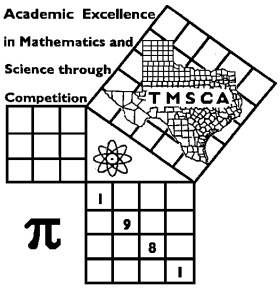


TMSCA HIGH SCHOOL MATHEMATICS TEST#10 © FEBRUARY 7, 2015

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2014-2015 TMSCA Mathematics Test Ten

1. What is $41.\overline{6}\%$ of the quotient of $\frac{2}{3}$ and $\frac{5}{2}$?

A) $\frac{1}{18}$ B) $\frac{1}{90}$ C) $\frac{1}{180}$ D) $\frac{2}{9}$ E) $\frac{1}{9}$

2. Mr. Green's statistics class passed out a survey to 108 seniors. Forty-five students agree with neither proposition. Of the students who agreed with propositions, 32 agreed with proposition one and 39 agreed with proposition two. How many students agreed with both propositions?

A) 8 B) 7 C) 13 D) 6 E) 11

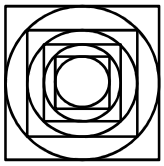
3. Java Joe sells two types of coffee. Type A costs \$11.00 per pound and Type B costs \$13.50 per pound. A customer wants to buy 2 pounds of a blend for \$24.00. How much of Type A should Joe put in the blend?

A) 1 pound B) 1.2 pounds C) 0.75 pounds D) 0.8 pounds E) 1.25 pounds

4. Which of the following is an equation of the line that is parallel to $2x-13y=27$ and includes $(-9,11)$?

A) $2x-13y=-143$ B) $13x+2y=-95$ C) $2x-13y=-161$ D) $13x+2y=-139$ E) $2x-13y=-125$

5. A circle is inscribed in a square, then a square is inscribed in the circle, then a circle is inscribed in the smaller square followed by a circle inscribed in the smaller square and so on. If this pattern is continued infinitely and one side of the largest square is 4 inches, what is the sum of the areas of all of the circles?



A) 16π in^2 B) 32π in^2 C) 8π in^2 D) $\frac{8\pi}{1-\sqrt{2}}$ in^2 E) $\frac{4\pi}{1-\sqrt{2}}$ in^2

6. A red die and a green die are both rolled and the top numbers on each are recorded. Given that the number on the red die is prime, what is the probability that the sum of the dice will be prime?

A) $\frac{7}{18}$ B) $\frac{1}{2}$ C) $\frac{2}{9}$ D) $\frac{5}{18}$ E) 0

7. What is the period of the graph of the function $f(x)=\frac{2}{3}\cos\left(3\left(\theta-\frac{\pi}{2}\right)\right)$?

A) $\frac{4\pi}{3}$ B) $\frac{2\pi}{3}$ C) $\frac{\pi}{2}$ D) π E) $\frac{2}{3}$

8. The repeating decimal $0.363636...$ in base 8 can be written as which of the following fractions in base 8?

A) $\frac{12}{25_8}$ B) $\frac{10}{21_8}$ C) $\frac{15}{32_8}$ D) $\frac{12}{21_8}$ E) $\frac{23}{52_8}$

9. The ratio of width to length in a rectangular yard is 3:7. If the area of the yard is 7581 yd^2 and fencing costs \$2.87 per yard, how much would it cost to completely fence the yard?

A) \$545.30 B) \$999.55 C) \$1090.60 D) \$817.95 E) \$1181.65

10. Barry has 10 books on his shelf. He likes to keep his 6 math books together, but doesn't care how the others are arranged. How many distinct arrangements of books can Barry make on his shelf?

A) 518400 B) 17280 C) 3628800 D) 86400 E) 151200

11. If $x-y=16$ and $xy=28$, then $x^2+y^2=$

A) 312 B) 200 C) 284 D) 228 E) 528

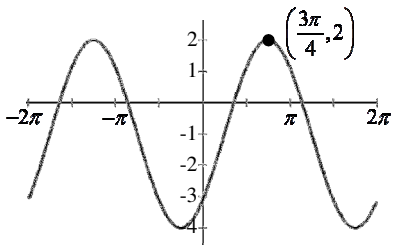
12. Larry wants to buy 4 new tires for his truck. The ones he wants usually sell for \$228 per tire, but he has a coupon for 10% off each tire. When he goes to buy his tires, his buddy gives him an additional 15% off his total cost. How much will Larry pay if tax is 8.25%?

