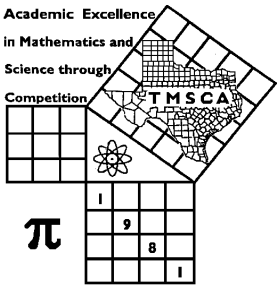


T M S C A H I G H S C H O O L

M A T H E M A T I C S

T E S T # 1 ©

OCTOBER 20, 2012

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2012-2013 TMSCA High School Mathematics Test 1

1. What is 11.11...% of $\left(\left(\frac{1}{8}\right)\div 0.3125\right)$?

(A) $\frac{1}{3}$ (B) $\frac{5}{1152}$ (C) $\frac{1}{45}$ (D) $\frac{2}{45}$ (E) $\frac{5}{9}$

2. The statement $\left(-\frac{1}{3}\right)(-3)=1$ is an example of

(A) distributive (B) identity (C) associative (D) inverse (E) commutative

3. Bonnie plans to buy two dozen cupcakes that usually cost $1.10 each. She receives a 15% discount on the first dozen, a 20% discount on the second dozen, and pays 8.25% sales tax on the total. How much will her order cost?

(A) $23.58 (B) $19.43 (C) $17.95 (D) $14.81 (E) $22.71

4. The points $P(7,12)$ and $Q(5,20)$ are on the line $\overleftrightarrow{PQ}$. The equation of the line $\perp$ to $\overleftrightarrow{PQ}$ through P is:

(A) $4x-y-16=0$ (B) $x-4y+41=0$ (C) $4x-y+5=0$ (D) $x-4y+5=0$ (E) $x-4y+16=0$

5. If $9x^3+3x^2-3x-1=(ax+1)(bx^2+c)$ where $a,b,c\in\mathbb{Z}$, then $a+b+c=$_____.

(A) 5 (B) 8 (C) 7 (D) 9 (E) 4

6. Find the range of the mean, median and mode of 3.75, 3.5, 4.25, 2.75, 3.5, 5.25, 4, 5.

(A) 2.25 (B) 4 (C) 2.75 (D) 3.5 (E) 0.5

7. Triangle *ABC* is a right triangle, and $\overline{BD}$ is the altitude to the hypotenuse. Which of the following pairs are complementary?

(A) $\angle ADB$, $\angle CDB$ (B) $\angle BCD$, $\angle ABD$ (C) $\angle BCD$, $\angle DBC$ (D) $\angle ABC$, $\angle ACB$ (E) $\angle BAD$, $\angle CBD$

8. Solve for *x*: $2xy+3x-5=y-x$

(A) $\frac{5-y}{6}$ (B) $\frac{y+5}{2y+4}$ (C) $\frac{y-5}{2y+2}$ (D) $\frac{y+5}{5y+1}$ (E) $\frac{5}{y+4}$

9. Given the sequence 3, 4, 7, 12…147, *k*, 199, find the value of *k*.

(A) 173 (B) 172 (C) 160 (D) 164 (E) 179

10. What is the smallest angle formed by the hour and minute hand on the clock at 11:15?

(A) 112.5° (B) 120° (C) 247.5° (D) 240° (E) 97.5°

11. Given *y* varies inversely with $(2x-1)$ and that $y=5$ when $x=4$, find the value of *y* when $x=11$.

(A) $\frac{15}{7}$ (B) $\frac{4}{209}$ (C) $\frac{3}{5}$ (D) $\frac{5}{3}$ (E) $\frac{7}{15}$

12. Abby and Bill can build a brick wall that is 4 feet long and 3 feet high in 3 hours together. How long will it take for them to build a wall twice as long and twice as high if they work at the same pace?

(A) 12 hours (B) 4 hours (C) 6 hours (D) 24 hours (E) 8 hours

13. What is the amplitude of $f(x)=1.3+2.5\cos(3(x-5))$?

(A) 1.3 (B) 15 (C) 2.5 (D) $\frac{5}{3}$ (E) 5

14. Blackbeard sailed 40 miles on a bearing of 72°, then 65 more miles on a bearing of 80°. Approximately how far north of his original position is his final destination?

