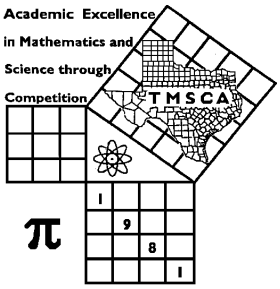


TMSCA HIGH SCHOOL MATHEMATICS TEST # 10 ©

FEBRUARY 9, 2013

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2012-2013 TMSCA High School Mathematics Test

1. Evaluate: $\dfrac{(8+6)\cdot 3}{7\cdot 2+7\cdot 5}$

(A) $\dfrac{337}{7}$ (B) $\dfrac{297}{7}$ (C) 47 (D) $\dfrac{26}{49}$ (E) $\dfrac{6}{7}$

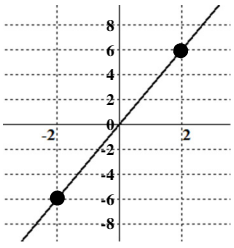
2. A craft store offers a weekly coupon for 30% off of one regularly priced item. Joan buys baking supplies that are all on sale for 20% off for the week. She buys four cookie cutters with a regular price of $1.99 each, two cans of colored icing with a regular price of $4.59 each, a decorator kit with a regular price of $22.50 and pays tax at a rate of 7.25% on her whole purchase. What is the smallest bill she can have if she can use the coupon on an item of her choice?

(A) $29.46 (B) $50.82 (C) $31.60 (D) $22.53 (E) $36.25

3. The distance driven on a road trip is directly proportional to the rate at which the car is moving. On Andy's vacation, when the rate is 66 mph, the distance driven is 231 miles. How far will he drive if he raises his rate to 75 mph?

(A) 225 miles (B) 150 miles (C) 187.5 miles (D) 262.5 miles (E) 175 miles

4. Which of the following is an equation of the line shown right?



(A) $y-2=3(x-6)$ (B) $y+2=3(x+6)$ (C) $y-2=\dfrac{1}{3}(x-6)$

(D) $y-6=3(x-2)$ (E) $y-2=\dfrac{1}{3}(x-6)$

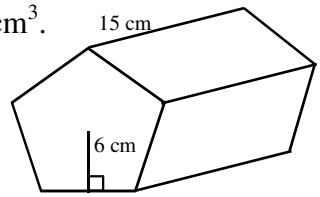
5. If $\dfrac{x^3+6x^2+12x+8}{x^4-8x^2+16}=\dfrac{ax+b}{x^2-4x+4}$, find $a+b$.

(A) 2 (B) 3 (C) -2 (D) 1 (E) -3

6. The intersection of the perpendicular bisectors of the sides of a triangle is the _____.

(A) incenter (B) median (C) centroid (D) circumcenter (E) orthocenter

7. Find the volume of the regular, right pentagonal prism shown right to the nearest cm^3.



(A) 844 cm^3 (B) 1425 cm^3 (C) 520 cm^3 (D) 3924 cm^3 (E) 1962 cm^3

8. The third term of an arithmetic sequence is -2, and the sum of the first fourteen terms is 98. Find the first term.

(A) 4 (B) -2 (C) 2 (D) -6 (E) -4

9. How many faces does an icosahedron have?

(A) 10 (B) 32 (C) 12 (D) 36 (E) 20

10. Four math books need to be shelved together on a shelf containing ten books. How many arrangements of books are possible?

(A) 120960 (B) 241920 (C) 40320 (D) 3628800 (E) 30240

11. If $g(x)=x-1$ and $f(x)=x^4$, find $g(f(x+1))$.

(A) $x^4+4x^3+6x^2+4x$ (B) $x^4+3x^3+3x^2+x$ (C) $x^4+4x^3+6x^2+4x-2$

(D) x^4-2 (E) x^4

12. A quadrilateral is inscribed in a circle. The measures of two angles opposite each other are $\left(x^2+100\right)^\circ$ and $\left(11x\right)^\circ$. Find the measure of the larger angle.

(A) 125° (B) 90° (C) 55° (D) 169° (E) 100°

13. Mrs. Jones is fifteen years older than twice the age of her son. Twenty years ago, her age was nine times that of her son. Find son's age.

