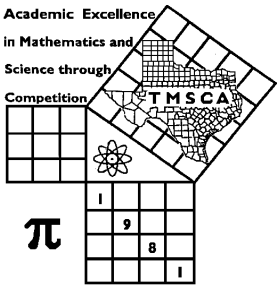


TMSCA HIGH SCHOOL MATHEMATICS TEST# 1 © OCTOBER 24, 2015

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2015-2016 TMSCA Mathematics Test One

1. Evaluate: $24\times15-4!-27\div(18-6)\times4+27$.

   A. 362.4 B. 374 C. 355.6 D. 402 E. 354

2. Caroline had a rope that was 15 feet long. She cut off three pieces such that the ratio of lengths of the pieces were 2:3:12 with 10 inches of string left over. How long was the shortest piece?

   A. 2 ft. 5 in. B. 1 ft. 6 in. C. 2 ft. 6 in. D. 1 ft. 10 in. E. 1 ft. 8 in.

3. Given that the data set $12,a,b,c,24,29$ is shown least to greatest and has a mean of 20, mode of 24 and median of 20. Calculate the value of $a+c$.

   A. 39 B. 48 C. 40 D. 31 E. 35

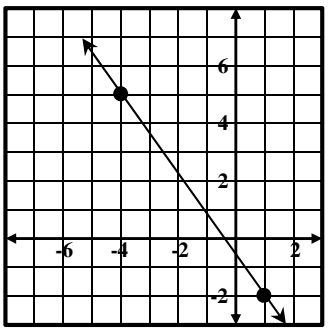
4. What is the mean of the first four abundant numbers?

   A. 14 B. 7 C. 18.5 D. 21 E. 21.5

5. Evaluate: $\dfrac{(x+2)!}{(x-3)!}\div\dfrac{x!}{(x-1)!}$.

   A. $x^6-5x^4+4x^2$ B. x^4+5 C. x^4-5x^2+4 D. x^6+4x^2 E. x^4-5x^2+5

6. Which of the following is the standard form of the equation of the line represented in this graph?



   A. $5x+7y=3$ C. $7x-5y=-10$ E. $7x+5y=13$

   B. $7x-5y=-17$ D. $7x+5y=-3$

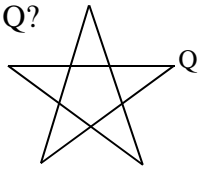
7. If $\theta=5\lambda$ and $\alpha+\theta=\varphi$, then $\alpha+5\lambda=\varphi$. This is an example of the _____ property.

   A. Substitution B. Transitive C. Commutative D. Associative E. Reflexive

8. Two consecutive angles in a pentagon are supplementary. The other three angles are congruent. What is the measure of one of the three congruent angles?

   A. 60° B. 120° C. 150° D. 90° E. 135°

9. The angles at each point on the star shown are congruent. What is the measure of the angle Q?



   A. 30° B. 27° C. 54° D. 72° E. 36°

10. What is the area of the region entirely bounded by the two functions $f(x)=x^2-4x+1$ and $g(x)=-4x+10$?

    A. 6 B. 36 C. 18 D. 108 E. 24

11. If $x+y=-3$, and $xy=-10$, then $x^3+y^3=$

    A. -117 B. -57 C. 33 D. 63 E. 3

12. The four brothers Lester, Morris, Nigel and Porter wanted to go on a road trip, but Lester had no money. Morris, Nigel and Porter each gave Lester one-fourth, one-fifth and one-third of his money respectively. If each gave Lester the same amount, what fraction of the money did Lester possess after the exchange?

    A. $\dfrac{3}{13}$ B. $\dfrac{3}{7}$ C. $\dfrac{1}{3}$ D. $\dfrac{3}{10}$ E. $\dfrac{1}{4}$

13. Find the value of the arithmetic mean for terms *a*, *b* and *c* in the geometric sequence: $3072, 3840, a, b, c, \ldots$.

A. 4880 B. 6100 C. 7495.68 D. 6000 E. 4800

14. $\tan\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}\right) \div \cot\left(\frac{5\pi}{3}\right)\csc\left(\frac{\pi}{6}\right) \div \cos\left(\frac{5\pi}{3}\right)\csc\left(\frac{5\pi}{3}\right) =$

A. $\frac{4}{3}$ B. 4 C. 2 D. $\frac{2\sqrt{3}}{3}$ E. $\frac{1}{2}$

15. The intersection of the medians of a triangle is called the _____.

A. Centroid B. Incenter C. Median D. Circumcenter E. Orthocenter

16. How many integral values of *n* exist such that $n > 3$, and $\frac{n!}{(n-3)!} \le 150$?

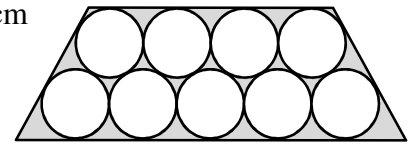
A. 0 B. 3 C. 1 D. 4 E. 2

17. There are two values of *k* for which $\det\begin{bmatrix} k-1 & 4 \\ -3 & 2k \end{bmatrix} = 0$. The sum of those two values is

A. 1 B. 5 C. 3 D. -2 E. -1

18. The radius of each circle is 2.5 cm. Find the perimeter of the trapezoid. (nearest tenth)

A. 68.1 cm B. 64.6 cm C. 88.1 cm D. 56.5 cm E. 58.9 cm



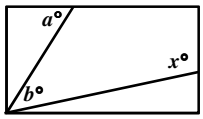
19. The number 478 in base 9 is equivalent to the number *k* in base 3. Find the sum of the digits in *k*.

A. 8 B. 19 C. 9 D. 7 E. 10

20. Find the mean value of $f(x) = 4x^3 - 6x^2 + 2x - 1$ for $[-1, 3]$.

A. 8 B. 19 C. 9 D. 18 E. 7

21. In the rectangle shown right, what is *x* in terms of *a* and *b*?



A. $90 - a - b$ B. $90 - a + b$ C. $a + b$ D. $180 - a - b$ E. $90 + a - b$

22. How many distinct arrangements can be formed using all of the letters in the words "FALL FESTIVAL"?

A. 39916800 B. 19958400 C. 967680 D. 20442240 E. 11880

23. If $g(x) = x - 1$ and $f(x) = x^4$, find $g(f(x+1))$.

A. $x^4 + 3x^3 + 3x^2 + x$ C. x^4 E. $x^4 + 4x^3 + 6x^2 + 4x - 2$

B. $x^4 + 4x^3 + 6x^2 + 4x$ D. $x^4 - 2$

24. A chemistry student needs to mix a 50 fluid ounce solution containing 54% glucose. The pharmacist has 30% and 90% solutions on hand. How much of the 30% solution should she use?