(A) 35 miles (B) 103 miles (C) 18 miles (D) 99 miles (E) 24 miles

15. Given $f(x)=7x^4+11x^3-5x^2+20x-9$, find the sum of the coefficients of the cubic and quadratic terms of $f'(x)$.

(A) 23 (B) 5 (C) 54 (D) 6 (E) 61

16. If the area of a trapezoid is 48 cm^2, and the height is 9 cm, what is the length of the median?

(A) $\frac{32}{3}$ (B) 6 (C) $\frac{16}{3}$ (D) $\frac{24}{3}$ (E) 10

17. How many solutions are there to the equation $6x+5y=108$ such that *x* and *y* are both non-negative integers?

(A) 4 (B) 3 (C) 11 (D) 5 (E) 0

18. Simplify: $\left(-2-3\sqrt{-5}\right)\left(4\sqrt{-8}\right)$.

(A) $-24\sqrt{10}-16i\sqrt{2}$ (B) $-24\sqrt{5}-16i\sqrt{2}$ (C) $24\sqrt{10}-16i\sqrt{2}$ (D) $-24\sqrt{7}-16i\sqrt{2}$ (E) $80i\sqrt{5}$

19. A particle's movement along the number line is defined by the function $f(t)=t^4-4t^3-26t^2+60t+25$. At which of the following times is the particle moving to the left?

(A) -1 (B) -3 (C) 1 (D) 3 (E) 7

20. The student council president wants to form a representative student committee consisting of 2 male athletes, 2 female athletes, 3 band members, and 2 academic UIL participants. If there are 4 female athletes, 5 male athletes, 6 band members and 5 academic UIL participants available to serve, how many distinct committees could be formed?

(A) 24 (B) 12000 (C) 600 (D) 576000 (E) 172

21. Mr. Anderson's AP Statistics class has 3 male students. What is the probability that all three male students were born on a Monday?

(A) $\frac{3}{343}$ (B) $\frac{1}{21}$ (C) $\frac{1}{3}$ (D) $\frac{3}{7}$ (E) $\frac{1}{343}$

22. Melanie has a wire that is 3 feet and 8 inches long. To the nearest square inch, what is the area of the largest closed triangle she can make by bending the wire?

(A) 87 in^2 (B) 124 in^2 (C) 44 in^2 (D) 93 in^2 (E) 65 in^2

23. The sides on an isosceles triangle are $2x, 7x$, and $7x$ respectively. What is the exact area of the triangle in terms of *x*?

(A) $4x^2\sqrt{3}$ (B) $7x^2\sqrt{2}$ (C) $3x^2\sqrt{5}$ (D) $2x^2\sqrt{3}$ (E) $1.5x^2\sqrt{5}$

24. What are the coordinates of the center of the circle $x^2+y^2-4x+6y=-4$?

(A) $(0,0)$ (B) $(2,-3)$ (C) $(-2,3)$ (D) $(0,-2)$ (E) $(1,3)$

25. Find the value of *k* in the sequence: 3, 3, 6, 9, 15…2961, *k*, 7752.

(A) 5356 (B) 4791 (C) 4558 (D) 6155 (E) 6271

26. Given $f(x)=2x+5$ and $g(x)=x^2-1$, find $g(f(x))$.

(A) $4x^2+24$ (B) $4x^2+20x+24$ (C) $2x^2+3$ (D) $4x^3+10x^2+6x+15$ (E) $2x^2+4$

27. What is the total surface area of a tetrahedron with one edge length of 7 cm?

(A) $49\sqrt{3}$ cm^2 (B) $\frac{196\sqrt{3}}{3}$ cm^2 (C) 98 cm^2 (D) $196\sqrt{3}$ cm^2 (E) 196 cm^2

28. Find $\frac{d}{dx}\left(\frac{2x+1}{x^2-2}\right)$.

(A) $-\frac{2x^2+2x+4}{x^4+4}$ (B) $-\frac{2x^2+2x+4}{x^4-4x^2+4}$ (C) $\frac{2x^2+2x+4}{x^4-4x+4}$ (D) $\frac{1}{x}$ (E) $\frac{2x^2+2x+4}{x^4+4}$

29. On one roll of two independent, fair tetrahedral die, what is the probability that the sum of the two dice is less than 5?

(A) $\frac{3}{5}$ (B) $\frac{5}{8}$ (C) $\frac{5}{3}$ (D) $\frac{3}{8}$ (E) $\frac{8}{5}$

30. If an integral factor of 72, not including 1 or 72 is chosen at random, what are the odds that it is a multiple of 3?