(A) 75 (B) 65 (C) 50 (D) 25 (E) 20

14. Determine the period of $y=\frac{3}{2}\sin\left(\frac{1}{3}x-7\right)+8$.

(A) $\frac{2\pi}{3}$ (B) 2π (C) $\frac{3\pi}{2}$ (D) 6π (E) $\frac{7\pi}{3}$

15. There are two values of *k* for which $\det\begin{bmatrix} k+1 & 5 \\ -2 & k \end{bmatrix}=22$. The sum of those two values is

(A) 0 (B) 1 (C) 7 (D) -1 (E) -7

16. When $3x^3+2x^2-7x+k$ is divided by $\left(x-2\right)$ the remainder is *m*. Find the value of *m* in terms of *k*.

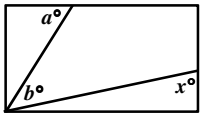
(A) $m=k-2$ (B) $m=k-30$ (C) $m=k+46$ (D) $m=k+30$ (E) $m=k+18$

17. On triangle *ABC*, $AB=16$ cm, $BC=14$ cm, and $m\angle A=60^\circ$. There are two possible lengths for $\overline{AC}$. The shorter of the two lengths is

(A) 2.4 cm (B) 10 cm (C) 4.6 cm (D) 6 cm (E) 25.4 cm

18. In the rectangle shown right, what is *x* in terms of *a* and *b*?

(A) $a+b$ (B) $b-a$ (C) $90-a+b$ (D) $a-b$ (E) $90+a-b$



19. Which of the following series converges?

(A) $\sum_{n=1}^{\infty}\frac{(-3)^n}{n!}$ (B) $\sum_{n=0}^{\infty}\frac{n+1}{2n+1}$ (C) $\sum_{n=1}^{\infty}\frac{n}{1000(n+1)}$ (D) $\sum_{n=1}^{\infty}\left(\frac{3}{2}\right)^n$ (E) $\sum_{n=1}^{\infty}\log n$

20. Let the region *R* be bounded in the first quadrant by the *x*-axis, *y*-axis and the graph of $f\left(x\right)=\sin 2x$. Find the volume of the solid generated by the revolution of *R* around the *x*-axis.

(A) 0.5 (B) 1.23 (C) 2.47 (D) 0.39 (E) 3.14

21. The intersection of two distinct planes is a _____.

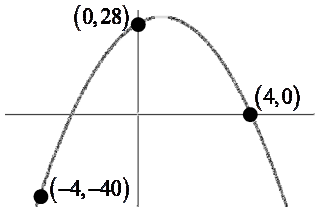
(A) Point (B) Triangle (C) Parallelogram (D) Line (E) Plane

22. Myrtle has bins containing 9 flavors of lollipops. In how many ways can she package 5 to sell if she can repeat flavors?

(A) 1001 (B) 2002 (C) 1287 (D) 715 (E) 3003

23. What are the coordinates of the other zero of the parabola shown right?

(A) $\left(-4,0\right)$ (B) $\left(-\frac{7}{3},0\right)$ (C) $\left(-3,0\right)$ (D) $\left(-\frac{9}{5},0\right)$ (E) $\left(-\frac{7}{4},0\right)$



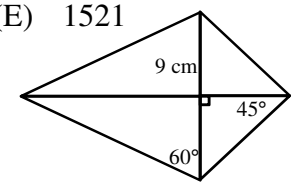
24. On an academic UIL team of 38 students, 16 practice for mathematics, 12 practice for number sense and 9 students practice for both. How many students do not do either mathematics or number sense?

(A) 10 (B) 22 (C) 19 (D) 21 (E) 15

25. Let $f(x) = x^3 + 3x + 1$, and $g(x) = x^2$. Find $f(g'(3))$.

(A) 37 (B) 235 (C) 226 (D) 441 (E) 1521

26. The area of the kite shown right is_____.



(A) $486\,\text{cm}^2$ (B) $162\sqrt{3}\ \text{cm}^2$ (C) $162+81\sqrt{3}\ \text{cm}^2$ (D) $81+81\sqrt{3}\ \text{cm}^2$ (E) $81+162\sqrt{3}\ \text{cm}^2$

27. If $y = \tan\theta$, for what value of θ does $\frac{dy}{dx} = \frac{dx}{dy}$?

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$ (E) $\frac{\pi}{6}$

28. The circumference of a great circle on a soap bubble is expanding at a rate of 11 cm/sec. When the radius is 5 cm, the surface area of the soap bubble is changing at a rate of_____ cm^2/sec.

