



**TMSCA HIGH SCHOOL
MATHEMATICS
TEST #10 ©
FEBRUARY 15, 2014**

GENERAL DIRECTIONS

1. About this test:
 - A. You will be given 40 minutes to take this test.
 - B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

1. Evaluate $3+7\times 3-6+(2+19)\div 21\times 9$.

- A) 33 B) -21 C) 27 D) -30 E) $19\frac{2}{9}$

2. If $2a+7b-4a+2=9$ then $-2a+7b+2=9$. This is an example of _____.

- A) Addition Property of Equality C) Substitution Property E) Subtraction Property of Equality
 B) Distributive Property D) Closure Property

3. A department store is having a special sale where customers get 15% off the first \$20 worth of purchases, 25% off the next \$50 worth of purchases and 40% off purchases over \$70. How much will a customer's final bill be if she buys \$110 worth of merchandise?

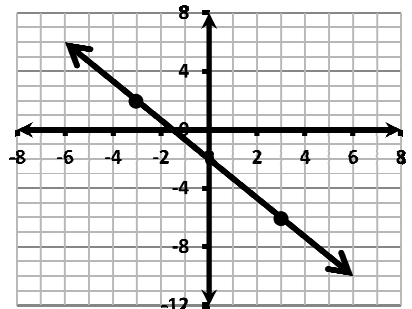
- A) \$66 B) \$78.50 C) \$70.50 D) \$64.50 E) \$53.50

4. If $U = \{a,b,c,d,e,f,g,h\}$, $A = \{a,c,e,g\}$, and $B = \{b,c,d,e\}$, find $A' \cap B'$.

- A) $\{a,f,g,h\}$ B) $\{a,b,c,d,e,g\}$ C) $\{b,c,d,e,f\}$ D) $\{a,b,d,f,g,h\}$ E) $\{f,h\}$

5. The line parallel to the line shown and through the point $(2,3)$ intersects the x-axis at $x = ?$

- A) $\frac{17}{3}$ B) $-\frac{4}{3}$ C) $\frac{4}{3}$ D) $\frac{13}{3}$ E) $\frac{17}{4}$



6. Simplify $\left(\frac{x^2+10x+25}{x^2-25}\right)\left(\frac{x^2-10x+25}{x-5}\right)$.

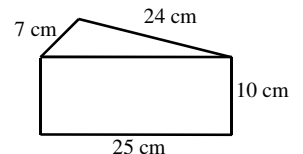
- A) $x+5$ B) x^2+25 C) $x^2+10x+25$ D) x^2-25 E) $x-5$

7. Angle A is complementary to angle B and supplementary to angle C. If $m\angle B = 4x - 1$ and $m\angle C = 12x + 1$. Find the measure of angle A.

- A) 43° B) 52° C) 47° D) 38° E) 55°

8. Calculate the total surface area of the triangular prism shown.

- A) 560 cm^2 B) 644 cm^2 C) 840 cm^2 D) 924 cm^2 E) 728 cm^2



9. Which of the following is a solution to the system of inequalities $x - y < 2$, $x > -2$ and $y \leq 3$?

- A) $\left(1, \frac{3}{2}\right)$ B) $(3,1)$ C) $\left(\frac{3}{2}, -1\right)$ D) $(-2, -4)$ E) $(1, -3)$

10. If $25^{x-y} = 125$ and $4^{x+y} = 128$, then $y = ?$

- A) 2.5 B) 1.75 C) 1 D) 2 E) 0.25

11. $f(x) = 2x + 1$ and $g(x) = x^2$. $g(f(x+2)) =$

- A) $4x^2 + 25$ B) $4x^2 + 12x + 9$ C) $4x^2 + 9$ D) $x^2 - 25$ E) $4x^2 + 20x + 25$

12. Abby and Bill can build a brick wall that is 4 feet long and 3 feet high in 6 hours together. How long will it take for them to build a wall twice as long and twice as high if they work at the same pace?

