TMSCA HIGH SCHOOL MATHEMATICS TEST#1 ©

OCTOBER 24, 2015

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2015-2016 TMSCA Mathematics Test One

1. Evaluate: $24\times 15-4!-27\div(18-6)\times 4+27$.

   A. 362.4 B. 374 C. 355.6 D. 402 E. 354

2. Caroline had a rope that was 15 feet long. She cut off three pieces such that the ratio of lengths of the pieces were 2:3:12 with 10 inches of string left over. How long was the shortest piece?

   A. 2 ft. 5 in. B. 1 ft. 6 in. C. 2 ft. 6 in. D. 1 ft. 10 in. E. 1 ft. 8 in.

3. Given that the data set $12, a, b, c, 24, 29$ is shown least to greatest and has a mean of 20, mode of 24 and median of 20. Calculate the value of $a+c$.

   A. 39 B. 48 C. 40 D. 31 E. 35

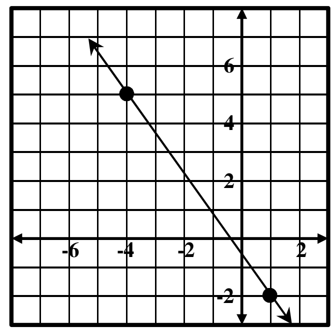
4. What is the mean of the first four abundant numbers?

   A. 14 B. 7 C. 18.5 D. 21 E. 21.5

5. Evaluate: $\frac{(x+2)!}{(x-3)!}\div\frac{x!}{(x-1)!}$.

   A. $x^6-5x^4+4x^2$ B. x^4+5 C. x^4-5x^2+4 D. x^6+4x^2 E. x^4-5x^2+5

6. Which of the following is the standard form of the equation of the line represented in this graph?



   A. $5x+7y=3$ C. $7x-5y=-10$ E. $7x+5y=13$

   B. $7x-5y=-17$ D. $7x+5y=-3$

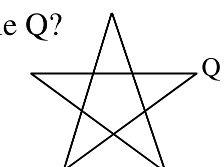
7. If $\theta=5\lambda$ and $\alpha+\theta=\varphi$, then $\alpha+5\lambda=\varphi$. This is an example of the __________ property.

   A. Substitution B. Transitive C. Commutative D. Associative E. Reflexive

8. Two consecutive angles in a pentagon are supplementary. The other three angles are congruent. What is the measure of one of the three congruent angles?

   A. 60° B. 120° C. 150° D. 90° E. 135°

9. The angles at each point on the star shown are congruent. What is the measure of the angle Q?



   A. 30° B. 27° C. 54° D. 72° E. 36°

10. What is the area of the region entirely bounded by the two functions $f(x)=x^2-4x+1$ and $g(x)=-4x+10$?

    A. 6 B. 36 C. 18 D. 108 E. 24

11. If $x+y=-3$, and $xy=-10$, then $x^3+y^3=$

    A. -117 B. -57 C. 33 D. 63 E. 3

12. The four brothers Lester, Morris, Nigel and Porter wanted to go on a road trip, but Lester had no money. Morris, Nigel and Porter each gave Lester one-fourth, one-fifth and one-third of his money respectively. If each gave Lester the same amount, what fraction of the money did Lester possess after the exchange?

    A. $\frac{3}{13}$ B. $\frac{3}{7}$ C. $\frac{1}{3}$ D. $\frac{3}{10}$ E. $\frac{1}{4}$

13. Find the value of the arithmetic mean for terms a, b and c in the geometric sequence: $3072, 3840, a, b, c....$

A. 4880 B. 6100 C. 7495.68 D. 6000 E. 4800

14. $\tan\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}\right) \div \cot\left(\frac{5\pi}{3}\right)\csc\left(\frac{\pi}{6}\right) \div \cos\left(\frac{5\pi}{3}\right)\csc\left(\frac{5\pi}{3}\right) =$