A) \$740.43 B) \$747.84 C) \$898.32 D) \$755.24 E) \$972.43

13. Let $8x^5-12x^4+6x^3-5x^2+2=0$. According to Descartes' Rule of Signs how many possible positive real roots are there?

A) 4, 2 or 0 B) 2 or 0 C) 0 D) 3 or 1 E) 1

14. The graph shows $f(x)=a\cos(x+b)+c$. $f(x)=$



A) $2\cos\left(x+\frac{\pi}{4}\right)-1$ B) $3\cos\left(x+\frac{3\pi}{4}\right)-1$ C) $-2\cos\left(x+\frac{3\pi}{4}\right)-1$ D) $-3\cos\left(x+\frac{3\pi}{4}\right)-1$ E) $-3\cos\left(x+\frac{\pi}{4}\right)-1$

15. A jar contains 48 coins worth \$5.23. It contains pennies, nickels, dimes and quarters. There are two more nickels than pennies and eight less quarters than dimes. How many dimes are in the jar?

A) 6 B) 11 C) 19 D) 10 E) 8

16. If $a_0=4$, $a_1=6$ and $a_{n+1}=(a_{n-2})^2-6a_{n-1}$, then $a_5=$

A) -536 B) 27552 C) 1632 D) 807424 E) 904

17. A and B are the roots of $f(x)=2x^2-11x+15$. Evaluate $A^5+5A^4B+10A^3B^2+10A^2B^3+5AB^4+B^5$.

A) $\frac{161051}{32}$ B) $\frac{161051}{1024}$ C) $-\frac{1}{32}$ D) $-\frac{161051}{32}$ E) $-\frac{161051}{1024}$

18. Given that the even integers continue in the pattern shown, find the sum of the numbers in the 8th row.

A) 504 B) 510 C) 990 D) 720 E) 750

2 4
6 8 10
12 14 16 18
20 22 24 26 28 ...

19. $\sin^4\alpha-\cos^4\alpha=$

A) $\sin 2\alpha$ B) $\tan^2\alpha$ C) $-\cos 2\alpha$ D) $\sin^2\alpha$ E) $\cos^2\alpha$

20. There are two values of k for which $\det\begin{pmatrix}k & 1\\ 3 & k+7\end{pmatrix}=95$. What is larger value of k?

A) 7 B) -7 C) -14 D) 21 E) 14

21. What is the sum of the quadratic and linear coefficients of the derivative of $f(x)=9x^4-3x^3+5x^2+6x-9$?

A) 27 B) 11 C) -27 D) 1 E) -37

22. What is the area of the region enclosed by the graphs of $f(x)=-4x^2+7x+15$ and $g(x)=3x+7$?

A) 6 B) 36 C) $\frac{14}{3}$ D) $\frac{32}{3}$ E) 18

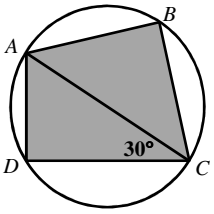
23. A satellite has eight solar power cells. There is a 40% that any single cell will fail in the first five years of operation. If the satellite needs at least two functional cells to continue operating, what is the probability that the satellite will be operational at the end of five years? (nearest thousandth)

A) 0.121 B) 0.984 C) 0.992 D) 0.994 E) 0.999

24. Simplify: $a^3b^3\div a^{-3}b^2\times a^5\div(a^7b^7)+a^4\div b^2$.

A) $\frac{a^4}{b^2}$ B) $\frac{2a^4}{b^2}$ C) $\frac{a^8}{b^2}$ D) $\frac{2a^4}{b^4}$ E) $\frac{1}{a^6b^8}$

25. The illustration shown is a quadrilateral inscribed in a circle. If $\overline{AC}$ is a diameter and $\overline{AB} \simeq \overline{BC}$, what is the probability that a dart landing randomly in the circle would land outside the shaded region? (nearest hundredth)



A) 0.39 B) 0.64 C) 0.61 D) 0.41 E) 0.36

26. $\left(3+\frac{1}{x^4}\right)\div\left(\frac{1}{x^2}-2\right)=$

A) $\frac{3x^4+2}{1-2x^2}$ B) $\frac{3x^4+2}{1-2x^4}$ C) $\frac{3x^2+1}{1-2x^4}$ D) $\frac{3x^4+1}{x^2-2x^4}$ E) $\frac{3x^2-1}{x^2+2x^4}$

27. The time Meredith spends in the car on the way to work varies inversely with the speed she drives. If it takes her 20 minutes driving 40 mph, how long will it take her if she drives 25 mph?