- A) 12 hours B) 4 hours C) 6 hours D) 24 hours E) 8 hours

13. What is the period of the function $f(x) = -3\sin\left(\frac{x}{3}\right) + 4$?

- A) 3 B) $\frac{2\pi}{3}$ C) 6π D) 3π E) $\frac{3\pi}{2}$

14. Caroline left her campsite to hike for 3 miles on a bearing of 273° . From there she hiked another 4.2 miles on a bearing of 194° . How much further will she hike if she goes directly back to camp? (nearest yard)

- A) 3574 yd B) 9870 yd C) 3786 yd D) 12672 yd E) 10260 yd

15. $A = \begin{pmatrix} a & 4 \\ -5 & a \end{pmatrix}$ and $A^2 = \begin{pmatrix} -16 & 16 \\ -20 & -16 \end{pmatrix}$. Find the value of a .

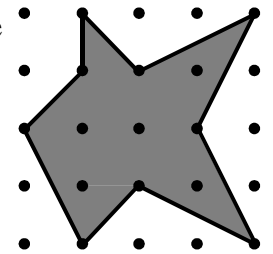
- A) -2 B) 4 C) -5 D) 2 E) -4

16. Given $\int_2^{11} f(x) dx = 27$, evaluate $\int_2^{11} \left[\frac{f(x)+5}{2} \right] dx$.

- A) 54 B) 58.5 C) 18.5 D) 36 E) 32

17. The dots in the diagram are 0.3 inches apart both vertically and horizontally. Calculate the area of the shaded region.

- A) 2.550 in^2 B) 1.035 in^2 C) 0.765 in^2 D) 0.311 in^2 E) 0.673 in^2

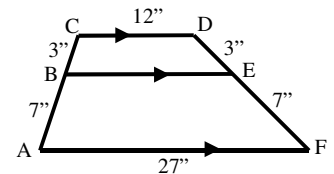


18. Burr's Ice Cream Shop serves 6 different flavors of ice cream with 3 container choices and 2 toppings. How many possible ways are there to order 2 scoops in a container with 1 topping?

- A) 126 B) 168 C) 216 D) 63 E) 108

19. Find BE.

- A) $19.5''$ B) $18''$ C) $19''$ D) $16.5''$ E) $13.5''$



20. What is the sum of all the numbers in row 13 of Pascal's triangle?

- A) 4096 B) 16384 C) 8192 D) 1716 E) 3003

21. If $f(x) = 3x^4 - 7x^3 + 5x^2 - 24x + 8$ then $f''(-1) =$

- A) 88 B) -67 C) -23 D) 4 E) 47

22. $14_{16} + 12_8 + 10_4 = \underline{\hspace{2cm}}_2$.

- A) 100000 B) 100101 C) 100010 D) 100001 E) 101001

23. If $g(x) \leq f(x) \leq h(x)$ for all x, k in $[a, b]$, where $x \neq k$, and $\lim_{x \rightarrow k} g(x) = L$ and $\lim_{x \rightarrow k} h(x) = L$ then

$\lim_{x \rightarrow k} f(x) = L$. This theorem is known as:

- (A) Sandwich Theorem (C) Rolle's Theorem (E) Fundamental Theorem of Calculus
 (B) Intermediate Value Theorem (D) Fundamental Theorem of Algebra

24. You commute 56 miles one way to work. The trip to work takes 10 minutes longer than the return. Your average speed on the trip home is 8 mph faster. How long does it take you to get home?

- A) 70 minutes B) 50 minutes C) 48 minutes D) 66 minutes E) 60 minutes

25. A full cylindrical water tank with a radius of 1 yard and a height of 3 yards springs a steady leak. If it is empty four hours after developing the leak, what is the rate of loss? (nearest gallon per minute)

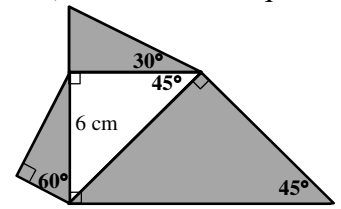
- A) 7 gal/min B) 8 gal/min C) 6 gal/min D) 2 gal/min E) 3 gal/min

26. The seven trapezoidal means are constructed as segments in a trapezoid. Which one is the longest?

- A) Heronian B) Contraharmonic C) Geometric D) Harmonic E) Root-mean square

27. Find the area of the shaded region. (nearest tenth)

- A) 69.8 cm² B) 43.6 cm² C) 54.2 cm² D) 61.6 cm² E) 56.7 cm²



28. Solve $\sqrt{17-2x}+1=x$ for x .

- A) ± 4 B) ± 8 C) 2 D) 4 E) 8

29. To play in a charity tournament, student council decides to form a basketball team with 10 players. In how many ways can they choose a team from a council of 9 girls and 6 boys if the team must have at least 6 girls?

