T M S C A H I G H S C H O O L

M A T H E M A T I C S

T E S T # 1 ©

O C T O B E R 2 6 , 2 0 1 3

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2013-2014 TMSCA Mathematics Test One

1. Evaluate: $27\bullet\left(0.\overline{6}\div 0.6\right)+0.375\div 0.\overline{2}\bullet 64$.

(A) 135 (B) 150 (C) $31.\overline{3}$ (D) 138 (E) $28.\overline{3}$

2. Karen borrowed \$1115 for 8 months at a simple interest rate of 3.5%. What will her monthly payments be to the nearest cent?

(A) \$171.90 (B) \$142.63 (C) \$144.25 (D) \$142.66 (E) \$139.38

3. There are 334 seniors at City High School taking math and/or science classes. If 197 students are taking science and 259 are taking math, how many students are taking both?

(A) 197 (B) 259 (C) 106 (D) 334 (E) 122

4. Which of the following is **not** an equation of the line parallel to $2x+7y=13$ through the point $(-2,3)$?

(A) $2x-7y=17$ (B) $4x+14y=34$ (C) $-2x-7y=-17$ (D) $2x+7y=17$ (E) $-6x-21y=-51$

5. Simplify: $\dfrac{4x^2-25}{x^2+10x+21}\bullet\dfrac{x^2+11x+24}{2x^2+19x+35}$.

(A) $\dfrac{2x^2+11x-40}{x^2+49}$ (B) $\dfrac{2x^2-11x+40}{x^2-49}$ (C) $\dfrac{2x^2+11x+40}{x^2-14x+49}$ (D) $\dfrac{2x^2-11x+40}{x^2+49}$ (E) $\dfrac{2x^2+11x-40}{x^2+14x+49}$

6. A boat travelling with the current can make a trip in 7 hours. The return trip takes 15 hours. If the current flows at a rate of 2 mh^{-1}, what is the speed of the boat in still water?

(A) 7.5 mh^{-1} (B) 5.5 mh^{-1} (C) 2 mh^{-1} (D) 3.5 mh^{-1} (E) 4 mh^{-1}

7. Which of the following sets of numbers is closed under subtraction?

(A) Irrational (B) Natural (C) Whole (D) Imaginary (E) Rational

8. A regular pentagon with vertices A, B, C, D and E respectively is inscribed in a circle. Find the measure of $\angle AEC$.

(A) 108° (B) 72° (C) 144° (D) 90° (E) 54°

9. Which of the following types of functions is the best model for the value of an investment with a fixed initial investment and fixed interest rate?

(A) Logarithmic (B) Quadratic (C) Trigonometric (D) Exponential (E) Logistic

10. The ratio of the width to the length of a rectangle is 5:7. The perimeter is 124.8 cm. What is the area of the rectangle?

(A) 35 cm^2 (B) 3785.6 cm^2 (C) 946.4 cm^2 (D) 876.6 cm^2 (E) 182 cm^2

11. Given $a_{n+2}=a_n\left(a_{n+1}\right)$, $a_1=3$ and $a_2=-4$ find a_6.

(A) -27648 (B) 576 (C) -576 (D) 15925248 (E) 27648

12. A 13-ft plank is used for a ramp up to the edge of a porch that stands 2.75 ft off the ground. Find the angle of elevation to the nearest second.

(A) 77° 44’ 15” (B) 12° 12’ 45” (C) 12° 47’ (D) 1° 21’ 27” (E) 11° 56’ 39”

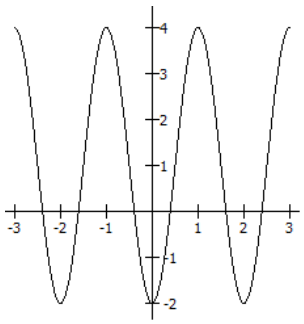
13. Find the total surface area of a right cone with base diameter 6.26 cm and height 10.20 cm.