A. $\frac{4}{3}$ B. 4 C. 2 D. $\frac{2\sqrt{3}}{3}$ E. $\frac{1}{2}$

15. The intersection of the medians of a triangle is called the ____________.

A. Centroid B. Incenter C. Median D. Circumcenter E. Orthocenter

16. How many integral values of n exist such that $n > 3$, and $\frac{n!}{(n-3)!} \le 150$?

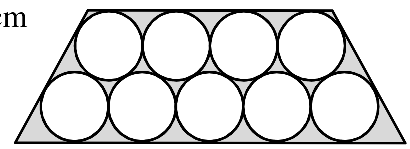
A. 0 B. 3 C. 1 D. 4 E. 2

17. There are two values of k for which $\det\begin{bmatrix} k-1 & 4 \\ -3 & 2k \end{bmatrix} = 0$. The sum of those two values is

A. 1 B. 5 C. 3 D. -2 E. -1

18. The radius of each circle is 2.5 cm. Find the perimeter of the trapezoid. (nearest tenth)

A. 68.1 cm B. 64.6 cm C. 88.1 cm D. 56.5 cm E. 58.9 cm



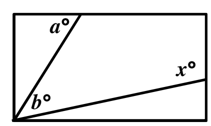
19. The number 478 in base 9 is equivalent to the number k in base 3. Find the sum of the digits in k.

A. 8 B. 19 C. 9 D. 7 E. 10

20. Find the mean value of $f(x) = 4x^3 - 6x^2 + 2x - 1$ for $[-1, 3]$.

A. 8 B. 19 C. 9 D. 18 E. 7

21. In the rectangle shown right, what is x in terms of a and b?



A. $90 - a - b$ B. $90 - a + b$ C. $a + b$ D. $180 - a - b$ E. $90 + a - b$

22. How many distinct arrangements can be formed using all of the letters in the words "FALL FESTIVAL"?

A. 39916800 B. 19958400 C. 967680 D. 20442240 E. 11880

23. If $g(x) = x - 1$ and $f(x) = x^4$, find $g(f(x+1))$.

A. $x^4 + 3x^3 + 3x^2 + x$
B. $x^4 + 4x^3 + 6x^2 + 4x$
C. x^4
D. $x^4 - 2$
E. $x^4 + 4x^3 + 6x^2 + 4x - 2$

24. A chemistry student needs to mix a 50 fluid ounce solution containing 54% glucose. The pharmacist has 30% and 90% solutions on hand. How much of the 30% solution should she use?

A. 30 ounces B. 27 ounces C. 20 ounces D. 23 ounces E. 25 ounces

25. Which of the following quadrants does not contain a solution to $5x + 3y \ge 9$?

A. QIII B. QI & QII C. QIV D. QI & QIV E. QI

26. A triangle with side lengths 12 cm, 11 cm and 22 cm is a(n)___________triangle.

A. isosceles acute B. scalene acute C. isosceles obtuse D. scalene obtuse E. scalene right

27. Which of the following is an equation of the tangent line of $f\left(x\right)=2x^2-x+\frac{4}{x}$ for $x=2$?

A. $6x-y=4$ B. $6x+y=-4$ C. $6x-y=-6$ D. $x+6y=-4$ E. $x+6y=8$

28. If $\log 9=P$, and $\log 5=Q$, then $\log 0.6=$

A. $P-Q^2$ B. $\frac{P}{2Q}$ C. $\frac{P-2Q}{2}$ D. $\frac{P}{Q^2}$ E. $\frac{P-Q}{2}$

29. If $U=\{a,b,c,d,e,f,g,h\}$, $A=\{a,c,e,g\}$, and $B=\{b,c,d,e\}$, find $A'\cap B'$.

A. $\{a,f,g,h\}$ B. $\{a,b,c,d,e,g\}$ C. $\{b,c,d,e,f\}$ D. $\{a,b,d,f,g,h\}$ E. $\{f,h\}$

30. If *P, Q* and *R* are real numbers such that $P+Q+R=8$, $R^2=P^2+Q^2$ and $PQ=8$, find the value of *R*.

A. 4 B. 8 C. 3 D. 6 E. 1

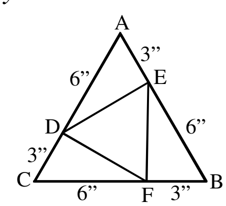
31. There are 6 girls and 8 boys in Ms. Angel's homeroom class. She must select a group of 2 girls and 2 boys to represent her class in a Veterans Day ceremony. How many distinct groups could does she have to choose from?