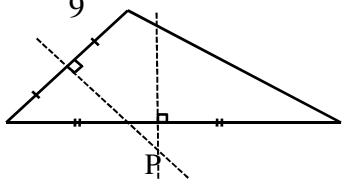
(A) $\frac{2}{3}$ (B) $\frac{3}{5}$ (C) $\frac{3}{2}$ (D) $\frac{5}{3}$ (E) $\frac{7}{3}$

31. Using the following array, determine the value of the mean of the 10th row.

2
2 2
2 4 2
2 6 6 2

(A) $\frac{256}{5}$ (B) $\frac{512}{5}$ (C) $\frac{254}{5}$ (D) $\frac{256}{9}$ (E) $\frac{512}{9}$

32. Point P is the _____ of the triangle.



(A) incenter (B) centroid (C) midpoint (D) circumcenter (E) orthocenter

33. Meridith invested $300 for a total of four years. In the first year, her investment grew 3.5%. In the second year, she lost 2%. In the third and fourth year she gained 5% and 1.75% respectively. What was her average rate of return over the four year period?

(A) 2.06% (B) 2.71% (C) 2.17% (D) 8.34% (E) 2.03%

34. Two pumps fill a cylindrical tank in one hour. The tank has a radius of 34 inches and a height of 41 inches. If the first pump pumps at a rate of 3 gallons per minute, which of the following is closest to the rate of the second pump?

(A) 10.74 gal/min (B) 4.74 gal/min (C) 9.95 gal/min (D) 7.74 gal/min (E) 6.95 gal/min

35. Evaluate: $\prod_{k=0}^{3}\left(2^{k-1}\right)$.

(A) 4 (B) 7.5 (C) 32 (D) 14.5 (E) 6.5

36. Find the value of k if $\int_{-2}^{k}(2x-1)\,dx = 14$, where $k > -2$.

(A) -9 (B) 5 (C) 6 (D) 9 (E) 8

37. A spherical balloon is inflated with gas at a rate of 500 cubic centimeters per minute. How fast is the circumference of a great circle on the balloon changing when the radius is 60 centimeters?

(A) $\frac{25}{6}$ cm/min (B) $\frac{5}{18}$ cm/min (C) $\frac{5}{72}$ cm/min (D) $\frac{50}{3}$ cm/min (E) $\frac{25}{36}$ cm/min

38. Let A and B be the roots of $3x^2+5x-2=0$. Find the value of $A^3+3A^2B+3AB^2+B^3$.

(A) $\frac{125}{27}$ (B) $\frac{343}{27}$ (C) $-\frac{125}{27}$ (D) $-\frac{64}{27}$ (E) $-\frac{343}{27}$

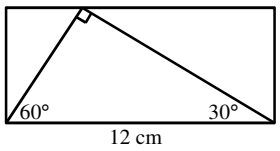
39. Simplify: $1-\frac{4}{2!}+\frac{16}{4!}-\frac{64}{6!}\ldots$.

(A) 0.909 (B) -0.416 (C) 7.39 (D) -0.422 (E) 0.933

40. Jordan made cookies. He gave 8 cookies to his younger siblings, then gave one-third of the remaining to his best friend. Next, he put aside half of the remaining cookies for his teacher, ate one fourth of those that he had left. He then gave the last 18 cookies to his mom. How many cookies did he bake?

(A) 96 (B) 72 (C) 60 (D) 68 (E) 80

41. Find the area of the rectangle shown.



(A) 36 cm^2 (B) 24 cm^2 (C) $48\sqrt{3}$ cm^2 (D) 72 cm^2 (E) $36\sqrt{3}$ cm^2

42. On Triangle ABC, $AB=8$, $BC=7$ and $m\angle A=59^\circ$. Which of the following is one of the two possible lengths of $\overline{AC}$ to the nearest hundredth of a unit?

(A) 2.71 (B) 5.55 (C) 3.87 (D) 6.17 (E) 5.78

43. Which of the following functions expresses the area, A, of an equilateral triangle in terms of the length of the apothem, a?

(A) $A=\frac{9a^2}{4}$ (B) $A=9a^2$ (C) $A=3a^2\sqrt{3}$ (D) $A=\frac{3a^2\sqrt{3}}{4}$ (E) $A=6a^2\sqrt{3}$

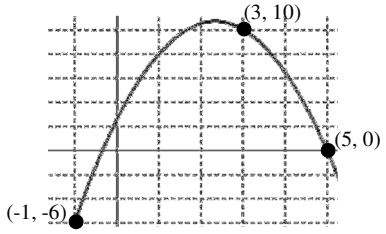
44. A waiter received a 15% tip from table one in the amount of \$7.00 and a 20% tip from table two in the amount of \$2.50. What was the difference in the bills from table one and two not including the tip?

