)$, $x - 28y = -387$

41. $\int 1 \cdot \ln x dx$, $u = \ln x$ and $dv = dx$ then $du = \frac{1}{x} dx$ and $v = x$

$\int u dv = uv - \int v du = x \ln x - \int 1 dx = x \ln x - x + C$

45. expansion = $\sin 4 + \cos 4 \approx -1.410446116$

46. $\frac{306}{6+11} = 18$, bucket contains 108 pink and 198 other

$\frac{108-p}{198} = \frac{1}{3}$, $p = 42$

47. definition of derivative: $\sec^2\left(\frac{\pi}{3}\right) = 4$

50. $A = 3(26^2)\sqrt{25+10\sqrt{5}} \approx 13957$

51. $\log_9\left(\frac{81}{9 \cdot 3}\right) = \log_9 x$, $\log_9(3) = \log_9 x$, $x = 3$

52. $\sqrt{1 - \left(-\frac{7}{25}\right)^2} = \pm \frac{24}{25}$, in domain sine is (+), so $\tan \theta = \frac{24}{25}$

56. let $z = ax^5 + bx^3 + cx$, $z + 9 = 15$, so $z = 6$,
 $f(-8) = -6 + 9 = 3$

57. $\binom{9}{3} (3x^2)^3 \left(-\frac{2}{x}\right)^6 = 145152$

59. $\frac{8!}{2! \cdot 2! \cdot 2!} = 5040$, the 8 is for the number of letters and the 2's are the number of each letter that repeats.

60. $(-7-4)^2 = 121$