A. 30 ounces B. 27 ounces C. 20 ounces D. 23 ounces E. 25 ounces

25. Which of the following quadrants does not contain a solution to $5x + 3y \ge 9$?

A. QIII B. QI & QII C. QIV D. QI & QIV E. QI

26. A triangle with side lengths 12 cm, 11 cm and 22 cm is a(n)_____triangle.

A. isosceles acute B. scalene acute C. isosceles obtuse D. scalene obtuse E. scalene right

27. Which of the following is an equation of the tangent line of $f(x)=2x^2-x+\frac{4}{x}$ for $x=2$?

A. $6x-y=4$ B. $6x+y=-4$ C. $6x-y=-6$ D. $x+6y=-4$ E. $x+6y=8$

28. If $\log 9=P$, and $\log 5=Q$, then $\log 0.6=$

A. $P-Q^2$ B. $\frac{P}{2Q}$ C. $\frac{P-2Q}{2}$ D. $\frac{P}{Q^2}$ E. $\frac{P-Q}{2}$

29. If $U=\{a,b,c,d,e,f,g,h\}$, $A=\{a,c,e,g\}$, and $B=\{b,c,d,e\}$, find $A'\cap B'$.

A. $\{a,f,g,h\}$ B. $\{a,b,c,d,e,g\}$ C. $\{b,c,d,e,f\}$ D. $\{a,b,d,f,g,h\}$ E. $\{f,h\}$

30. If *P, Q* and *R* are real numbers such that $P+Q+R=8$, $R^2=P^2+Q^2$ and $PQ=8$, find the value of *R*.

A. 4 B. 8 C. 3 D. 6 E. 1

31. There are 6 girls and 8 boys in Ms. Angel's homeroom class. She must select a group of 2 girls and 2 boys to represent her class in a Veterans Day ceremony. How many distinct groups could does she have to choose from?

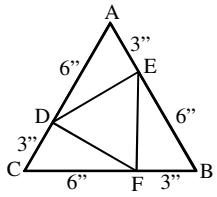
A. 1680 B. 43 C. 420 D. 225 E. 1001

32. Which of the following equations in rectangular form can be written as $r-6\sin\theta=0$ in polar form?

A. $x^2+y^2=9$ C. $x^2+y^2=3$ E. $x^2+y^2-6y=0$

B. $x^2-6x+y^2=0$ D. $x^2+y^2=2\sqrt{3}$

33. Find the area of $\triangle DEF$. (nearest tenth)



A. 9.7 in^2 B. 20.8 in^2 C. 6.8 in^2 D. 7.8 in^2 E. 11.7 in^2

34. Find the remainder when $f(x)=6x^3-x^2-7x+5$ is divided by $x-3$.

A. 137 B. 143 C. -133 D. -145 E. 155

35. A sales clerk is packaging blue, red and black pens for a back-to-school sale. How many different packages of 6 pens can he make?

A. 84 B. 126 C. 28 D. 56 E. 35

36. Two roots of $f(x)=x^3+bx^2+cx+d$ are 4 and $3+i$. Find $b+c+d$.

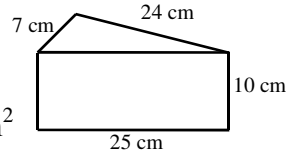
A. 54 B. -26 C. -16 D. 64 E. -4

37. If $g(x)\le f(x)\le h(x)$ for all *x*, *k* in $[a,b]$, where $x\neq k$, and $\lim_{x\to k} g(x)=L$ and $\lim_{x\to k} h(x)=L$ then $\lim_{x\to k} f(x)=L$. This theorem is known as:

A. Sandwich Theorem C. Rolle's Theorem E. Fundamental Theorem of Calculus

B. Intermediate Value Theorem D. Fundamental Theorem of Algebra

38. Calculate the total surface area of the triangular prism shown.



A. 728 cm^2 B. 644 cm^2 C. 840 cm^2 D. 924 cm^2 E. 560 cm^2

39. Find the sum of all the three digit numbers whose digits have a sum of eight and whose digits can all be used to form a perfect cube.

A. 1925 B. 1776 C. 861 D. 915 E. 1420

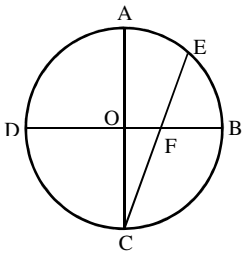
40. The ratio of length to width of a rectangle is 13:3 and the perimeter is 1536 in. What is the area of the rectangle?
A. 359424 ft^2 B. 1248 ft^2 C. 179712 ft^2 D. 624 ft^2 E. 2496 ft^2

41. The function $f(x)=\dfrac{2x^3}{x^2-3}$ is increasing at which of the following values of x?
A. -3 B. -4 C. -1 D. 0 E. 2

42. How many distinct solutions exist for $2\sin^2 x = 1+2\sin x$, where $0 \le x < 2\pi$?
A. 0 B. 1 C. 2 D. 3 E. 4

43. Meredith set out to row on a lake. She rowed 500 m on a bearing of 75°, then 200 m on a bearing of 25°, then 350 m on a bearing of 52°. How far is she from her original starting point?
A. 1050 m B. 615 m C. 775 m D. 526 m E. 994 m

44. Circle O has perpendicular diameters and a chord, find AE if CF = 7 inches and EF = 6 inches. (nearest tenth)
A. 4.2 in B. 3.6 in C. 3.8 in D. 4.5 in E. 3.2 in



45. What is the harmonic mean of the roots of the function $f(x)=6x^2-11x-72$?
A. $\dfrac{11}{12}$ B. $-\dfrac{144}{11}$ C. $\dfrac{7}{2}$ D. $\dfrac{12}{11}$ E. $-\dfrac{11}{144}$

46. Find y as a function of x given that $\dfrac{d^2y}{dx^2}=4-6x$ and that when $x=2$, $\dfrac{dy}{dx}=-4$ and $y=7$.
A. $y=7+2x^2-x^3$ C. $y=7+3x^2-x^3$ E. $y=9+3x^2-x^3$
B. $y=1+2x^2-x^3$ D. $y=23+3x^2-x^3$

47. What is the constant term in the binomial expansion of $\left(3x^3-\dfrac{2}{x}\right)^8$?
A. 576 B. 72576 C. 1296 D. 145152 E. 16128

48. A contestant on a game show rolls a single, fair, standard die. The player loses \$100 if an odd number is rolled. If he rolls an even prime, he gets a \$500 payout. If he rolls a perfect number, he gets \$1000 payout. Otherwise, nothing happens. What are his expected winnings?
A. \$200 B. \$250 C. \$240 D. \$150 E. \$275

49. The point $(6,-2)$ lies on a circle whose center is $(0,8)$. Where does the point $(8,13)$ lie in reference to the circle?
A. Inside B. Outside C. On the Circle D. Q II E. Unknowable

50. How many solutions are there for the equation $2x+5y=125$ where both x and y are non-negative integers?
A. 12 B. 11 C. 13 D. 10 E. 14

51. Sixty-five percent of homes in a town have pets. If four homes are chosen at random for a survey, find the probability that all four have pets. (nearest percent)
A. 26% B. 11% C. 13% D. 15% E. 18%

52. If $\frac{7x+13}{x^2+2x-3}=\frac{A}{x+3}+\frac{B}{x-1}$, then $AB=$

A. 7 B. -6 C. -3 D. 6 E. 10

53. Given that the set of natural numbers continue in the triangular pattern shown below, find the median of the numbers in row 9.