(A) 11 (B) 275 (C) 44 (D) 55 (E) 220

29. Let $a_1 = 7$, $a_2 = -2$ and $a_n = 3a_{n-2} - 2a_{n-1}$. Find a_6.

(A) -13 (B) -22 (C) 187 (D) -542 (E) 298

30. Find the sum of the series to the nearest ten thousandth: $3 - \frac{1}{6} + \frac{1}{120} - \frac{1}{5040} \ldots$.

(A) 2 (B) 2.8415 (C) 4.7183 (D) 2.5403 (E) 3.5574

31. Two fair tetrahedral dice are rolled. What are the odds that the sum of the bottom faces is a prime number?

(A) 9:16 (B) 1:1 (C) 1:2 (D) 7:9 (E) 9:7

32. If $\frac{x+2}{x-7} + \frac{x-7}{x+2} = A + \frac{B}{(x+2)(x-7)}$, then $B =$

(A) 25 (B) 5 (C) 81 (D) -5 (E) -9

33. The Real value solution set of $|3x-1|+3<11$ is

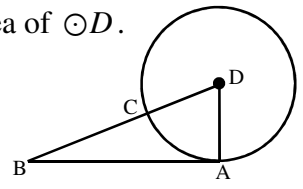
(A) $\left\{x \middle| -\frac{7}{3} < x < 3\right\}$ (B) $\left\{x \middle| -3 < x < -\frac{7}{3}\right\}$ (C) $\left\{x \middle| \{x<-3\} \cup \left\{x > \frac{7}{3}\right\}\right\}$ (D) $\left\{x \middle| -3 < x < \frac{7}{3}\right\}$ (E) $\left\{x \middle| \left\{x < -\frac{7}{3}\right\} \cup \{x>3\}\right\}$

34. Two workers can paint a fence in three hours. How long would it take five workers to paint a fence twice as long and twice as high if they each paint at the same rate?

(A) 4 hr 48 min (B) 6 hr (C) 1 hr 12 min (D) 2 hr 24 min (E) 4 hr 8 min

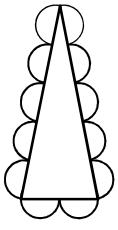
35. On the diagram shown right, $\overline{AB}$ is tangent to $\odot D$, $AB = 12$ and $BC = 8$. Find the area of $\odot D$.

(A) 64π (B) 16π (C) 10π (D) 24π (E) 25π



36. On the figure shown right, all of the semicircles are congruent. If the area of the triangle is $4\sqrt{6}$ then the diameter of one circle is

(A) 2 (B) $2\sqrt{2}$ (C) 1 (D) $\sqrt{2}$ (E) 4



37. Find the sum of the infinite series: $-1.2 + 0.9 - 0.675 + 0.50625 \ldots$.

(A) -3.2812 (B) -1.6 (C) -2.4 (D) -4.8 (E) -0.68571

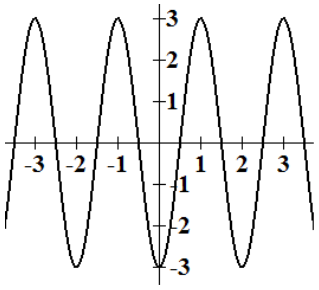
38. Classify the graph of the equation: $4x^2+8x+4=y^2-6y$

(A) lemniscate (B) parabola (C) ellipse (D) hyperbola (E) circle

39. Find the acute angle that the line $3x+4y=18$ forms with the *y*-axis to the nearest hundredth of a degree.

(A) 41.41° (B) 48.59° (C) 33.69° (D) 36.87° (E) 53.13°

40. The function $f(x)=$_____ will produce this graph.



(A) $3\cos(\pi x+1)$ (B) $3\cos(x+\pi)$ (C) $3\sin(\pi(x+1))$

(D) $3\cos(\pi x-1)$ (E) $3\cos(\pi(x-1))$

41. If $f(x)=\cos x$ then $\lim_{h\to 0}\dfrac{f(\pi+h)-f(\pi)}{h}$ is

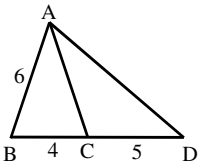
(A) 0 (B) 1 (C) undefined (D) -1 (E) $\sqrt{3}$

42. $\left(1-i\sqrt{3}\right)^9=$

(A) 512 (B) $512i$ (C) $-512-512i$ (D) -512 (E) $-512+512i$

43. Given that $\overline{AB}\cong\overline{AC}$, find the perimeter of triangle *ABD*.

(A) 18 (B) 24 (C) 22 (D) 25 (E) 23



44. If $\int_{-2}^{4} f(x)\,dx=10.5$ then $\int_{-2}^{4}(2f(x)+3)\,dx=$

(A) 27 (B) 28.5 (C) 24 (D) 28 (E) 39

45. Simplify: $\left(3-2\sqrt{-8}\right)\left(1+3\sqrt{-50}\right)$

(A) $-117+41i\sqrt{2}$ (B) $-117-41i\sqrt{2}$ (C) $123+41i\sqrt{2}$ (D) 123 (E) $123-41i\sqrt{2}$