A. 1680 B. 43 C. 420 D. 225 E. 1001

32. Which of the following equations in rectangular form can be written as $r-6\sin\theta=0$ in polar form?

A. $x^2+y^2=9$ C. $x^2+y^2=3$ E. $x^2+y^2-6y=0$

B. $x^2-6x+y^2=0$ D. $x^2+y^2=2\sqrt{3}$



33. Find the area of $\triangle DEF$. (nearest tenth)

A. 9.7 in^2 B. 20.8 in^2 C. 6.8 in^2 D. 7.8 in^2 E. 11.7 in^2

34. Find the remainder when $f\left(x\right)=6x^3-x^2-7x+5$ is divided by $x-3$.

A. 137 B. 143 C. -133 D. -145 E. 155

35. A sales clerk is packaging blue, red and black pens for a back-to-school sale. How many different packages of 6 pens can he make?

A. 84 B. 126 C. 28 D. 56 E. 35

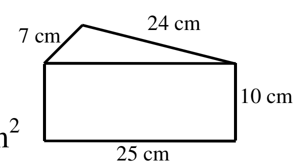
36. Two roots of $f\left(x\right)=x^3+bx^2+cx+d$ are 4 and $3+i$. Find $b+c+d$.

A. 54 B. -26 C. -16 D. 64 E. -4

37. If $g\left(x\right)\le f\left(x\right)\le h\left(x\right)$ for all *x*, *k* in $[a,b]$, where $x\ne k$, and $\lim_{x\to k} g\left(x\right)=L$ and $\lim_{x\to k} h\left(x\right)=L$ then $\lim_{x\to k} f\left(x\right)=L$. This theorem is known as:

A. Sandwich Theorem C. Rolle's Theorem E. Fundamental Theorem of Calculus

B. Intermediate Value Theorem D. Fundamental Theorem of Algebra



38. Calculate the total surface area of the triangular prism shown.

A. 728 cm^2 B. 644 cm^2 C. 840 cm^2 D. 924 cm^2 E. 560 cm^2

39. Find the sum of all the three digit numbers whose digits have a sum of eight and whose digits can all be used to form a perfect cube.

A. 1925 B. 1776 C. 861 D. 915 E. 1420

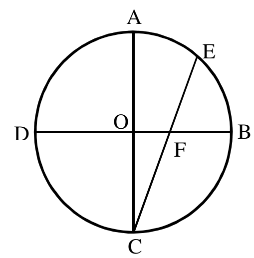
40. The ratio of length to width of a rectangle is 13:3 and the perimeter is 1536 in. What is the area of the rectangle?
A. 359424 ft^2 B. 1248 ft^2 C. 179712 ft^2 D. 624 ft^2 E. 2496 ft^2

41. The function $f(x)=\dfrac{2x^3}{x^2-3}$ is increasing at which of the following values of x?
A. -3 B. -4 C. -1 D. 0 E. 2

42. How many distinct solutions exist for $2\sin^2 x = 1 + 2\sin x$, where $0 \le x < 2\pi$?
A. 0 B. 1 C. 2 D. 3 E. 4

43. Meredith set out to row on a lake. She rowed 500 m on a bearing of 75°, then 200 m on a bearing of 25°, then 350 m on a bearing of 52°. How far is she from her original starting point?
A. 1050 m B. 615 m C. 775 m D. 526 m E. 994 m

44. Circle O has perpendicular diameters and a chord, find AE if CF = 7 inches and EF = 6 inches. (nearest tenth)
A. 4.2 in B. 3.6 in C. 3.8 in D. 4.5 in E. 3.2 in



45. What is the harmonic mean of the roots of the function $f(x)=6x^2-11x-72$?
A. $\dfrac{11}{12}$ B. $-\dfrac{144}{11}$ C. $\dfrac{7}{2}$ D. $\dfrac{12}{11}$ E. $-\dfrac{11}{144}$

46. Find y as a function of x given that $\dfrac{d^2y}{dx^2}=4-6x$ and that when $x=2$, $\dfrac{dy}{dx}=-4$ and $y=7$.
A. $y=7+2x^2-x^3$ C. $y=7+3x^2-x^3$ E. $y=9+3x^2-x^3$
B. $y=1+2x^2-x^3$ D. $y=23+3x^2-x^3$