			1				(row 1)
		2	3	4			(row 2)
	5	6	7	8	9		(row 3)
10	11	12	13	14	15	16	(row 4)
			…				(…)

A. 83 B. 73 C. 77 D. 67 E. 85

54. The square root of 1013 in base 6 is:

A. 111_6 B. 23_6 C. 35_6 D. 25_6 E. 151_6

55. If $y^2=5-12i$ and $y^3=-9-46i$ where $y=a+bi$ then $a+b=$

A. 1 B. -38 C. 5 D. -62 E. 6

56. $3^3+4^3+5^3+...+12^3+13^3+14^3=$

A. 11016 B. 11017 C. 289570 D. 11025 E. 2744

57. What is the area of a regular hexagon in terms of the length, *s*, of one side?

A. $\frac{3s^2\sqrt{3}}{4}$ B. $\frac{4s^2\sqrt{3}}{3}$ C. $2s^2\sqrt{3}$ D. $\frac{3s^2\sqrt{3}}{2}$ E. $3s^2\sqrt{3}$

58. Find the units digit of 17^{2015}.

A. 3 B. 1 C. 7 D. 0 E. 9

59. Simplify to the nearest ten-thousandth place: $1+(1.3)+\frac{(1.3)^2}{2!}+\frac{(1.3)^3}{3!}+\frac{(1.3)^4}{4!}+....$

A. 0.2624 B. 0.2675 C. 3.6693 D. 3.6302 E. 0.9636

60. The function f is such that $\int_{-1}^{8} f(x)\,dx=9$. What is the value of $\int_{-1}^{8}(f(x)+3)\,dx$?

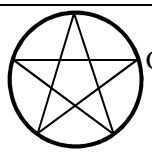
A. 12 B. 36 C. 27 D. 32 E. 18

2015-2016 TMSCA Mathematics Test One Answers

1. E	21. E	41. B
2. E	22. B	42. C
3. A	23. B	43. E
4. C	24. A	44. B
5. C	25. A	45. B
6. D	26. D	46. A
7. A	27. A	47. E
8. B	28. C	48. A
9. E	29. E	49. A
10. B	30. C	50. C
11. A	31. C	51. E
12. E	32. E	52. E
13. B	33. E	53. B
14. B	34. A	54. B
15. A	35. C	55. A
16. B	36. C	56. A
17. A	37. A	57. D
18. A	38. A	58. A
19. C	39. B	59. C
20. E	40. D	60. B

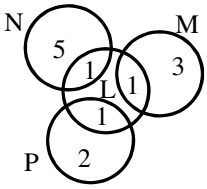
2015-2016 TMSCA Mathematics Test One Selected Solutions

9. Angle Q is an inscribed angle with an intercepted arc $\frac{360^\circ}{5}=72^\circ$, so $m\angle Q=36^\circ$.



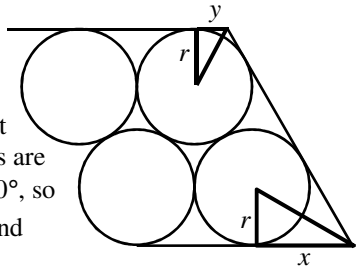
11. $x^3+y^3=(x+y)(x^2-xy+y^2)=$
$(x+y)\left[(x+y)^2-3xy\right]=-3(9+30)$

12. If each gives Lester a dollar, use a Venn diagram to show the fraction relationships then Lester has $\frac{3}{13}$ of the money in the end.



16. $n(n-1)(n-2)\le 150$. For a starting point, evaluate $\sqrt[3]{150}\approx 5.31$. Try 5 for the $(n-1)$ value. $6\bullet5\bullet4=120$, while $7\bullet6\bullet5=210$, so *n* values can be 6, 5, and 4.

18. The small triangles at the corners are 30°-60°-90°, so $x=r\sqrt{3}$ and $y=\frac{r\sqrt{3}}{3}$. As part of the perimeter, *x* and *y* each appear 4 times, so area is $4x+4y+18r$ or approximately 68.1 when $r=2.5$.



22.There are 12 total letters with the "F" repeated twice, the "A" repeated twice and the "L" repeated thrice, so the total number of distinct arrangements is $\frac{12!}{(2!)(2!)(3!)}=19958400$.

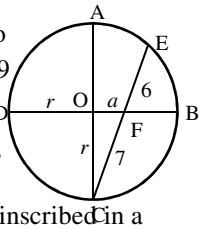
23. $g(f(x+1))=(x+1)^4-1$. Use binomial theorem or Pascal's triangle to get $(x^4+4x^3+6x^2+4x+1)-1$ for a final answer $x^4+4x^3+6x^2+4x$.

30. With just a bit of algebra, $(P+Q)^2=(8-R)^2$ so, $P^2+2PQ+Q^2=64-16R+R^2$ then $R^2+2PQ=64-16R+R^2$ or $2PQ=64-16R$ therefore, $2(8)=64-16R$ for a solution of $R=3$.

35. There are three different types to choose from and packages of six, so the number of possibilities is ${}_{3+6-1}C_6=28$.

39. The two perfect 3-digit cubes whose digits have a sum of 8 are 125 and 512. There are six possible 3-digit numbers that can be formed with the digits 1, 2 and 5. $125+152+251+215+521+512=1776$

44. To find *r*, use the two relationships $a^2+r^2=49$ and $(r+a)(r-a)=42$. Adding the two together, $2r^2=91$ and. Then, because triangle AEC is inscribed in a semi-circle, it has a right angle at E and $13^2+(AE)^2=(2r)^2$ and $AE=\sqrt{4r^2-169}$ or $AE=\sqrt{2(91)-169}\approx 3.6$ inches.



47. The constant term will be ${}_8C_2\left(3x^3\right)^2\left(-\frac{2}{x}\right)^6$ because there *x* will have a power of 6 in both the numerator and denominator. The constant left will be 16128.

52. Multiplying all terms of the equation by the LCD, will yield the equation: $7x+13=A(x-1)+B(x+3)$. Let $x=1$ to find the value of *B* using the equation $20=4B$ so $B=5$. Similarly, let $x=-3$ to find the value of *A* using the equation $-8=-4A$ so $A=2$ and $AB=10$.

53. The median of any row in the arrangement shown will always be the center number. The 1, 3, 7, 13…can either be used to develop a quadratic regression $y=x^2-x+1$ where *x* is the row number and *y* is the median. For the 9th row the median will be 73. An alternative would be to use the differences in the center numbers and continue the pattern
1+2=3
3+4=7
7+6=13
13+8=21……57+16=73.

56. The formula for the sum of the first *n* cubes is $\left(\frac{n(n+1)}{2}\right)^2$, so the sum of the series will be $\left(\frac{14(15)}{2}\right)^2-1-8=11016$.

59. The series shown is the McClaurin series for the function $f(x)=e^x$ when $x=1.3$, so $e^{1.3}\approx 3.6692967$.

T M S C A H I G H S C H O O L

M A T H E M A T I C S

T E S T # 2 ©

O C T O B E R 3 1, 2 0 1 5

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2015-2016 TMSCA Mathematics Test Two

1. Evaluate: $1\div(1+3)^{-1}\times 4-\frac{5}{8}+17\times(32)^{0}$.

   A. $13\frac{3}{8}$ B. $32\frac{5}{8}$ C. $32\frac{3}{8}$ D. $20\frac{3}{8}$ E. $20\frac{5}{8}$

2. $\angle A$ and $\angle B$ are supplementary. If the $m\angle A : m\angle B$ is 5:7, find the measure of the complement of $\angle A$.

   A. 24° B. 21° C. 15° D. 18° E. 36°

3. What is the greatest common factor of 204, 510 and 646?

   A. 17 B. 34 C. 19 D. 51 E. 13

4. Leroy walked from his house to the school to pick up his bicycle at an average rate of 6 mph. He rode his bike home at an average rate of 26 mph. The total trip took 32 minutes. How far does Leroy live from the school?