(A) 209.83 cm^2 (B) 323.71 cm^2 (C) 104.64 cm^2 (D) 135.69 cm^2 (E) 418.58 cm^2

14. What is the sum of the coefficients of the expansion of $(x-3)^4$?

(A) -2 (B) -108 (C) 16 (D) -12 (E) 81

15. Which of the functions below generates the graph shown?



(A) $f(x) = -3\sin(\pi(\theta - 2))$ (C) $f(x) = 3\cos(\pi(\theta - 2)) + 1$ (E) $f(x) = \sin(\pi(\theta - 2)) + 3$

(B) $f(x) = -3\cos(\pi(\theta - 2)) + 1$ (D) $f(x) = \cos(\pi(\theta - 2)) + 3$

16. If $f(x) = \frac{x-3}{x+7}$ find the value of $f^{-1}(2)$.

(A) 10 (B) $-\frac{1}{9}$ (C) -11 (D) -17 (E) $-\frac{10}{3}$

17. Let $f(x) = \frac{5x^2 - 2x + 7}{x+3}$ and $s(x)$ be the slant asymptote. Find the value of $s(3)$.

(A) -2 (B) 7 (C) 8 (D) 28 (E) -17

18. Let $f(x) = ax^3 - bx + 20$ where a and b are constants. If $f(6) = 9$ then $f(-6) =$

(A) 49 (B) 31 (C) 11 (D) 29 (E) -49

19. How many degrees are in $7\frac{2}{3}\pi$ radians?

(A) 1380° (B) 1260° (C) 8280° (D) 2760° (E) 2520°

20. $\int \frac{x+5}{3} dx =$ _____ $+ C$ where C is some arbitrary constant.

(A) $\frac{x^2 + 5x}{6}$ (B) $\frac{x^2 + 5x}{3}$ (C) $\frac{2x^2 + 5x}{6}$ (D) $\frac{x(2x+5)}{3}$ (E) $\frac{x(x+10)}{6}$

21. Carrie drew 8 hearts, 4 diamonds, 3 clubs and 9 spades from a standard deck of cards. She shuffled the cards she drew then dealt two onto the table. What is the probability that the first card she dealt was a spade and the second was a heart? (nearest %)

(A) 10% (B) 71% (C) 13% (D) 90% (E) 68%

22. What is the sum of all the numbers in the first seven rows of Pascal's triangle ?

(A) 128 (B) 255 (C) 64 (D) 254 (E) 127

23. Point A has polar coordinates $\left(6, \frac{2}{3}\pi\right)$ and rectangular coordinates

(A) $(-3\sqrt{3}, 3)$ (B) $(3, 3\sqrt{3})$ (C) $(3\sqrt{3}, -3)$ (D) $(-3, -3\sqrt{3})$ (E) $(-3, 3\sqrt{3})$

24. How many distinct arrangements can be made from the letters of the word LICORICE?

(A) 40320 (B) 10080 (C) 1260 (D) 180 (E) 720

25. If $f'(x)=12x^3+6x^2-10x+2$ and $f(-1)=3$ then $f(1)=$

(A) 10 (B) 11 (C) 6 (D) 2 (E) 8

26. The length of one edge of a regular tetrahedron is 8 cm. Find the volume.

(A) $\frac{128\sqrt{2}}{3}\text{cm}^3$ (B) $\frac{256\sqrt{3}}{9}\text{cm}^3$ (C) $48\sqrt{3}\text{ cm}^3$ (D) $64\sqrt{3}\text{ cm}^3$ (E) $\frac{256\sqrt{3}}{3}\text{cm}^3$

27. The points $(-6,1)$ and $(8,a)$ are equidistant from the point $(2,7)$. The greatest possible value of a is?

(A) 15 (B) $\sqrt{113}$ (C) 36 (D) $\sqrt{15}$ (E) 10

28. When Jerry received her paycheck, she immediately paid 1/3 of it for rent and \$120 to the phone company. The next day, she paid 25% of what was left for her car payment. Finally, she put half of the remaining money in savings and was left with \$225. How much was she paid?

(A) \$630 (B) \$855 (C) \$760 (D) \$1080 (E) \$1440

29. Let $x^2-y^2=27$. Find $D_x y$.

(A) $\frac{x}{y}$ (B) $x+y$ (C) $x-y$ (D) $\frac{y}{x}$ (E) $\frac{2x-27}{2}$

30. The real solution set to $|2x-7|\le 9$ is

(A) $\{x|\{x\le -1\}\cup\{x\ge 8\}\}$ (B) $\{x|\{x\le -8\}\cup\{x\ge 1\}\}$ (C) $\{x|-1\le x\le 8\}$ (D) $\{x|-8\le x\le 1\}$ (E) $\{x|\{x\le -8\}\cup\{x\ge -1\}\}$