(A) \$8.45 (B) \$18.33 (C) \$34.17 (D) \$4.50 (E) \$26.25

45. Areas of two similar heptagons are 252 m^2 and 175 m^2 respectively. If the perimeter of the larger heptagon is 51 m, what is the perimeter of the smaller heptagon?

(A) 36.83 m (B) 48.15 m (C) 7.39 m (D) 6.07 m (E) 42.5 m

46. Find the x-coordinate of the vertex on the parabola shown on the right.



(A) $\frac{7}{3}$ (B) $\frac{32}{3}$ (C) $\frac{5}{2}$ (D) $\frac{85}{8}$ (E) $\frac{2}{1}$

47. Given $y=\frac{x-1}{x+3}$, at which of the following values of x is $\frac{dy}{dx}=\frac{dx}{dy}$?

(A) 1 (B) -5 (C) 2 (D) -3 (E) 3

48. Given a sequence with Fibonacci characteristics 3, a, b, 11, find the value of $a+b$.

(A) 7 (B) 2.67 (C) 14 (D) 11.76 (E) 11

49. What is the sum of the coefficients of the quadratic and linear terms in the expansion of $(2x+3)^7$?

(A) 28 (B) 30618 (C) 147 (D) 20412 (E) 1542

50. Which of the following is an equation of the tangent to $f(x)=x^5-2x^4+x^3+3x^2+7x-1$ at $x=2$?

(A) $47x-y-14=0$ (B) $x+47y-1553=0$ (C) $x-y+35=0$ (D) $47x-y-61=0$ (E) $x+47y-1549=0$

51. In how many ways can eight people be seated at a round table?

(A) 72 (B) 40320 (C) 10080 (D) 20160 (E) 5040

52. An ice cream parlor offers nine flavors of ice cream and two types of cones. How many different ways are there to order a two scoop cone?

(A) 45 (B) 90 (C) 55 (D) 56 (E) 110

53. The reaction times of human beings are normally distributed with a mean of 0.76 seconds and a standard deviation of 0.06 seconds. What is the probability that the reaction time of a person chosen at random will be greater than 0.70 seconds?

(A) 0.159 (B) 0.841 (C) 1 (D) 0.683 (E) 0.819

54. Simplify $a^3\times b^3\div a^{-1}\times b^{-3}\times a^2\div b^2\div b^{-3}$.

(A) a^6b^7 (B) $\frac{a^5}{b^2}$ (C) $\frac{a^6}{b^5}$ (D) a^6b (E) $\frac{a^6}{b^6}$

55. Given $A=\begin{bmatrix}3 & 2\\ 2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}-1 & -2\\ 1 & 2\end{bmatrix}$, find AB.

(A) $\begin{bmatrix}-3 & -4\\ 2 & 2\end{bmatrix}$ (B) $\begin{bmatrix}-7 & 7\\ -4 & 4\end{bmatrix}$ (C) $\begin{bmatrix}-6 & -24\\ -2 & -8\end{bmatrix}$ (D) $\begin{bmatrix}-7 & -4\\ 7 & 4\end{bmatrix}$ (E) $\begin{bmatrix}-1 & -2\\ -1 & -2\end{bmatrix}$

56. Simplify $\frac{(x+2)!}{(x-3)!}\div\frac{x!}{(x-1)!}$.

(A) x^4+5x^2+4 (B) $\frac{x}{x^2-3x+2}$ (C) $x^6-5x^4+4x^2$ (D) $\frac{x^4-5x^2+4}{x^2}$ (E) x^4-5x^2+4

57. When $f(x)=x^4+3x^3+px^2-2x+q$ is divided by $(x-2)$ the remainder is 15, and $(x+3)$ is a factor of $f(x)$. Find the value of $p+q$.

(A) -30 (B) -33 (C) -54 (D) -60 (E) -36

58. Triangle *ABC* is a right triangle with $\overline{AC}\perp\overline{CB}$. Point *D* is on $\overline{AC}$ such that the ratio of *AD* to *CD* is 2:1. If $CB=12$ and $DB=13$ find the length of $\overline{AB}$.