- A) 1260 B) 3960 C) 5580 D) 1980 E) 2121

30. In a survey of 34 academic UIL team members, 18 liked water to drink on trips while 23 liked soda. If 4 students didn't like either, how many students liked both?

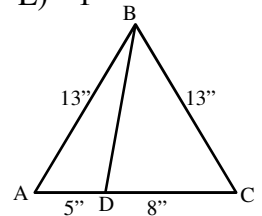
- A) 30 B) 18 C) 14 D) 11 E) 4

31. If P , Q and R are real numbers such that $P+Q+R=8$, $R^2=P^2+Q^2$ and $PQ=8$, find the value of R .

- A) 4 B) 8 C) 3 D) 6 E) 1

32. Find BD . (nearest quarter inch)

- A) $9\frac{1}{2}$ B) 10 C) $10\frac{1}{4}$ D) $10\frac{3}{4}$ E) $11\frac{1}{4}$



33. How many points of intersection occur when $r=3\sin\theta-1$ and $r=3$ are graphed on the polar coordinate system?

- A) 0 B) 1 C) 2 D) 3 E) 4

34. Train A is travelling north out of Chicago at a rate of 85 mph and Train B is travelling east out of Chicago at a rate of 92 mph. How fast is the distance between trains A and B changing when they are 15.4 miles and 52.8 miles out of Chicago respectively?

- A) 112.12 mph B) 131 mph C) 120.63 mph D) 107.36 mph E) 119.18 mph

35. The length of one edge of a regular octahedron is $2\sqrt{3}$ dm. Calculate the surface area.

- A) $24\sqrt{3}$ dm² B) $36\sqrt{3}$ dm² C) $72\sqrt{3}$ dm² D) $48\sqrt{3}$ dm² E) $60\sqrt{3}$ dm²

36. Find the sum of all the three digit numbers whose digits have a sum of eight and whose digits can all be used to form a perfect cube.

- A) 1776 B) 1925 C) 861 D) 915 E) 1420

37. Two fair six-sided dice are rolled. What are the odds that the sum of the two top faces will be an abundant number?

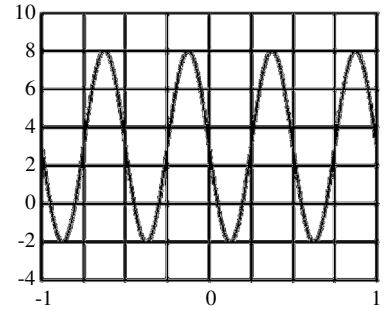
- A) 1:5 B) 1:35 C) 1:17 D) 1:11 E) 1:10

38. The polynomial $x^2 - 4x + 3$ is a factor of $x^3 + (a - 4)x^2 + (3 - 4a)x + 3$. Find the value of the constant a .

- A) $\frac{1}{32}$ B) -6 C) 0 D) $\frac{43}{216}$ E) 1

39. Which of the following is the function represented in the graph?

- A) $f(x) = 3 - 5\sin(4\pi x)$ C) $f(x) = 5 - 3\cos(4\pi x)$ E) $f(x) = 5 - 4\sin(3\pi x)$
 B) $f(x) = 5 - 3\sin\left(\frac{x}{2\pi}\right)$ D) $f(x) = 3 - 5\cos\left(\frac{x}{4\pi}\right)$



40. $12_4 + 23_5 + 34_6 + 45_7 + 56_8 + 67_9 = \text{---}_{10}$.

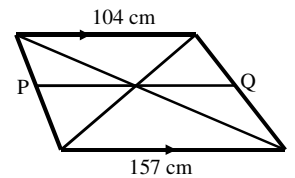
- A) 214 B) 175 C) 237 D) 154 E) 181

41. $\csc^2 x + \sec^2 x =$

- A) $2\sec x \csc x$ B) $\csc 2x$ C) $2\sec x$ D) $2\csc(2x)$ E) $\sec^2 x \csc^2 x$

42. Find PQ. (nearest tenth centimeter)

- A) 127.8 cm B) 129.6 cm C) 125.1 cm D) 132.3 cm E) 133.2 cm



43. The first and fifth term of an infinite geometric sequence are 512 and $\frac{81}{8}$ respectively. Calculate the sum of the sequence.