   A. 2.6 mi B. 2.8 mi C. 1.6 mi D. 1.9 mi E. 2.4 mi

5. At Hobby Stop the price of a tube of oil paint is $7.85 and the price of a paint brush is $3.95. Crafty Carl has a 30% off coupon to use for the paint and the brushes are on sale for 15% off. If Carl buys 8 tubes of paint and 4 brushes, what will his cost be after the 8.5% sales tax has been applied?

   A. $35.01 B. $50.27 C. $52.99 D. $57.39 E. $62.27

6. Simplify $a^{-3}\div b^{3}\times a^{-5}\div b^{-5}\times a^{-3}\div b^{5}$.

   A. $\frac{b^{3}}{a^{11}}$ B. $\frac{1}{a^{11}b^{3}}$ C. $\frac{1}{ab^{7}}$ D. $\frac{b^{7}}{a^{11}}$ E. $\frac{a}{b^{7}}$

7. Two parallel lines are cut by a transversal to form two consecutive interior angles with measures $(x^{2}-26)°$ and $(5x+2)°$. What is the measure of the acute angle?

   A. 62° B. 17° C. 50° D. 87° E. 57°

8. The value of *y* varies inversely with $5x^{2}$. If $y=1.2$ when $x=2$, what is the value of *y* when $x=4$?

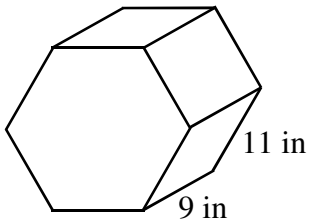
   A. 1.2 B. 0.15 C. 4.8 D. 2.4 E. 0.30

9. The graph of the function $f(x)=x^{4}-12x^{3}+48x^{2}-64x$ has points of inflection when $x=a$ and $x=b$ where $a<b$. What is the value of *b*?

   A. -1 B. 4 C. -2 D. 3 E. 1

10. Find the lateral surface area of the regular hexagonal prism shown. (nearest square inch)

    A. 495 in^2 B. 594 in^2 C. 1015 in^2 D. 2314 in^2 E. 1157 in^2



11. Find the sum of the arithmetic mean, median, mode and range of 3, 19, 5, 2, 10, 35, & 3.

    A. 49 B. 48 C. 55 D. 52 E. 53

12. Which of the following is an equation of the line passing through $(-3.2,1.5)$ parallel to $2x+3y=9$?

    A. $20x+30y=141$ C. $20x-30y=-19$ E. $20x-30y=-141$

    B. $20x+30y=-19$ D. $20x-30y=19$

13. A piggy bank contains a total of 82 dimes and quarters worth \$16.45. How many dimes are in the bank?

A. 26 B. 54 C. 27 D. 41 E. 55

14. The number of integers between 1 and 54 that are relatively prime to 54 is:

A. 15 B. 17 C. 18 D. 20 E. 24

15. Classify the triangle with side lengths 17 cm, 20 cm and 32 cm.

A. Isosceles Acute B. Isosceles Obtuse C. Scalene Acute D. Scalene Obtuse E. None of these

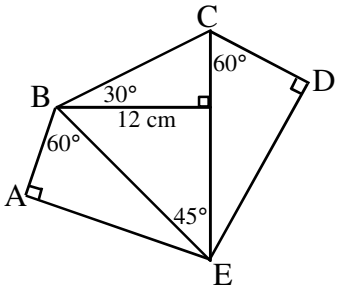
16. What is the sum of the arithmetic series $11+8.5+6+...+(-26.5)$?

A. -116 B. -124 C. 48.5 D. -270 E. -288

17. Solve $\sin 2x = -\cos x$, where $0 \le x < 2\pi$.

A. $\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ B. $\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$ C. $\left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\right\}$ D. $\left\{\frac{\pi}{2}, \frac{\pi}{3}, \frac{3\pi}{2}, \frac{2\pi}{3}\right\}$ E. $\left\{\frac{\pi}{3}, \frac{2\pi}{3}\right\}$

18. Find the perimeter of pentagon ABCDE. (nearest tenth)



A. 80.1 cm B. 68.2 cm C. 62.9 cm D. 71.0 cm E. 74.8 cm

19. $\frac{d}{d\theta}\sin\left(3\theta^2\right) =$

A. $6\theta\cos\left(3\theta^2\right)$ B. $\cos(6\theta)$ C. $-3\theta\cos(6\theta)$ D. $3\theta\cos\left(3\theta^2\right)$ E. $-3\theta\cos\left(3\theta^2\right)$

20. Simplify $\frac{2x^2+x-6}{x^2+4x-5} \times \frac{x^3-3x^2+2x}{4x^2-6x}$.

A. $\frac{x^2-4}{2x+10}$ B. $\frac{x^2-2x}{x+10}$ C. $\frac{x^2-4}{x+5}$ D. $\frac{x-2}{2x+10}$ E. $\frac{x^2-2x}{x+5}$

21. Find the average rate of change for $f(x) = 3x^3 - 2x^2 + 5$ on the interval $[2,7]$

A. 77 B. 54 C. 50 D. 183 E. 55

22. What is the arithmetic mean of the first six triangular numbers?

A. $15\frac{1}{6}$ B. 56 C. $9\frac{1}{3}$ D. $3\frac{1}{2}$ E. $13\frac{5}{6}$

23. How many distinguishable arrangements can be made from the letters in the word "SASSAFRASS"?

A. 181440 B. 90720 C. 5040 D. 10080 E. 720

24. $876_9 = k_3$. Find the sum of the digits in *k*.

A. 9 B. 8 C. 7 D. 10 E. 6

25. Which of the following quadrant(s) does not contain a solution to $7x - 5y \le -49$?

A. QIV B. QI & QII C. QIII &QIV D. QIII E. QI

26. Phil is filling up his empty conical water tank. The diameter of the base is 18 feet and the height of the tank is 6 ft. What is the least number of whole gallons of water he will need to fill the tank?

A. 11421 gal B. 3808 gal C. 4847 gal D. 952 gal E. 318 gal

27. Given $a_{n+1} = a_{n-1} + (a_n)^2$, $a_0 = 2$ and $a_1 = 3$, find a_4.

A. 2608 B. 15387 C. 124 D. 6898 E. 1611

28. Jill can paint a room alone in 8 hours. She painted for the first hour by herself then her friend Kylie joined her. They finished in a total of 6 hours instead of 8. How many hours would it have taken Kylie to do the whole job alone?

A. 20 hours B. 18 hours C. 15 hours D. 12 hours E. 10 hours

29. $3\log a^2b^2 + \frac{1}{2}\log a^3b - 3\log ab =$

A. $\log a^3b^4\sqrt{ab}$ B. $-\log\frac{a^3b^3}{2}$ C. $\log a^4b^3\sqrt{ab}$ D. $\log\frac{1}{a^3b^3}$ E. $\log a^8b^7\sqrt{ab}$

30. If $\int_{-3}^{8} f(x)\,dx = 42$, then $\int_{-3}^{8}\left[2+5f(x)\right]dx =$

A. 210 B. 212 C. 139 D. 218 E. 232

31. The equation $5x^2 - 6x + k = 0$ has two positive solutions when which of the following is true?

A. $-1.2 < k < 1.2$ B. $k > 1.2$ C. $k < 1.8$ D. $-1.2 < k < 1.8$ E. $0 < k < 1.8$

32. There are 84 students in the senior class in Texas HS senior class. The ratio of boys to girls is 7:5. If the number of boys remains constant, how many new girls would have to enroll to change the ratio to 1:1?