(A) $4\sqrt{61}$ (B) 20 (C) $3\sqrt{41}$ (D) $2\sqrt{61}$ (E) $9\sqrt{41}$

59. How many positive perfect cubes are factors of $(3!)(4!)(6!)$?

(A) 3 (B) 5 (C) 4 (D) 8 (E) 6

60. Let $a=\log x$, $b=\log y$ and $c=\log z$. Write $\log\left(\frac{x^2\sqrt{y}}{z^3}\right)$ in terms of *a*, *b*, and *c*.

(A) $ab-3c$ (B) $\frac{a^2\sqrt{b}}{c^3}$ (C) $\frac{-3ab}{c}$ (D) $a^2+\sqrt{b}-c^3$ (E) $2a+\frac{1}{2}b-3c$

2012-2013 TMSCA High School Mathematics Test 1 Key

1. D	21. E	41. E
2. D	22. D	42. A
3. A	23. A	43. C
4. B	24. B	44. C
5. A	25. B	45. E
6. E	26. B	46. A
7. C	27. A	47. B
8. B	28. B	48. E
9. B	29. D	49. B
10. A	30. E	50. D
11. D	31. B	51. E
12. A	32. D	52. B
13. C	33. E	53. B
14. E	34. D	54. D
15. E	35. A	55. E
16. C	36. B	56. E
17. A	37. C	57. A
18. C	38. C	58. C
19. D	39. B	59. E
20. B	40. E	60. E

1. $\frac{1}{9}\cdot\frac{1}{8}\cdot\frac{16}{5}=\frac{2}{45}$

2. Definition of multiplicative inverse

3. $1.0825(1.1)(12)\big((.85)+(.8)\big)=\$23.58$

4. $y-12=\frac{1}{4}(x-7)\rightarrow x-4y+41=0$

5. $\left(9x^3+3x^2\right)-(3x+1)$

   $3x^2(3x+1)-1(3x+1)$

   $(3x+1)\left(3x^2-1\right)$

6. mean = 4, median = 3.875, mode = 3.5
   Range of m, m, & m = 0.5

7. definition of complementary

8. $2xy+3x+x=y+5$

   $x(2y+4)=y+5\therefore x=\frac{y+5}{2y+4}$

9. quad reg: $y=x^2-2x+4$, use table to find missing value.

10. $360-(30\cdot11.25-6\cdot15)$

11. $5=\frac{k}{2(4)-1}$, $k=35$, $y=\frac{35}{2\cdot11-1}=\frac{5}{3}$

12. $2\cdot3\cdot r=12$, so $r=2$.
    $2\cdot t\cdot2=48$, so $t=12$.

13. coefficient of $\cos x=2.5=$ amplitude

14. reference angle for 72° bearing is 18°
    reference angle for 80° bearing is 10°
    $40\sin18+65\sin10=23.65\approx24$ miles

15. $f'(x)=28x^3+33x^2-10x+20$

    $28+33=61$

16. $A=med\cdot height$, $med=\frac{48}{9}=\frac{16}{3}$

17. $y=\frac{(108-6x)}{5}$. Use table function.

18. $\left(-2-3i\sqrt{5}\right)\left(8i\sqrt{2}\right)=-16i\sqrt{2}-24i^2\sqrt{10}$

    $=24\sqrt{10}-16i\sqrt{2}$

19. $f'(t)=4t^3-12t^2-52t+60$. Use table.

    The derivative is $(-)$ @ $x=3$.

20. $\binom{4}{2}\binom{5}{2}\binom{6}{3}\binom{5}{2}=12000$

21. $\left(\frac{1}{7}\right)^3=\frac{1}{343}$

22. $3\cdot12+8=44$ inches, so $s=\frac{44}{3}$ in.