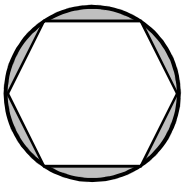
31. The measure of one interior angle of a regular dodecagon is

(A) 108° (B) 144° (C) 136° (D) 150° (E) 156°

32. $\sum_{n=1}^{10}\left[2n(n+1)\right]=$

(A) 490 (B) 220 (C) 110 (D) 880 (E) 770

33. A regular hexagon is inscribed in a circle with a radius of 8 cm. Find the area of the shaded region.



(A) 20.64 cm^2 (B) 166.28 cm^2 (C) 173.35 cm^2 (D) 34.79 cm^2 (E) 22.55 cm^2

34. A circle is circumscribed about a triangle. The center of the circle is the____ of the triangle.

(A) Centroid (B) Circumcenter (C) Incenter (D) Orthocenter (E) Foci

35. The probability of rolling each number one through six on a weighted die is shown in the table below. Find the expected value of a single roll.

x	1	2	3	4	5	6
$p(x)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{7}$	a	$\frac{1}{3}$	$\frac{1}{7}$

(A) $\frac{305}{63}$ (B) $\frac{221}{63}$ (C) $\frac{257}{63}$ (D) $\frac{233}{63}$ (E) $\frac{295}{63}$

36. Triangle PQR is such that $m\angle R=60^\circ$, $PR=8$ and $PQ=7$. There are two possible values for QR. Find the sum of the two values.

(A) 7 (B) 5 (C) 15 (D) 12 (E) 8

37. Four brothers each paint at the same rate. If three of the brothers can paint a fence in 5 hours, how long would it take all four brothers to paint a fence that is twice as long and twice as tall?

(A) 7 hrs 30 mins (B) 15 hrs (C) 10 hrs (D) 20 hrs (E) 12 hrs 20 mins

38. A particular model of car has an advertised gas mileage of 35 mpg for in-town driving. Upon further investigation, a consumer discovers that the gas mileage is normally distributed with a standard deviation of 2.5 mpg. What is the probability that the driver will get over 37 mpg for in-town driving?

(A) 29% (B) 79% (C) 21% (D) 33% (E) 31%

39. Ned left his front porch and travelled 548 yards on a bearing of 52° then turned and travelled 372 yards on a bearing of 184°. How far will Ned have to travel to go directly back to his porch? (nearest foot)

(A) 2760 ft. (B) 1222 ft. (C) 920 ft. (D) 1380 ft. (E) 407 ft.

40. Find the sum of the infinite geometric series $2.7+1.5+0.8\overline{3}\ldots$.

(A) 3.375 (B) 2.144 (C) 3.950 (D) 13.500 (E) 6.075

41. Let P and Q be the roots of $6x^2-25x+14=0$. Find $P^4+4P^3Q+6P^2Q^2+4PQ^3+Q^4$.

(A) $\frac{390625}{1296}$ (B) $\frac{83521}{1296}$ (C) $\frac{390625}{81}$ (D) $\frac{50625}{16}$ (E) $\frac{194737}{1296}$

42. Simplify $a^2\times b^3\times a^{-2}\div b^{-3}\div a^3\times b^{-5}$.

(A) $\frac{1}{a^3b}$ (B) a^3b (C) $\frac{a^3}{b^5}$ (D) a^3b^5 (E) $\frac{b}{a^3}$

43. Which of the following is an equation of the tangent line to $f(x)=7x^3+2x-5$ when $x=-3$?

(A) $y-200=191(x-3)$ (C) $y+191=200(x-3)$ (E) $y-191=-200(x-3)$

(B) $y-191=-200(x+3)$ (D) $y+200=191(x+3)$

44. Texas HS ends every school day at 3:22 pm. What is the smallest angle between the hour and minute hands of a clock at 3:22 pm? (nearest degree)

(A) 42° (B) 31° (C) 46° (D) 30° (E) 37°

45. Simplify $(5+3i)^4$.

(A) $-724+960i$ (B) $-644-960i$ (C) $-724-960i$ (D) $724-960i$ (E) $-644+960i$

46. Let $\|v_1\|=7$ and $\|v_2\|=24$, when the direction angles of v_1 and v_2 are 36° and 77° respectively. Find the direction angle of $\|v_1+v_2\|$. (nearest degree)