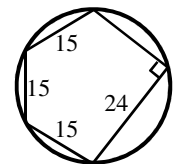
- A) $\frac{4177}{8}$ B) $\frac{6505}{8}$ C) $\frac{5341}{5}$ D) $\frac{4731}{8}$ E) $\frac{4096}{5}$

44. If $\frac{x-20}{x+20} + \frac{x+20}{x-20} = A\frac{B}{C}$ then $B =$

- A) 400 B) 1600 C) 3200 D) 1000 E) 2400

45. Find the area of the pentagon inscribed in the circle. (nearest square unit)

- A) 441 units² B) 532 units² C) 508 units² D) 465 units² E) 706 units²



46. Find the slope of the tangent to the graph of $2x^2 + 7y^2 = 39$ at the point $(-4, 1)$.

- A) $\frac{8}{7}$ B) $\frac{55}{14}$ C) $-\frac{32}{7}$ D) $-\frac{5}{4}$ E) $\frac{25}{4}$

47. A basketball player consistently makes 2 out of 3 free throw shots he takes. If he takes 6 shots in a game, what is the probability that he will make at least 4 of them?

- A) $\frac{496}{729}$ B) $\frac{80}{243}$ C) $\frac{112}{729}$ D) $\frac{256}{729}$ E) $\frac{16}{729}$

48. In triangle PQR the three medians intersect at M. If the median QA has length 5.4 cm then QM = ?

- A) 1.8 cm B) 2.7 cm C) 4.5 cm D) 3.6 cm E) 2.1 cm

49. Box A contains 6 red balls and 2 green balls. Box B contains 4 red balls and 3 green balls. A fair die is thrown. If an even number is obtained, a ball is selected from box A; if an odd number is obtained, a ball is selected from box B. If a red ball is selected, what is the probability that it came from box B?

- A) 0.568 B) 1.313 C) 0.762 D) 0.432 E) 0.276

50. Simplify to the nearest ten-thousandth place: $-1 + \frac{(1.5)^2}{2!} - \frac{(1.5)^4}{4!} + \frac{(1.5)^6}{6!} - \dots$

- A) 4.4817 B) 0.4055 C) -0.0707 D) -0.9975 E) 0.0262

51. The operation € is defined as $A\text{€}B = A^3 + 3A + B^3$. Compute $3\text{€}(1\text{€}2)$.

- A) 756 B) 1764 C) 12 D) 42876 E) 36

52. How many solutions are there to $7x + 8y = 221$ such that $x, y \in \mathbb{Z}^+$.

- A) 4 B) 24 C) 7 D) 18 E) 8

53. Let $a = \log x$, $b = \log y$ and $c = \log z$. Write $\log\left(\frac{x^2\sqrt{y^3}}{z^3}\right)$ in terms of a , b , and c .

- A) $\frac{3ab}{2} - 3c$ B) $\frac{a^2\sqrt{b^3}}{c^3}$ C) $\frac{-3ab}{2c}$ D) $a^2 + \sqrt{b^3} - c^3$ E) $2a + \frac{3}{2}b - 3c$

54. If $\frac{7x+13}{x^2+2x-3} = \frac{A}{x+3} + \frac{B}{x-1}$, then $AB =$

- A) 7 B) -6 C) 10 D) -3 E) 6

55. If $y^2 = 5 - 12i$ and $y^3 = -9 - 46i$ where $y = a + bi$ then $a + b =$

- A) -38 B) 5 C) -62 D) 1 E) 6

56. Let P and Q be the roots of $2x^2 + 4x - 11$. Find $P^5 + 5P^4Q + 10P^3Q^2 + 10P^2Q^3 + 5PQ^4 + Q^5$.

- A) -1 B) -17 C) -22 D) -32 E) -16

57. How many positive perfect cubes are factors of $(3!)(4!)(7!)$?

- A) 5 B) 7 C) 4 D) 6 E) 8

58. If $f(x) = 2\cos^2 x$, $g(x) = x^2 + 9$, and $h(x) = g(f'(x))$, find $h\left(\frac{\pi}{6}\right)$.

- A) 21.00 B) 9.25 C) 18.00 D) 13.91 E) 12.00

59. Find the constant term in the expansion of $\left(2x^2 - \frac{3}{x}\right)^9$.

- A) 19683 B) -145152 C) 979776 D) 489888 E) -326592

60. The areas of the bases of a frustum are A_1 and A_2 and the height is 13 inches. The Heronian mean of A_1 and A_2 is $144\pi \text{ in}^2$. Find the volume of the frustum.