A. 24 B. 35 C. 21 D. 7 E. 14

33. Let x and y exist such that $8 < x < y$. If $8, x, 24$ form an arithmetic sequence and $x, y, 25$ form a geometric sequence, then $xy = ?$

A. 320 B. 400 C. 256 D. 160 E. 324

34. Find C if the remainder of $2x^3 - 7x^2 + 3x + C$ divided by $x - 4$ is 16.

A. -236 B. 35 C. -12 D. 7 E. 14

35. $A = \begin{bmatrix} 3 & 5 \\ -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 7 \\ 3 & -4 \end{bmatrix}$, so $\det BA =$

A. -376 B. 186 C. -377 D. -324 E. 203

36. The chords $\overline{AD}$ and $\overline{BC}$ intersect inside circle O at point P. If $AD = 23$, $AP = 5$ and $BP = 3$, find CB.

A. 30 B. 27 C. $\frac{124}{3}$ D. 33 E. $\frac{115}{3}$

37. The altitudes of a triangle intersect at the

A. Orthocenter B. Incenter C. Circumcenter D. Median E. Origin

38. The graph of $9x^2 - y^2 - 36x - 6y + 18 = 0$ is a

A. Circle B. Hyperbola C. Parabola D. Ellipse E. Cartoid

39. John's Ice Cream carries eight gourmet flavors of home-made ice cream every day. If they also carry four choices of cones, how many different 2-scoop cone choices are there every day?

A. 180 B. 45 C. 220 D. 210 E. 144

40. Two roots of $f(x) = x^3 + bx^2 + cx + d$ are 7 and $3 + i$. Find $b + c + d$.

A. -34 B. -31 C. -1 D. -10 E. 17

41. The graph of the parametric equations $x = 3\cos t$ and $y = 5\sin t$ is a(n) _____.

A. Ellipse B. Hyperbola C. Circle D. Parabola E. Line

42. If $4^x \cdot 16^{2y} = 1$ and $7^{5x} \cdot 49^y = \frac{1}{49}$, find the value of $x + y$.

A. $\frac{4}{9}$ B. $\frac{1}{9}$ C. $-\frac{4}{9}$ D. $\frac{1}{3}$ E. $-\frac{1}{3}$

43. Find the digit in the millionths place of the sum of the series $1 - \frac{2}{1!} + \frac{4}{2!} - \frac{8}{3!} + \frac{16}{4!} - \ldots$.

A. 6 B. 5 C. 7 D. 4 E. 3

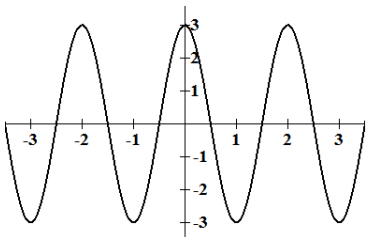
44. Let *a*, *b* and *c* be real numbers such that $c = a + b + 8$, $c^2 = a^2 + b^2$ and $ab = 8$. Find the value of $6c$.

A. 3 B. 18 C. 30 D. -24 E. -30

45. What is the angle between the hour and minute hands on a clock at 6:23 pm?

A. 53.5° B. 35.5° C. 42° D. 56.5° E. 47.5°

46. Which of the following functions yields the graph shown?



A. $y = 3\sin(\pi(x-2))$ C. $y = 6\sin(\pi x - 2)$ E. $y = 3\cos(\pi(x-2))$

B. $y = 3\sin(\pi x - 2)$ D. $y = 6\cos(\pi(x-2))$

47. The Real value solution for $x^2 - 2x - 3 < 0$ is?

A. $\{x | -1 < x < 3\}$ C. $\{x | \{x < -3\} \cup \{x > 1\}\}$ E. $\{x | x > -1\}$

B. $\{x | \{x < -1\} \cup \{x > 3\}\}$ D. $\{x | -3 < x < 1\}$

48. Find the smallest positive angle between the vectors $v_1 = \langle -16, 3 \rangle$ and $v_2 = \langle 11, 9 \rangle$. (nearest degree)

A. 29° B. 23° C. 130° D. 40° E. 140°

49. Find the slope of $2x^2 + 3y^2 = 29$ at the point $(-1, 3)$.

A. $\frac{9}{2}$ B. $-\frac{9}{2}$ C. $-\frac{8}{3}$ D. $-\frac{2}{9}$ E. $\frac{2}{9}$

50. Given triangle ABC such that $m\angle A = 60°$, $AC = 17$ cm and $BC = 15$ cm there are two possible lengths for the side $\overline{AB}$. Find the sum of these two lengths.

A. $17\sqrt{3}$ B. 17 C. $15\sqrt{3}$ D. 15 E. $\frac{17\sqrt{3}}{2}$

51. Let $f(x) = \frac{2x^2 - 7x + 5}{x - 8}$ and $s(x)$ be the slant asymptote of *f*. Find the value of $s(-6)$.

A. 12 B. -3 C. 77 D. -35 E. -12

52. Find the value of $A + B + C$, where *A*, *B* and *C* are non-negative integers such that: $\frac{41}{8} = A + \left(\frac{1}{B + \frac{1}{C+1}} \right)$

A. 12 B. 7 C. 15 D. 18 E. 9

53. Carmen has white marbles and yellow marbles in a bag. The probability that she will select a white marble is $\frac{1}{4}$. If 30 white marbles are added to the bag, the probability of selecting a white marble becomes $\frac{1}{3}$. How many yellow marbles are in the bag?

A. 180 B. 60 C. 90 D. 240 E. 270

54. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, …be the terms of the Fibonacci sequence. Find $(f_7)^2 + (f_8)^2$.

A. 233 B. 1597 C. 34 D. 610 E. 55

55. If $f(x) = ax^5 + bx^3 - 10$ and $f(3) = -40$, then $f(-3) =$

A. 20 B. -30 C. -20 D. -50 E. 30

56. Find the area of the convex quadrilateral with vertices $(2,7)$, $(4,1)$, $(1,-5)$ and $(-6,2)$.

A. 24 B. 27.5 C. 60.5 D. 59.5 E. 30.5

57. How many solutions (x, y) are there to $5x + 3y = 900$ where x and y are both positive integers?

A. 61 B. 58 C. 60 D. 59 E. 62

58. If $y = 9 - x$ and $xy = 16$ then $|x - y| =$

A. $3\sqrt{7}$ B. $\sqrt{17}$ C. 4 D. 7 E. $\sqrt{33}$

59. $\tan\theta > 0$ and $\sin\theta < 0$. Where will θ terminate?

A. QI B. QII C. QIII D. QIV E. y - axis

60. $444_5 + 777_8 - 222_3 =$ _____$_{10}$.

A. 400 B. 609 C. 630 D. 398 E. 611

2015-2016 TMSCA Mathematics Test Two Answers

1. C	21. D	41. A
2. C	22. C	42. E
3. B	23. C	43. B
4. A	24. A	44. B
5. E	25. A	45. A
6. B	26. B	46. E
7. A	27. B	47. A
8. E	28. A	48. C
9. B	29. C	49. E
10. B	30. E	50. B
11. D	31. E	51. B
12. B	32. E	52. A
13. C	33. A	53. A
14. B	34. C	54. D
15. D	35. C	55. A
16. B	36. D	56. C
17. C	37. A	57. D
18. C	38. B	58. B
19. A	39. E	59. C
20. A	40. B	60. B

2015-2016 TMSCA Mathematics Test Two Selected Solutions

4. $\frac{d}{6}+\frac{d}{26}=\frac{32}{60}$ because the two fractions on the right side of the equation represent the times for the trip and return trip. The solution to the equation is 2.6 miles.