A) 30 minutes B) 32 minutes C) 24 minutes D) 36 minutes E) 28 minutes

28. Given $\sin\theta=-\frac{1}{2}$ and $\frac{\pi}{2}\le\theta\le\frac{3\pi}{2}$, calculate $\cos 2\theta$.

A) $\frac{\sqrt{3}}{2}$ B) $-\frac{1}{2}$ C) $\frac{1}{2}$ D) 1 E) $-\frac{\sqrt{3}}{2}$

29. The operation ∂ is defined so that $a\partial b=\frac{a+b}{a^2+b^2}$. Evaluate $2\partial(-1\partial 3)$.

A) $\frac{55}{101}$ B) $\frac{10}{3}$ C) $\frac{11}{20}$ D) $\frac{7}{29}$ E) $\frac{13}{5}$

30. Find the value of $A+B+C$, where *A, B* and *C* are positive integers such that $\frac{33}{10}=A+\frac{1}{B+\frac{1}{C+1}}$.

A) 8 B) 7 C) 18 D) 5 E) 10

31. The matrix multiplication $\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$ results in a 270° counter-clockwise rotation of the point (x, y) around the origin. $\begin{pmatrix}a & b\\ c & d\end{pmatrix}=$

A) $\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$ B) $\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$ C) $\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$ D) $\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$ E) $\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$

32. What is the digit in the tens place in the sum: $(1!)+(3!)+(9!)+(27!).....+(729!)$?

A) 8 B) 2 C) 5 D) 4 E) 7

33. Norman invests \$10,000 in an account with a fixed interest rate of 4.25% that is compounded quarterly. How long will it take for him to have a total of \$14,500 in the account? (nearest quarter of a year)

A) 8.75 B) 8.25 C) 10.50 D) 9.25 E) 8.50

34. $9\frac{1}{3}\%$ of $\left(\frac{1}{4}\div 0.58333...\right)=$

A) 4 B) $\frac{1}{25}$ C) $\frac{21}{500}$ D) $\frac{7}{250}$ E) $\frac{3}{50}$

35. $223_4+425_6+205_8=$ _____$_{10}$

A) 853 B) 273 C) 522 D) 337 E) 373

36. If $\frac{A}{2x+5}+\frac{B}{x-13}=\frac{16x-22}{2x^2-21x-65}$, then $A+B=$

A) 14 B) 8 C) 10 D) 6 E) 12

37. The total surface area of a regular tetrahedron is $147\sqrt{3}\text{ cm}^2$. What is the perimeter of one face of the tetrahedron?

A) 21 cm B) $21\sqrt{3}$ cm C) $7\sqrt{3}$ cm D) 7 cm E) $14\sqrt{3}$ cm

38. $\angle A$ and $\angle B$ are supplementary. If $m\angle A=(4x)^\circ$ and $m\angle B=(5x+18)^\circ$, find $m\angle B$.

A) 18° B) 108° C) 36° D) 90° E) 72°

39. Given $f(x)=2x-1$ and $g(x)=x^3-8$, find $g(f(x))$.

A) $8x^3-4x^2+2x-9$ C) $8x^3-12x^2+6x-1$ E) $8x^3-10$

B) $8x^3-1$ D) $8x^3-12x^2+6x-9$

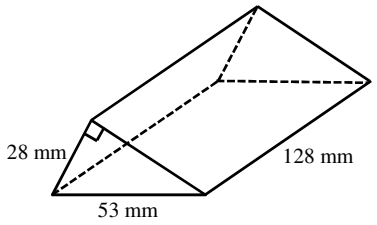
40. A large cylindrical tank holds 6500 gallons of water. If the height of the tank is 1 yard, what is the radius of the tank to the nearest inch?

A) 362 in. B) 136 in. C) 115 in. D) 427 in. E) 284 in.