47. What is the constant term in the binomial expansion of $\left(3x^3-\dfrac{2}{x}\right)^8$?
A. 576 B. 72576 C. 1296 D. 145152 E. 16128

48. A contestant on a game show rolls a single, fair, standard die. The player loses \$100 if an odd number is rolled. If he rolls an even prime, he gets a \$500 payout. If he rolls a perfect number, he gets \$1000 payout. Otherwise, nothing happens. What are his expected winnings?
A. \$200 B. \$250 C. \$240 D. \$150 E. \$275

49. The point $(6,-2)$ lies on a circle whose center is $(0,8)$. Where does the point $(8,13)$ lie in reference to the circle?
A. Inside B. Outside C. On the Circle D. Q II E. Unknowable

50. How many solutions are there for the equation $2x+5y=125$ where both x and y are non-negative integers?
A. 12 B. 11 C. 13 D. 10 E. 14

51. Sixty-five percent of homes in a town have pets. If four homes are chosen at random for a survey, find the probability that all four have pets. (nearest percent)
A. 26% B. 11% C. 13% D. 15% E. 18%

52. If $\frac{7x+13}{x^2+2x-3}=\frac{A}{x+3}+\frac{B}{x-1}$, then $AB=$

A. 7 B. -6 C. -3 D. 6 E. 10

53. Given that the set of natural numbers continue in the triangular pattern shown below, find the median of the numbers in row 9.

			1				(row 1)
		2	3	4			(row 2)
	5	6	7	8	9		(row 3)
10	11	12	13	14	15	16	(row 4)
			…				(…)

A. 83 B. 73 C. 77 D. 67 E. 85

54. The square root of 1013 in base 6 is:

A. 111_6 B. 23_6 C. 35_6 D. 25_6 E. 151_6

55. If $y^2=5-12i$ and $y^3=-9-46i$ where $y=a+bi$ then $a+b=$

A. 1 B. -38 C. 5 D. -62 E. 6

56. $3^3+4^3+5^3+...+12^3+13^3+14^3=$

A. 11016 B. 11017 C. 289570 D. 11025 E. 2744

57. What is the area of a regular hexagon in terms of the length, s, of one side?

A. $\frac{3s^2\sqrt{3}}{4}$ B. $\frac{4s^2\sqrt{3}}{3}$ C. $2s^2\sqrt{3}$ D. $\frac{3s^2\sqrt{3}}{2}$ E. $3s^2\sqrt{3}$

58. Find the units digit of 17^{2015}.

A. 3 B. 1 C. 7 D. 0 E. 9

59. Simplify to the nearest ten-thousandth place: $1+(1.3)+\frac{(1.3)^2}{2!}+\frac{(1.3)^3}{3!}+\frac{(1.3)^4}{4!}+\ldots$

A. 0.2624 B. 0.2675 C. 3.6693 D. 3.6302 E. 0.9636

60. The function f is such that $\int_{-1}^{8} f(x)\,dx=9$. What is the value of $\int_{-1}^{8}\left(f(x)+3\right)dx$?

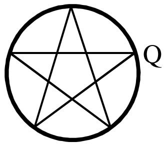
A. 12 B. 36 C. 27 D. 32 E. 18

2015-2016 TMSCA Mathematics Test One Answers

1. E	21. E	41. B
2. E	22. B	42. C
3. A	23. B	43. E
4. C	24. A	44. B
5. C	25. A	45. B
6. D	26. D	46. A
7. A	27. A	47. E
8. B	28. C	48. A
9. E	29. E	49. A
10. B	30. C	50. C
11. A	31. C	51. E
12. E	32. E	52. E
13. B	33. E	53. B
14. B	34. A	54. B
15. A	35. C	55. A
16. B	36. C	56. A
17. A	37. A	57. D
18. A	38. A	58. A
19. C	39. B	59. C
20. E	40. D	60. B

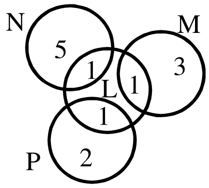
2015-2016 TMSCA Mathematics Test One Selected Solutions