(A) 22° (B) 113° (C) 41° (D) 68° (E) 66°

47. Use the Fibonacci type sequence $a,b,2,c,11$ to find the value of $a+b+c$.

(A) 2 (B) 11 (C) 7 (D) 10 (E) 9

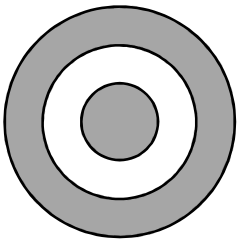
48. Find $f(2)+f(4)+f(-2)$ if $f(x)=\begin{cases} x+3, & x<0 \\ x^2, & 0\le x\le 3 \\ \frac{x}{2}, & x>3 \end{cases}$

(A) 5 (B) 7 (C) 6 (D) 3 (E) 4

49. Given $f(x)=x+7$ and $g(x)=(x-2)^3$, find $g(f(2))\bullet f(g(1))$.

(A) 1512 (B) 2744 (C) 1728 (D) 1807 (E) 2058

50. The concentric circles shown have radii of 2 cm, 4 cm and 6 cm. What are the odds that a particle hitting the circles at random would hit the shaded regions?



(A) 3:1 (B) 2:1 (C) 3:2 (D) 1:3 (E) 1:2

51. Let $f(x)=\dfrac{1}{\sqrt{9-x^2}}$. At which of the following intervals is f continuous?

(A) $(-3,3)$ (B) $(-9,9)$ (C) $[0,3]$ (D) $[-3,0)$ (E) $[-9,9]$

52. If $\sin\theta>0$ and $\tan\theta<0$ then in where will θ terminate?

(A) QI (B) QII (C) QIII (D) QIV (E) y - axis

53. The medians of triangle JKL intersect at M. If the median through K has a length of 7.2 cm, find MK.

(A) 3.6 cm (B) 2.4 cm (C) 1.8 cm (D) 4.8 cm (E) 6 cm

54. Gillian's average speeds for each morning commute during a work week are 42 mph, 47 mph, 32 mph, 50 mph and 45 mph. What was her average speed for the week?

(A) 43.20 mph (B) 42.71 mph (C) 43.57 mph (D) 42.75 mph (E) 42.17 mph

55. If $\ln 2=x$, $\ln 3=y$ and $\ln 5=z$ then $\ln 7.5=$

(A) $\dfrac{yz}{x}$ (B) $yz-x$ (C) $\dfrac{xy}{z}$ (D) $y+z-x$ (E) $\dfrac{y+z}{x}$

56. Let $f(x)=ax^7-bx^3-cx+7$. If $f(3)=15$ then $f(-3)=$

(A) -15 (B) -1 (C) 9 (D) -20 (E) -3

57. The infinite series $1-\dfrac{1}{2}+\dfrac{1}{3}-\dfrac{1}{4}\ldots$ converges to

(A) $\ln 2$ (B) e^2 (C) e (D) 0 (E) $\dfrac{2}{3}$

58. Ms. Angle must send 3 girls and 4 boys to the counselor for a survey. If her class has 8 girls and 9 boys, how many distinct groups could she send?

(A) 182 (B) 3360 (C) 1016064 (D) 19448 (E) 7056

59. If $y=x^{2x}$ find $\dfrac{dy}{dx}$.

(A) $2x^{2x}(1+\ln x)$ (B) $2x^{2x}$ (C) $4x^{2x}$ (D) $2x^{2x-1}$ (E) $x^{2x}(2+\ln x)$

60. How many solutions are there for $2x+5y=1342$ where x and y are both positive integers?

(A) 135 (B) 137 (C) 136 (D) 134 (E) 133

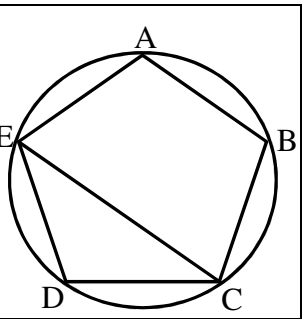
2013-2014 TMSCA Mathematics Test One Answers