41. Box A contains 5 pieces of paper numbered 1, 3, 5, 7 and 9. Box B contains 3 pieces of paper numbered 1, 4 and 9. One piece of paper is drawn at random from each box. What is the probability that the two numbers obtained will have a sum that is divisible by 3?

A) $\frac{2}{15}$ B) $\frac{1}{5}$ C) $\frac{7}{15}$ D) $\frac{4}{15}$ E) $\frac{2}{5}$

42. The volume of the right triangular prism shown is _____mm^3.



A) 80640 B) 17612 C) 19096 D) 94976 E) 11852

43. Solve $\log_9 81+\log_9\left(\frac{1}{9}\right)-\log_9 3=\log_9 x$ for x.

A) 3 B) 1 C) $\sqrt{3}$ D) 9 E) 27

44. What is domain of the relation $(x-2)^2+(y-3)^2=25$?

A) $[2,3]$ B) $(-2,8)$ C) $[-2,8]$ D) $[-3,7]$ E) $(-3,7)$

45. Evaluate $\lim_{h\to 0}\frac{\tan\left(\frac{\pi}{4}+h\right)-\tan\left(\frac{\pi}{4}\right)}{h}$.

A) $\frac{\sqrt{2}}{2}$ B) $\sqrt{2}$ C) 2 D) 1 E) $\frac{1}{2}$

46. If $f(x)=3^x$, find $f'(x)$.

A) $3x$ B) $3^x\ln 3$ C) $x\cdot 3^{x-1}$ D) 3^x E) $\frac{3^x}{\ln 3}$

47. The odds of drawing a pink raffle ticket at random from a bucket 495 tickets are 4:7. How many pink tickets would have to be removed from the bucket to reduce the odds to 1:3?

A) 15 B) 65 C) 105 D) 95 E) 75

48. What is the area of the convex quadrilateral with the vertices $(4,10)$, $(9,7)$, $(11,2)$ and $(2,2)$.

A) 83 B) 45.5 C) 75 D) 9 E) 20.25

49. Given a sequence with Fibonacci characteristics $2, a, b, 0, 1, c...$ find the value of $a+b+c$.

A) -1 B) 2 C) 0 D) -2 E) 1

50. Mr. Thompson has 12 students on his math team. He would like to form a study group including 5 students, but only one of his top three students. How many distinct study groups could he form?

A) 378 B) 264 C) 126 D) 210 E) 756

51. Given $\int_{-8}^{5} f(x)\,dx = 17$, find $\int_{-8}^{5} \left(2f(x)+8\right)dx$

A) 73 B) 138 C) 42 D) 10 E) 121

52. If $\cos\theta = -\frac{11}{61}$ and $0 \le \theta \le \pi$, then $\tan\theta =$

A) $\frac{60}{11}$ B) $\frac{11}{61}$ C) $-\frac{60}{61}$ D) $-\frac{60}{11}$ E) $\frac{60}{61}$

53. $f(x) = ax^5 + bx^3 + cx + 12$ and $f(8) = 19$, find $f(-8)$.

A) 7 B) 19 C) 24 D) 26 E) 5

54. Find the constant term in the expansion of $\left(3x^2 - \frac{5}{x}\right)^6$.

A) 5625 B) -20000 C) 84375 D) -1000 E) 15625

55. How many distinct arrangements are there of three letters chosen from the words "MATH TEAM"?

A) 72 B) 144 C) 42 D) 96 E) 56

56. If $\frac{x-8}{x+21} + \frac{x+21}{x-8}$ is equal to the mixed number $A\frac{B}{(x+21)(x-8)}$, then $B =$

A) 484 B) 169 C) 168 D) 841 E) 505

57. $f(x) = 1 + x - \frac{x^2}{2} - \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} - \frac{x^6}{6!} \ldots$. Find the 10^{-8} place of $f(4)$.

A) 6 B) 9 C) 5 D) 2 E) 1

58. A curve has equation $xy^3 + 2xy^2 = 3$. Find the slope of the tangent to this curve at the point (1, 1).

A) 0 B) $-\frac{3}{7}$ C) -1 D) $\frac{3}{7}$ E) $-\frac{3}{4}1$

59. The length of a rectangular picture is three times the width. The picture is surrounded by a frame which is 5 inches wide. If the perimeter of the outside of the frame is 96 inches, what is the length of the picture in inches?