9. Angle Q is an inscribed angle with an intercepted arc $\frac{360^\circ}{5}=72^\circ$, so $m\angle Q=36^\circ$.



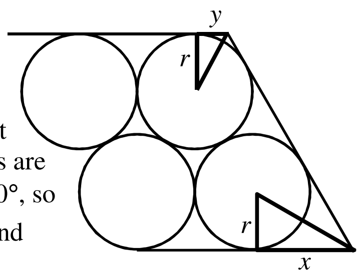
11. $x^3+y^3=(x+y)(x^2-xy+y^2)=$
$(x+y)\left[(x+y)^2-3xy\right]=-3(9+30)$

12. If each gives Lester a dollar, use a Venn diagram to show the fraction relationships then Lester has $\frac{3}{13}$ of the money in the end.



16. $n(n-1)(n-2)\le 150$. For a starting point, evaluate $\sqrt[3]{150}\approx 5.31$. Try 5 for the $(n-1)$ value. $6\bullet 5\bullet 4=120$, while $7\bullet 6\bullet 5=210$, so n values can be 6, 5, and 4.

18. The small triangles at the corners are 30°-60°-90°, so $x=r\sqrt{3}$ and $y=\frac{r\sqrt{3}}{3}$. As part of the perimeter, x and y each appear 4 times, so area is $4x+4y+18r$ or approximately 68.1 when $r=2.5$.



22.There are 12 total letters with the "F" repeated twice, the "A" repeated twice and the "L" repeated thrice, so the total number of distinct arrangements is $\frac{12!}{(2!)(2!)(3!)}=19958400$.

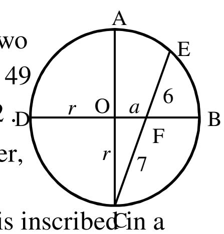
23. $g(f(x+1))=(x+1)^4-1$. Use binomial theorem or Pascal's triangle to get $(x^4+4x^3+6x^2+4x+1)-1$ for a final answer $x^4+4x^3+6x^2+4x$.

30. With just a bit of algebra, $(P+Q)^2=(8-R)^2$ so,
$P^2+2PQ+Q^2=64-16R+R^2$ then
$R^2+2PQ=64-16R+R^2$ or
$2PQ=64-16R$ therefore,
$2(8)=64-16R$ for a solution of $R=3$.

35. There are three different types to choose from and packages of six, so the number of possibilities is ${}_{3+6-1}C_6=28$.

39. The two perfect 3-digit cubes whose digits have a sum of 8 are 125 and 512. There are six possible 3-digit numbers that can be formed with the digits 1, 2 and 5.
$125+152+251+215+521+512=1776$

44. To find r, use the two relationships $a^2+r^2=49$ and $(r+a)(r-a)=42$. Adding the two together, $2r^2=91$ and. Then, because triangle AEC is inscribed in a semi-circle, it has a right angle at E and $13^2+(AE)^2=(2r)^2$ and
$AE=\sqrt{4r^2-169}$ or
$AE=\sqrt{2(91)-169}\approx 3.6$ inches.



47. The constant term will be ${}_8C_2(3x^3)^2\left(-\frac{2}{x}\right)^6$ because there x will have a power of 6 in both the numerator and denominator. The constant left will be 16128.

52. Multiplying all terms of the equation by the LCD, will yield the equation: $7x+13=A(x-1)+B(x+3)$. Let $x=1$ to find the value of B using the equation $20=4B$ so $B=5$. Similarly, let $x=-3$ to find the value of A using the equation $-8=-4A$ so $A=2$ and $AB=10$.

53. The median of any row in the arrangement shown will always be the center number. The 1, 3, 7, 13…can either be used to develop a quadratic regression $y=x^2-x+1$ where x is the row number and y is the median. For the 9th row the median will be 73. An alternative would be to use the differences in the center numbers and continue the pattern
1+2=3
3+4=7
7+6=13
13+8=21………57+16=73.

56. The formula for the sum of the first n cubes is $\left(\frac{n(n+1)}{2}\right)^2$, so the sum of the series will be $\left(\frac{14(15)}{2}\right)^2-1-8=11016$.

59. The series shown is the McClaurin series for the function $f(x)=e^x$ when $x=1.3$, so $e^{1.3}\approx 3.6692967$.