21. The average rate of change will just be the slope of the secant line connecting the endpoints of the function on the given interval. $m=\frac{f(7)-f(2)}{7-5}=183$.

23. There are a total of 10 letters with "S" repeating 5 times and "A" repeating thrice, so the number of distinguishable arrangements will be: $\frac{10!}{(5!)(3!)}=5040$.

28. Jill's rate is $\frac{1}{8}$ of a room per hour. Let Kylie's rate be r.
$\frac{1}{8}+5\left(\frac{1}{8}+r\right)=1$. Solving for *r* gives a rate for Kylie of $\frac{1}{20}$ of a room per hour, so it will take her 20 hours to do the job alone.

30.
$\int_{-3}^{8}\left[2+5f(x)\right]dx=\int_{-3}^{8}2dx+5\int_{-3}^{8}f(x)dx=$
$22+5(42)=232$.

33. Because of the definition of an arithmetic sequence, $x=\frac{8+24}{2}=16$. We can find the ratio of the terms in the geometric sequence using $16r^2=25$, so $r=\frac{5}{4}$ and $y=16\left(\frac{5}{4}\right)=20$. $xy=320$.

39. There are 8 flavors and we are going to order 2 scoops, so the number of choices for ice cream is ${}_{8+2-1}C_2=36$. There are also 4 choices for cones, so the total number of options is $4(36)=144$.

43. This is the power series for $f(x)=e^x$ when $x=-2$. $e^{-2}\approx 0.1353352832$ with the 5 being the digit in the millionths place.

44. A little bit of algebraic manipulation of the first equation yields:
$(c-8)^2=(a+b)^2$ then
$c^2-16c+64=a^2+2ab+b^2$ using substitution from the equation $c^2=a^2+b^2$
$c^2-16c+64=c^2+2ab$ then
$-16c+64=2ab\rightarrow -16c+64=16$, so
$c=3$ and $6c=18$.

48. $\cos\theta=\frac{(-16)(11)+(3)(9)}{\sqrt{(256+9)(121+81)}}$, so
$\theta\approx 130.09^\circ$.

49. $4x+6y\frac{dy}{dx}=0$, so $-4+18\frac{dy}{dx}=0$.
Solving for $\frac{dy}{dx}=\frac{2}{9}$.

50. Use the law of cosines to set up the quadratic equation:
$225=289+x^2-34x\cos 60$, where $x=AB$. Arranging the equation to have a 0 on one side: $0=x^2-17x+64$, so the sum of the roots (or two possible lengths) is 17.

52. $\frac{41}{8}=5+\frac{1}{8}=5+\frac{1}{7+\frac{1}{0+1}}$, so
$5+7+0=12$.

55. $f(3)=ax^5+bx^3-10=-40$, so for $x=3$, $ax^5+bx^3=-30$. Because both terms have odd exponents, for $x=-3$, $ax^5+bx^3=30$, and $f(-3)=30-10=20$.

58. $(x+y)^2=x^2+2xy+y^2=81$, and $(x-y)^2=x^2-2xy+y^2$, or $(x+y)^2-4xy$ so that $(x-y)^2=81-4(16)=17$ and $|x-y|=\sqrt{17}$.

T M S C A H I G H S C H O O L

M A T H E M A T I C S

T E S T # 3 ©

N O V E M B E R 7, 2 0 1 5

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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2015-2016 TMSCA Mathematics Test Three

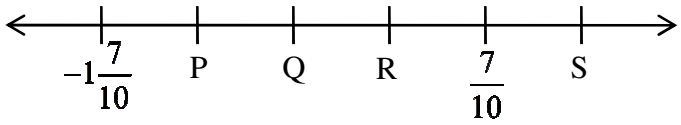
1. Evaluate: $5!+12(3-10)\div 20\times 15$.

   A. 183 B. 119 C. 120 D. 70 E. 57

2. Caroline had a rope that was 15 feet long. She cut off three pieces such that the ratio of lengths of the pieces were 2:5:10 with 10 inches of string left over. How long was the longest piece?

   A. 8 ft. 4 in. B. 7 ft. 10 in. C. 11 ft. 6 in. D. 8 ft. 10 in. E. 10 ft. 8 in.

3. The distances between the hash marks (|) are equal. Find $P+Q+R+S$.



   A. 2.4 B. 4.9 C. -0.5 D. 0.3 E. -0.2

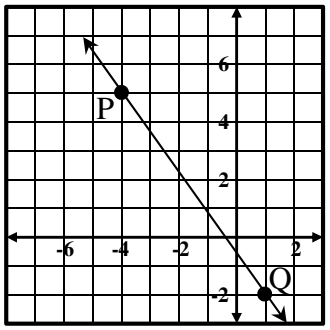
4. What is the median of the first five perfect numbers?

   A. 262 B. 4312 C. 6711798 D. 496 E. 8128

5. Evaluate: $\dfrac{(x+2)!}{(x-2)!}\div\dfrac{x!}{(x-1)!}$.

   A. $x^5+2x^4-x^3-2x^2$ B. x^3+2x^2-x-2 C. x^4-5x^2+4 D. x^6+4x^2 E. x^4-5x^2+5

6. Which of the following is the standard form of the equation of the perpendicular bisector of $\overline{PQ}$?



   A. $5x+7y=3$ C. $7x-5y=-10$ E. $7x+5y=-3$

   B. $7x-5y=-17$ D. $5x-7y=-18$

7. Which of the following properties, if any is not used in this example?

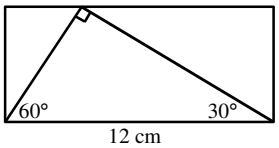
$$\begin{aligned}
5a-5a+7\times\left(3\times\frac{4}{7}\right) &= a(5-5)+7\times\left(3\times\frac{4}{7}\right)\\
&= a\times 0+7\times\left(3\times\frac{4}{7}\right)\\
&= a\times 0+7\times\left(3\times\frac{4}{7}\right)\\
&= 0+7\times\left(3\times\frac{4}{7}\right)\\
&= 0+7\times\left(\frac{4}{7}\times 3\right)\\
&= 0+\left(7\times\frac{4}{7}\right)\times 3\\
&= 0+4\times 3\\
&= 0+12\\
&= 12
\end{aligned}$$

   A. Distributive B. Commutative C. Associative D. Substitution E. All are used

8. The point of intersection of the three perpendicular bisectors of a triangle is called the_____.

   A. Center B. Circumcenter C. Centroid D. Incenter E. Orthocenter

9. Find the area of the rectangle shown.

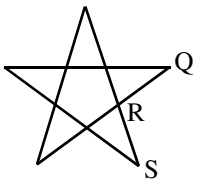


A. 36 cm^2 B. $36\sqrt{3}\text{ cm}^2$ C. 24 cm^2 D. $48\sqrt{3}\text{ cm}^2$ E. 72 cm^2

10. Two consecutive angles in a hexagon are supplementary. Another one of the angles is a right angle. The remaining three angles are congruent. What is the measure of one of the three congruent angles?