A) 7 in. B) 21 in. C) 24 in. D) 8 in. E) 30 in.

60. How many positive perfect cubes are factors of $(4!)(5!)(6!)$?

A) 6 B) 4 C) 8 D) 5 E) 7

2014-2015 TMSCA Mathematics Test Ten Answers

1. E	21. D	41. D
2. A	22. E	42. A
3. B	23. C	43. A
4. C	24. B	44. D
5. C	25. D	45. C
6. A	26. D	46. B
7. B	27. B	47. E
8. A	28. C	48. B
9. C	29. A	49. E
10. D	30. A	50. A
11. A	31. E	51. B
12. D	32. A	52. D
13. A	33. A	53. E
14. E	34. B	54. C
15. C	35. D	55. D
16. B	36. C	56. D
17. A	37. B	57. E
18. D	38. B	58. B
19. C	39. D	59. B
20. A	40. C	60. C

2014-2015 TMSCA Mathematics Test Ten Select Solutions

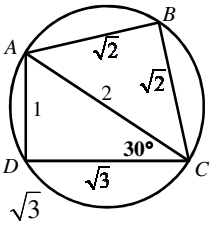
5. The radius of the circle (2) is also half the diagonal. Use this and the special triangle to find the radius of the second circle. The areas of the circles form a geometric sequence with the first two terms 4π and 2π, so the sum of the infinite sequence is $\frac{4\pi}{1-\frac{1}{2}} = 8\pi$.

8. This fraction is $\frac{36}{77_8} = \frac{30}{63_{10}} = \frac{10}{21_{10}} = \frac{12}{25_8}$.

17. The $A^5 + 5A^4B.....$ expression is the binomial expansion of $(A+B)^5$, so the sum of the roots to the 5th power is $\left(\frac{11}{2}\right)^5 = \frac{161051}{32}$.

18. $\left(\sin^2\alpha + \cos^2\alpha\right)\left(\sin^2\alpha - \cos^2\alpha\right) = 1\left(-\cos 2\alpha\right) = -\cos 2\alpha$.

25. Let the diameter of the circle be 2. Since each of the triangles are inscribed in semicircles, $\angle B$ and $\angle D$ are right triangles with the side lengths shown. The probability that a dart would land inside the quadrilateral is $\frac{1+\frac{\sqrt{3}}{2}}{\pi}$, so the probability of a dart landing outside would be $1-\frac{1+\frac{\sqrt{3}}{2}}{\pi}$.



30. $3\frac{3}{10} = 3+\frac{1}{\frac{10}{3}} = 3+\frac{1}{3+\frac{1}{3}}$, so $A=3$, $B=3$ and $C=2$. The sum is 8.

32. There are no changes in the 10's place after the 9! term is added, so the tens digit in $1!+3!+9!$ equals the 10's digit for the whole expression or 8.

35. $2\left(4^2\right)+2(4)+3+4\left(6^2\right)+2(6)+5+2\left(8^2\right)+5 = 337$

36. Multiply the whole equation by the common denominator to get $A(x-13)+B(2x+5) = 16x-22$, so solve the system $A+2B=16$ and $-13A+5B=-22$ (when $x=0$) to get $B=6$ and $A=4$ for a sum of 10.

46. The derivative of $f(x)=a^x$ is $f'(x)=a^x\ln a$, so the derivative of $f(x)=3^x$ will be $f'(x)=3^x\ln 3$.

53. Let $g(x)=ax^5+bx^3+x$, then $g(8)=7$ and $f(-8)=-g(8)+12=-7+12=5$.

54. The constant term in the expansion is ${}_6C_2\left(3x^2\right)^2\left(\frac{-5}{x}\right)^4 = 84375$ because the variables all divide to 1.

55. If there are 2-T's then there are 4 other choices for the last letter and 3 arrangements for each set of 3 letters, so there are 12 arrangements with 2-T's. Similarly, there are 12 arrangements with 2-A's and 2-M's. Also, there are $5\cdot4\cdot3=60$ arrangements with no repeated letters. In total there are $3(12)+60=96$ possible arrangements.

57. This is the MacClaurin series expansion of $f(x)=\sin x+\cos x$, so $f(4)=\sin 4+\cos 4\approx -1.410446116$ and the 10^{-8} place is 1