    $A=\frac{s^2\sqrt{3}}{4}\approx93\text{ in}^2$

23. The altitude to $2x$ is $\sqrt{(7x)^2-(1x^2)}$

    $=\sqrt{48x^2}=4x\sqrt{3}$, so $A=\frac{2x\cdot4x\sqrt{3}}{2}$

24. $x^2-4x+4+y^2+6y+9=-4+4+9$

    $(x-2)^2+(y+3)=9$, center @ $(2,-3)$

25. $3(Fibonacci)=3(1,1,2...,987,1597,2584)$

    $=(3,3,6,...,2961,4791,7752)\therefore k=4791$

26. $g(f(x))=(2x+5)^2-1$

    $=4x^2+20x+25-1=4x^2+20x+24$

27. $SA=4\cdot\frac{7^2\sqrt{3}}{4}=49\sqrt{3}$

28. $\frac{(x^2-2)(2)-(2x+1)(2x)}{(x^2-2)^2}$

    $\frac{2x^2-4-4x^2-2x}{x^4-4x^2+4}=-\frac{2x^2+2x+4}{x^4-4x^2+4}$

29. Table of possible values and ways to roll.

2	3	4	5	6	7	8
1	2	3	4	3	2	1

    $p(roll<5)=\frac{6}{16}=\frac{3}{8}$

30. Factors of 72: 2, **3**, 4, **6**, 8, **9**, **12**, **18**, **24**, **36**
    Odds = 7:3

31. $\frac{2(2^9)}{10}=\frac{512}{5}$

32. Definition of circumcenter

33. $\sqrt[4]{\left[(1.035)(0.98)(1.05)(1.0175)\right]}\approx2.03\%$

34. $rate=\frac{\pi(34)^2\cdot41}{231\cdot60}\approx10.74$

    $10.74-3=7.74$ gal/min

35. $(2^{-1})(2^0)(2^1)(2^2)=4$

36. $\left[x^2-x\right]_{-2}^{k}=k^2-k-(4+2)$

    $k^2-k-6=(k-3)(k+2)$, so $x=-2,3$.

37. $500=4\pi60^2\frac{dr}{dt}$, $\frac{dC}{dt}=2\pi\frac{dr}{dt}=\frac{500}{2\cdot60^2}$

    $\frac{dC}{dt}=\frac{5}{72}$ cm/min

38. $\left(-\frac{5}{3}\right)^3=-\frac{125}{27}$

39. $\cos2\approx-0.416$

40. $18\cdot\frac{4}{3}\cdot2\cdot\frac{3}{2}+8=80$

41. $A=12\cdot3\sqrt{3}=36\sqrt{3}$

42. $7^2=8^2+x^2-2\cdot8x\cos59^\circ$
    Graph and find intersections

43. $s=2a\sqrt{3}$, so $\frac{\left(2a\sqrt{3}\right)^2\sqrt{3}}{4}=3a^2\sqrt{3}$

44. $d=\frac{7}{0.15}-\frac{2.5}{0.2}=\$34.17$

45. ratio of perimeters = $\sqrt{\frac{175}{252}}=\frac{5}{6}$

    Solve: $\frac{5}{6}=\frac{p}{51}$, $p=42.5$ m

46. quad regression: $y=-1.5x^2+7x+2.5$

    $x=\frac{-b}{2a}=\frac{7}{3}$

47. $\frac{dy}{dx}=\frac{(x+3)-(x-1)}{(x+3)^2}=\frac{4}{(x+3)^2}=\pm1$

    when $x=-1,-5$

48. definition of Fibonacci

49. $\binom{7}{2}(2)^2(3)^5+\binom{7}{1}(2)^1(3)^6=30618$

50. $f(2)=33$ and $f'(x)=47$

    $y-33=47(x-2)$, $47x-y-61=0$

51. $\frac{8!}{8}=5040$

52. $2\cdot\binom{9+2-1}{2}=90$

53. $z=\frac{0.7-0.76}{0.06}=-1$ cumulative normal distribution. $p(x>0.7)\approx0.841$

54. $a^3b^3a^1b^{-3}a^2b^{-2}b^3=a^6b$

55. $AB=\begin{bmatrix}-3+2 & -6+4\\ -2+1 & -4+2\end{bmatrix}=\begin{bmatrix}-1 & -2\\ -1 & -2\end{bmatrix}$

56. $\frac{(x+2)!}{x!}\frac{(x-1)!}{(x-3)!}=(x+1)(x+2)\cdot(x-1)(x-2)$

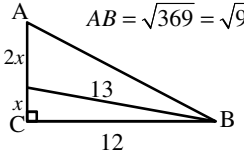
    $\left(x^2-1\right)\left(x^2-4\right)=x^4-5x^2+4$

57. $f(2)=36+4p+q=15$

    $f(-3)=6+36p+q=0$. Solve system

    $p+q=-30$

58. $x=5$, $AC=15$



    $AB=\sqrt{369}=\sqrt{9\cdot41}=3\sqrt{41}$

59. $2^8\cdot3^4\cdot5$, divisible by 1^4, 2^4, 3^4, 4^4, 6^4, 12^4

60. $\log\left(\frac{x^2\sqrt{y}}{z^3}\right)=2\log x+\frac{1}{2}\log y-3\log z$

    $=2a+\frac{1}{2}b-3c$.