A. 60° B. 120° C. 135° D. 90° E. 150°

11. The angles at each point on the star shown are congruent. What is the measure of the angle QRS?



A. 144° B. 135° C. 120° D. 96° E. 108°

12. How many integral values of n exist such that $n > 2$ and $\frac{n!}{(n-2)!} \le 150$?

A. 10 B. 9 C. 11 D. 8 E. 7

13. The chess club consists of 6 girls and 4 boys. How many different 3-person teams can the coach form if any team must have at least one boy?

A. 100 B. 35 C. 60 D. 95 E. 72

14. What is the area of the region entirely bounded by the two functions $f(x) = 2x^2 - 3x + 8$ and $g(x) = -x + 20$?

A. $\frac{125}{6}$ B. 30 C. 40 D. $\frac{125}{3}$ E. $\frac{43}{2}$

15. If $x + y = -5$, and $xy = -8$, then $x^3 + y^3 =$

A. 5 B. -320 C. -85 D. 63 E. -245

16. $\tan\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) \div \cot\left(\frac{5\pi}{4}\right)\csc\left(\frac{\pi}{4}\right) \div \cos\left(\frac{5\pi}{4}\right)\csc\left(\frac{5\pi}{4}\right) =$

A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. 4 D. 2 E. 1

17. There are two values of k for which $\det\begin{bmatrix} -k & 6 \\ 3 & k+3 \end{bmatrix} = -58$. The sum of those two values is

A. -5 B. -3 C. 5 D. -13 E. 3

18. What are the odds that a factor of 144 is a multiple of 4?

A. 3 to 2 B. 9 to 7 C. 3 to 5 D. 9 to 16 E. 3 to 8

19. $(3-2i)^5 =$

A. $-597-122i$ B. $243-32i$ C. $-597+122i$ D. $-243-32i$ E. $-275+211i$

20. How many 3-digit numbers exist such that the sum of their digits equals 5?

A. 10 B. 9 C. 15 D. 12 E. 14

21. Find the digit in the millionths place of the sum of the series: $4+8+\frac{64}{3!}+\frac{256}{4!}+\frac{1024}{5!}...$

A. 5 B. 0 C. 2 D. 4 E. 8

22. How many distinct arrangements can be formed using all of the letters in the words "APPLE PIE"?
A. 6720 B. 1260 C. 3360 D. 20160 E. 4480

23. Which of the following statements is a false statement for $f(x)=\begin{cases}3x^2-3, & x\le 2\\ 5x-1, & x>2\end{cases}$
A. $f(2)$ exists C. $\lim_{x\to 2^-} f(x)$ exists E. None of them
B. $\lim_{x\to 2^+} f(x)$ exists D. f is continuous

24. If $g(x)=x-1$ and $f(x)=x^4$, find $f(g(x+2))$.
A. $x^4+3x^3+3x^2+x$ C. x^4 E. $x^4+4x^3+6x^2+4x-1$
B. $x^4+4x^3+6x^2+4x+1$ D. x^4+1

25. The Beanery would like to market a mix of two of its most popular coffees. The Good Morning coffee sells for \$6 per pound, while the Dark Rich coffee sells for \$8 per pound. How much Good Morning coffee should be used to create one pound of a mix that sells for \$6.50 per pound?
A. 2.5 oz. B. 12 oz. C. 8 oz. D. 4 oz. E. 9.5 oz.

26. Given $\begin{bmatrix}a & 2\\ 7 & b\end{bmatrix}\times\begin{bmatrix}2b & 4\\ -a & 1\end{bmatrix}=\begin{bmatrix}24 & 14\\ 55 & 33\end{bmatrix}$, find the value of *a*.
A. 5 B. 6 C. 3 D. 2 E. 10

27. A triangle with side lengths 12 cm, 12cm and 22 cm is a(n)_____triangle.
A. scalene acute B. scalene obtuse C. isosceles acute D. isosceles obtuse E. scalene right

28. A particle is moving along a straight line with a function $f(t)=6t^2-5t-11$, where $f(t)$ is the distance in in meters per second. Find the instantaneous rate of change at a time of 2 seconds.
A. 19 m/s B. 3 m/s C. 1 m/s D. $\frac{11}{6}$ m/s E. $\frac{5}{6}$ m/s

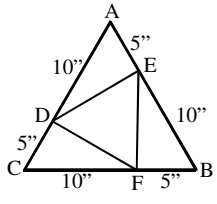
29. If $\log 9=P$, and $\log 5=Q$, then $\log 16.2=$
A. $2P-Q$ B. $\frac{2Q}{P}$ C. $\frac{Q^2}{P}$ D. $\frac{P}{Q^2}$ E. $\frac{P-Q}{5}$

30. If *P, Q* and *R* represent digits in $RPQ_4+QRP_3-PQR_2$ has a numeric value in base 10 of:
A. $9P+12Q+20R$ B. $7P-6Q+14R$ C. $21P+11Q+6R$ D. $P+8Q+18R$ E. $21P-11Q+6R$

31. If *P, Q* and *R* are real numbers such that $P+Q+R=10$, $R^2=P^2+Q^2$ and $PQ=6$, find the value of *R*.
A. 4.1 B. 5.2 C. 4.4 D. 5 E. 5.1

32. Which of the following equations in rectangular form can be written as $r-12\sin\theta=0$ in polar form?
A. $x^2+y^2=36$ C. $x^2+y^2=0$ E. $x^2+y^2-6y=0$
B. $x^2-6x+y^2=0$ D. $x^2+y^2-12y=0$

33. Find the area of $\triangle DEF$. (nearest tenth)
A. 21.7 in^2 B. 43.3 in^2 C. 28.9 in^2 D. 30.7 in^2 E. 32.5 in^2



34. Find the remainder when $f(x)=6x^3-x^2+7x+5$ is divided by $x-7$.
A. -2151 B. 2063 C. 2161 D. -2053 E. 1965

35. A sales clerk is packaging blue, red and black pens for a back-to-school sale. How many different packages of 5 pens can he make?

A. 21 B. 56 C. 28 D. 56 E. 35

36. Two roots of $f(x) = x^3 + bx^2 + cx + d$ are 5 and $3 + i$. Find $b + c + d$.

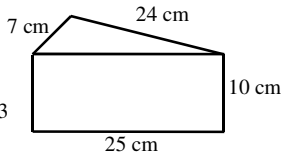
A. 15 B. -11 C. -21 D. -36 E. 26

37. Let f be continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b). If $f(a) = f(b)$, then there is at least one number c in (a,b) such that $f'(c) = 0$.

A. Sandwich Theorem C. Rolle's Theorem E. Fundamental Theorem of Calculus

B. Intermediate Value Theorem D. Fundamental Theorem of Algebra

38. Calculate the total volume of the triangular prism shown.



A. 728 cm^3 B. 644 cm^3 C. 924 cm^3 D. 840 cm^3 E. 560 cm^3

39. Line *m* has a slope of -5 and passes through the point $(-4,6)$. Line *n* passes through the points $(1,-1)$ and $(4,8)$. Line *m* intersects line *n* at (x, y). Find $x + y$.

A. 6.5 B. -6.5 C. -5.25 D. -10.25 E. -9

40. Given that the set of natural numbers continue in the triangular pattern shown below, find the median of the numbers in row 9.

			2				(row 1)
		4	6	8			(row 2)
	10	12	14	16	18		(row 3)
20	22	24	26	28	30	32	(row 4)
			…				(…)

A. 166 B. 134 C. 154 D. 146 E. 170

41. The ratio of length to width of a rectangle is 5:3 and the area is 43.35 in^2. What is the perimeter of the rectangle?