- A) $624\pi \text{ in}^3$ B) $1872\pi \text{ in}^3$ C) $1248\pi \text{ in}^3$ D) $2808\pi \text{ in}^3$ E) $1404\pi \text{ in}^3$

2013-2014 TMSCA Mathematics Test Nine Answers

1. C	21. A	41. E
2. C	22. C	42. C
3. B	23. A	43. E
4. E	24. E	44. B
5. E	25. B	45. C
6. A	26. B	46. A
7. C	27. C	47. A
8. E	28. D	48. D
9. A	29. E	49. D
10. C	30. D	50. C
11. E	31. C	51. B
12. D	32. E	52. A
13. C	33. C	53. E
14. B	34. A	54. C
15. D	35. A	55. D
16. D	36. A	56. D
17. C	37. B	57. D
18. A	38. E	58. E
19. D	39. A	59. D
20. C	40. E	60. B

10. Use $5^{2x-2y} = 5^3$ and $2^{2x+2y} = 2^7$ to form the system of equations $2x - 2y = 3$ and $2x + 2y = 7$. The solution point to the system is $\left(\frac{5}{2}, 1\right)$.

15.

$$\begin{pmatrix} a & 4 \\ -5 & a \end{pmatrix} \begin{pmatrix} a & 4 \\ -5 & a \end{pmatrix} = \begin{pmatrix} a^2 - 20 & 8a \\ -10a & a^2 - 20 \end{pmatrix}$$
 so $a = 2$

16.

$$\int_2^{11} \left[\frac{f(x)+5}{2} \right] dx = \frac{1}{2} \int_2^{11} f(x) dx + \int_2^{11} \frac{5}{2} dx$$

$$= \frac{1}{2}(27) + \frac{5}{2}(11-2) = 36$$

17. $\frac{10+9}{2} - 1 = 8.5$ square units on the graph, but each linear unit on the graph is 0.3 in, so each square unit on the graph is 0.09 in^2 . $8.5(0.3)^2 = 0.765 \text{ in}^2$.

18. ${}_{6+2-1}C_2 \cdot 3 \cdot 2 = 126$

19. The difference in the bases is 15".

$$BE = 12 + \frac{3}{3+7}(15) = 16.5 \text{ '' or}$$

$$BE = 27 - \frac{7}{3+7}(15) = 16.5 \text{ ''}$$

26. The contraharmonic mean is the longest of the trapezoidal means.

31. $P + Q = 8 - R$, so

$$P^2 + 2PQ + Q^2 = 64 - 16R + R^2$$

Substitute to generate the equation

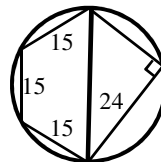
$$2(8) + R^2 = 64 - 16R + R^2, \text{ so } R = 3.$$

32. $BD = \sqrt{13^2 - 5(8)} \approx 11.358$

36. The two perfect cubes whose digits have a sum of 8 are 125 and 512.
 $125 + 152 + 251 + 215 + 521 + 512 = 1776$

42. The harmonic mean of the lengths of the bases is the length of the parallel segment through the intersection of the diagonals. So $PQ = \frac{2(104)(157)}{104+157} \approx 125.1$

45. Because of the right angle, the extra line drawn is a diameter.



Also, because the 15's are all the same, their central angles will be 60° and the length of the diameter will be 30. The other leg of the right triangle is 18. Using this information,

$$A = \frac{15^2 \sqrt{3}}{4}(3) + \frac{1}{2}(24)(18) \approx 508 \text{ units}^2$$

56. The expression is the binomial

expansion of $(P + Q)^5 = \left(-\frac{4}{2}\right)^5 = -32$

60. The volume of a frustum is the Heronian mean of the areas of the bases multiplied by the height of the frustum, so $V = 144\pi(13) = 1872\pi$.