1. D	21. C	41. A
2. B	22. E	42. E
3. E	23. E	43. D
4. A	24. B	44. B
5. E	25. B	45. E
6. B	26. A	46. D
7. E	27. A	47. B
8. B	28. D	48. B
9. D	29. A	49. E
10. C	30. C	50. B
11. A	31. D	51. A
12. B	32. D	52. B
13. D	33. D	53. D
14. C	34. B	54. E
15. B	35. D	55. D
16. D	36. E	56. B
17. A	37. B	57. A
18. B	38. C	58. E
19. A	39. B	59. A
20. E	40. E	60. D

2013-2014 TMSCA Mathematics Test One Select Solutions

8. $m\overset{\frown}{AC} = \frac{2}{5}(360^\circ) = 144^\circ$

$m\angle AEC = \frac{1}{2} m\overset{\frown}{AC} = 72^\circ$



15. $f(\theta) = -a\cos(b(\theta - c)) + d$ (cosine graph with reflection over x – axis.

$a = \frac{\max - \min}{2} = \frac{4-(-2)}{2} = 3,$

Period = 2, so $\frac{2\pi}{b} = 2,\ b = \pi$

No apparent horizontal shift, so c is a multiple of the period.

$d = \frac{\max + \min}{2} = \frac{4+(-2)}{2} = 1$

$f(\theta) = -3\cos(\pi(\theta - 2)) + 1$

18. Let $g(x) = ax^3 - bx$ which is an odd function, so

$g(-x) = -g(x).$

$f(6) = 9 = g(6) + 20 \therefore g(6) = -11$

$f(-6) = -g(6) + 20 = 31$

22. $2^0 + 2^1 + 2^2 + \ldots + 2^6 = 2^7 - 1 = 127$

24. 8 letters, 2 – "I"s and 2 – "C"s

number of distinct arrangements = $\frac{8!}{(2!)(2!)}$

26. $V = \frac{a^3\sqrt{2}}{12} = \frac{8^3\sqrt{2}}{12} = \frac{128\sqrt{2}}{3}$

32. $\sum_{n=1}^{10}\left[2n(n+1)\right] = \sum_{n=1}^{10}\left(2n^2 + 2n\right) =$

$\frac{2(10)(11)(21)}{6} + \frac{2(10)(11)}{2} = 880$

35. $a = 1 - \left(\frac{1}{9} + \frac{2}{9} + \frac{1}{7} + \frac{1}{3} + \frac{1}{7}\right) = \frac{1}{21}$

$E(x) = 1\left(\frac{1}{9}\right) + 2\left(\frac{2}{9}\right) + 3\left(\frac{1}{7}\right) + \ldots + 6\left(\frac{1}{7}\right) = \frac{233}{63}$

36. $49 = 64 + x^2 - 2(8)x\cos 60$

$0 = x^2 - 8x + 15$

sum of the roots = $-\frac{b}{a} = \frac{8}{1} = 8$

37. Let r be the rate of one brother.

$3 \bullet 5 \bullet r = 1\,\text{fence}\ \therefore r = \frac{1}{15}\frac{fence}{hour}$

$4 \bullet \frac{1}{15} t = 4$ (the fence now has 4 times the area

$t = 15$ hrs

39. Let $v_1 = \langle 548\cos 52, 548\sin 52\rangle = \langle x_1, y_1\rangle$

and $v_2 = \langle 372\cos 184, 372\sin 184\rangle = \langle x_2, y_2\rangle$

$\|v_1 + v_2\| = \sqrt{(x_1 + x_2)^2 + (y_1 + y_2)^2} \approx 1222\,\text{ft}$

41. $(P+Q)^4 = \left(-\frac{b}{a}\right)^4 = \frac{390625}{1296}$

54. Let 1 trip = 1 mile

week's time (T) = $\frac{1}{42} + \frac{1}{47} + \frac{1}{32} + \frac{1}{50} + \frac{1}{45}$

Average rate for 5 trips = $\frac{5}{T} \approx 42.17\,\text{mh}^{-1}$

57. $\ln x = (x-1) - \frac{(x-1)}{2} + \frac{(x-1)}{3} - \frac{(x-1)}{4} \ldots$

when $0 < x \le 2$

$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \ldots$

59. $\ln y = \ln x^{2x} = 2x\ln x$

$\frac{1}{y}\frac{dy}{dx} = 2x \bullet \frac{1}{x} + \ln x \bullet 2$

$\frac{dy}{dx} = 2(1 + \ln x)\, y = 2x^{2x}(1 + \ln x)$