46. Solve: $\log_2 x+\log_2(x-6)=4$

(A) 9 (B) 8 (C) 6 (D) -3 (E) 4

47. The surface area of an octahedron with an edge length of one?

(A) $\dfrac{\sqrt{3}}{2}$ (B) $\dfrac{8\sqrt{2}}{3}$ (C) $\dfrac{\sqrt{2}}{3}$ (D) $2\sqrt{3}$ (E) $\dfrac{8\sqrt{3}}{2}$

48. Set $S=\{1,2,3,4,5,6,7\}$. How many 5-element subsets of set *S* are there?

(A) 21 (B) 32 (C) 128 (D) 49 (E) 35

49. If $y^2=-45-28i$ and $y^3=-286+259i$, where $y=a+bi$, then $a+b=$

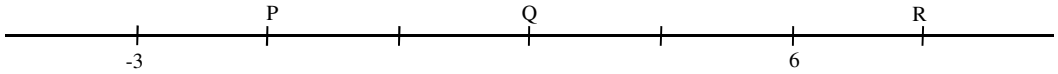
(A) -14 (B) -5 (C) 9 (D) 5 (E) -9

50. For the function shown on the right, $f'(x)$ exist when $x=$

(A) -3 (B) -2 (C) 0 (D) 2 (E) -7

$$f(x)=\begin{cases}\dfrac{x^2+2x-35}{x+7}, & x\le -3\\ |x|+1, & -3<x<2\\ x^2+2 & x\ge 2\end{cases}$$

51. The distances between the hash marks (|) are equal. Find P + Q + R.



(A) 3 (B) -3 (C) 6 (D) -3.44 (E) 9

52. If $\frac{x-1}{2x^2+11x+15}=\frac{A}{x+3}+\frac{B}{2x+5}$, then $AB=$

(A) -28 (B) -6 (C) 28 (D) -3 (E) 6

53. The letters in the world POLYGON are arranged in a line. How many of distinct arrangements are possible?

(A) 2520 (B) 21 (C) 10080 (D) 5040 (E) 720

54. $\frac{\cot t}{\sec t}=$

(A) $\sec t+\cos t$ (B) $\sec t-\cos t$ (C) $\csc t+\sin t$ (D) $\sec t-\sin t$ (E) $\csc t-\sin t$

55. How many petals does the graph of the curve $r=2\cos 5\theta$ have?

(A) 4 (B) 5 (C) 10 (D) 2 (E) 7

56. $\frac{x^2-9}{x^2-4}\div\frac{x^2+6x+9}{x^2+x-6}=$

(A) $\frac{x-2}{x+3}$ (B) $\frac{x-3}{x-2}$ (C) x^2-x-6 (D) $\frac{x-3}{x+2}$ (E) x^2+x-6

57. How many solutions are there to the equation $11x+9y=216$, where x and y are whole numbers?

(A) 0 (B) 2 (C) 4 (D) 3 (E) 1

58. Over time, it is observed that the arrival time for people attending an exhibition is normally distributed with a mean of 3 hours and 48 minutes. If the doors open at 9 am and 90% of people have arrived by 1:55 pm, what is the standard deviation of the distribution?

(A) 57 min (B) 52 min (C) 87 min (D) 42 min (E) 67 min

59. The faces of an icosahedron are

(A) Triangles (B) Squares (C) Hexagons (D) Pentagons (E) Octagons

60. How many numbers in the form a^4, where $a\in\mathbb{Z}^+$ are factors of $(3!)(7!)(9!)$?

(A) 5 (B) 7 (C) 9 (D) 6 (E) 8

2012-2013 TMSCA High School Mathematics Test 10 Key

1. E	21. D	41. A
2. C	22. C	42. D
3. D	23. B	43. B
4. D	24. C	44. E
5. B	25. B	45. C
6. D	26. D	46. B
7. E	27. A	47. D
8. D	28. E	48. A
9. E	29. D	49. B
10. A	30. B	50. B
11. A	31. E	51. E
12. A	32. C	52. A
13. D	33. A	53. A
14. D	34. A	54. E
15. D	35. E	55. B
16. E	36. D	56. D
17. D	37. E	57. D
18. C	38. D	58. B
19. A	39. E	59. A
20. C	40. E	60. E

2012 – 2013 TMSCA Solutions Mathematics Test Ten

2. $\left[0.8(4\cdot 1.99+2\cdot 4.59)+0.7(22.50)\right]=29.462$

$29.462(1.0725)\approx 31.60$

3. $231=66k$, $k=3.5$, $d=3.5(75)=262.5$

4. $m=3$, using $(2,6)$, $y-6=3(x-2)$

5. $\dfrac{(x+2)^3}{(x+2)^2(x-2)^2}=\dfrac{x+2}{(x-2)^2}$, $a+b=3$

7. $r=\dfrac{6}{\cos 36}$, $B=5\left(\dfrac{1}{2}\right)r^2\sin 72$,

$B(15)\approx 1961.66$

8. $a+2d=-2$, $7(2a+13d)=98$, $a=-6$,

$d=2$

10. treat the four as a unit $(7!)(4!)=120960$,

the four factorial is the arrangements of the 4 together

11. use binomial expansion of $(x+1)^4$

$x^4+4x^3+6x^2+4x+1-1$

12. $x^2+100+11x=180$, $(x-5)(x+16)=0$

Largest angle = $5^2+100=125$

13. $j=2s+15$, $j-20=9(s-20)$, $s=25$

14. $p=\dfrac{2\pi}{1/3}=6\pi$

15. $k^2+k+10=22$, $(k+4)(k-3)=0$,

$-4+3=-1$

16. $3(2)^3+2(2)^2-7(2)+k=m=k+18$

17. $14^2=16^2+x^2-2\cdot 16x\cos 60$, graph or factor $x=6,10$

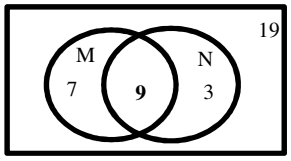
18. $90-a+b+90-x=90$, $x=90-a+b$

19. alternating series test for convergence

20. $\int_0^{\pi/2}\pi\sin^2 2x\,dx\approx 2.47$

22. $\binom{9+5-1}{5}=1287$

24.



25. $g'(3)=6$, $6^3+3\cdot 6+1=235$

26. $d_1=18$, $d_2=9+9\sqrt{3}$,

$A=\dfrac{1}{2}d_1d_2=81+81\sqrt{3}$

27. $\dfrac{dy}{dx}=\sec^2 x=\dfrac{dx}{dy}$, when $\cos x=\pm 1$, $x=\pi$

28. $\dfrac{dc}{dt}=11=2\pi\dfrac{dr}{dt}$, $\dfrac{dr}{dt}=\dfrac{11}{2\pi}$, $\dfrac{da}{dt}=8\pi r\dfrac{dr}{dt}$

$\dfrac{da}{dt}=40\pi\dfrac{11}{2\pi}=220$

29. 7, -2, 25, -56, 187, -542

30. expansion is $2+\sin 1\approx 2.8415$

31. The number of ways to roll each sum

sum	2	3	4	5	6	7	8
ways	1	2	3	4	3	2	1

9 ways to roll a prime vs. 7 for composite

32. $\left(2-(-7)\right)^2=81$

33. solve $|3x-1|<8$

$3x-1>-8$	$3x-1<8$
$x>-\dfrac{7}{3}$	$x<3$

34. $2\cdot r\cdot 3=1$, $5\left(\dfrac{1}{6}\right)t=4$, $t=4.8$ hours

35. $12^2+x^2=(x+8)^2$, $x=5$, $a=25\pi$

36. $h^2=25d^2-d^2$, $h=2d\sqrt{6}$, $2d^2\sqrt{6}=4\sqrt{6}$

$d=\sqrt{2}$

37. $S=\dfrac{-1.2}{1-(-0.75)}\approx -0.6857$

39. Find x and y intercepts, $\theta=\arctan\left(\dfrac{6}{4.5}\right)$

41. definition $f'(\pi)=-\sin\pi=0$

44. $\int_{-2}^{4}2f(x)\,dx+\int_{-2}^{4}3dx=21+18=39$

46. $2^4=x^2-6x$, $(x-8)(x+2)=0$, only 8 is in the domain of the logarithms

48. $\binom{7}{5}=21$

49. $y=\dfrac{-286+259i}{-45-28i}=2-7i$

50. The function is discontinuous @ $x=-7$

52. $x-1=A(2x+5)+B(x+3)$

let $x=-3$, $A=4$, let $x=-2.5$, $B=-7$

53. $\dfrac{7!}{2!}=2520$

54. $\dfrac{\cos t}{\sin t}\cdot\cos t=\dfrac{1-\sin^2 t}{\sin t}=\csc t-\sin t$

58. $z\approx 1.28155$, $z=\dfrac{4.9167-3.8}{\sigma}$, $\sigma\approx 52$

60. $2^{12}\cdot 3^7\cdot 5^2\cdot 7^2$ has factors 1^4, 2^4, 3^4, 4^4, 6^4, 8^4, 12^4, and 24^4