A. 13.6 in B. 26.3 in C. 21.7 in D. 13.2 in E. 27.2 in

42. The function $f(x) = \dfrac{2x^3}{x^2 - 8}$ is increasing at which of the following values of *x*?

A. -5 B. -4 C. -1 D. 0 E. 2

43. If $\dfrac{2x-5}{5x-2} - \dfrac{5x+2}{2x-5} = \dfrac{Ax^2 + Bx + C}{Px^2 + Qx + R}$, then $\dfrac{A+B+C}{P+Q+R} =$

A. $-57\dfrac{1}{10}$ B. $1\dfrac{1}{3}$ C. $\dfrac{2}{5}$ D. $-\dfrac{8}{9}$ E. $2\dfrac{2}{9}$

44. Carmen Cents has 75 nickels, dimes and quarters. She has three times as many nickels as dimes and two more nickels than quarters. How much money does she have?

A. \$10.90 B. \$10.50 C. \$11.60 D. \$11.90 E. \$7.60

45. The triangles ABC and PQR exist such that $\angle ABC \cong \angle QRP$, $\angle ACB \cong \angle QPR$, $AB = 12$, $BC = 27$, $QP = 48$ and $RP = 36$. Find $AC + QR$.

A. 36 B. 42 C. 48 D. 52 E. 54

46. The point $(3,-11)$ lies on a circle with center $(8,-5)$. Which of the following points lies inside the circle?

A. $(15,2)$ B. $(13,3)$ C. $(16,-5)$ D. $(16,0)$ E. $(15,-8)$

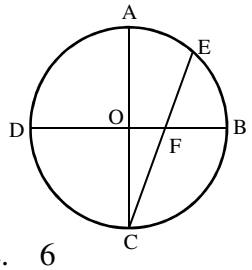
47. Find the area of the triangle with vertices $(-3,5)$, $(6,12)$ and $(4,-1)$.

A. 51.5 B. 46.5 C. 49 D. 47.5 E. 48

48. Meredith set out to row on a lake. She rowed 750 m on a bearing of 75°, then 250 m on a bearing of 25°, then 800 m on a bearing of 52°. How far is she from her original starting point? (nearest meter)

A. 1800 m B. 933 m C. 1723 m D. 1328 m E. 1524

49. Circle O has perpendicular diameters and a chord, find AE if CF =11 inches and EF = 9 inches. (nearest tenth)



A. 7 in B. 9.9 in C. 6.3 in D. 8.7 in E. 5.6 in

50. If $9^{x+y} = 6561$ and $27^{x-y} = 729$, then $x^2 - y^2 =$

A. 20 B. 8 C. 12 D. 4 E. 6

51. Cornelius has a bag that contains 6 blue chips, 5 red chips and 2 green chips. If he selects two chips without replacement, what is the probability that he will draw two of the same color?

A. $\frac{65}{132}$ B. $\frac{5}{13}$ C. $\frac{4}{13}$ D. $\frac{13}{36}$ E. $\frac{1}{3}$

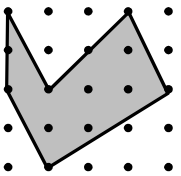
52. What is the constant term in the binomial expansion of $\left(3x^3 - \frac{1}{x}\right)^8$?

A. 5670 B. 70 C. 28 D. 6561 E. 252

53. Classify the graph of $3x^2 - y^2 + 18x + 6y = 9$

A. Parabola B. Ellipse C. Hyperbola D. Circle E. None of These

54. The dots are 3 units apart vertically and horizontally. Find the area of the shaded region.



A. 36 units^2 B. 72 units^2 C. 34 units^2 D. 64 units^2 E. 39 units^2

55. How many solutions are there for the equation $3x + 5y = 125$ where both x and y are non-negative integers?

A. 9 B. 10 C. 8 D. 7 E. 6

56. If $\frac{4x+13}{x^2+2x-3} = \frac{A}{x+3} + \frac{B}{x-1}$, then $AB =$

A. 4 B. $-\frac{17}{16}$ C. $-\frac{17}{4}$ D. $\frac{17}{4}$ E. $\frac{17}{16}$

57. The square root of 207 in base 9 is:

A. 15_9 B. 17_9 C. 16_9 D. 14_9 E. 13_9

58. If $y^2 = -5 - 12i$ and $y^3 = 46 + 9i$ where $y = a + bi$ then $a + b =$

A. 1 B. -38 C. 5 D. -62 E. 6

59. $4^3 + 5^3 + \ldots + 12^3 + 13^3 + 14^3 + 15^3$

A. 14400 B. 10989 C. 11025 D. 14364 E. 2744

60. What is the area of a regular hexagon in terms of the length of the apothem, a?

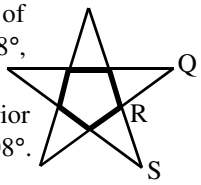
A. $2a^2\sqrt{3}$ B. $\frac{4a^2\sqrt{3}}{3}$ C. $\frac{3a^2\sqrt{3}}{4}$ D. $\frac{3a^2\sqrt{3}}{2}$ E. $3a^2\sqrt{3}$

2015-2016 TMSCA Mathematics Test Three Answers

1. E	21. B	41. E
2. A	22. C	42. A
3. E	23. E	43. B
4. D	24. B	44. B
5. B	25. B	45. D
6. D	26. C	46. E
7. E	27. D	47. A
8. B	28. A	48. C
9. B	29. A	49. C
10. E	30. D	50. B
11. E	31. C	51. E
12. A	32. D	52. E
13. A	33. E	53. C
14. D	34. B	54. B
15. E	35. A	55. A
16. D	36. C	56. B
17. B	37. C	57. D
18. A	38. D	58. A
19. A	39. E	59. D
20. C	40. D	60. A

2015-2016 TMSCA Mathematics Select Solutions

11. Each interior angle of the bold pentagon is 108°, and $\angle QRS$ is a vertical angle to one of the interior angles so $m\angle QRS = 108°$.



12. The left of the equation simplifies to $n(n-1)$, so estimate using $\sqrt{150} \approx 12.25$. $12(11) = 132$ while $13(12) = 156$, so 12 is the largest possible value of n. All the values are 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 or ten total values.

15. $x^3 + y^3 = (x+y)\left((x+y)^2 - 3xy\right)$ or $(-5)(25-3(-8)) = -245$.

21. This is the power series for the function $f(x) = e^x - 1$ when $x = 4$. $f(4) \approx 53.59815003$. The digit in the millionths place is 0.

22. There are 8 letters total with "P" repeated thrice and "E" repeated twice, so the number of distinct arrangements is: $\frac{8!}{(3!)(2!)} = 3360$.

24. $f\left(g(x+2)\right) = f(x+1) = (x+1)^4$, use coefficients from the binomial theorem or from Pascal's triangle to get $x^4 + 4x^3 + 6x^2 + 4x + 1$.

30. Re-written in base 10 the number will be $(16R + 4P + Q) + (9Q + 3R + P) - (4P + 2Q + R)$ or $P + 8Q + 18R$.

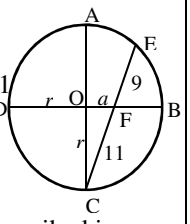
31. $(P+Q)^2 = (10-R)^2$, so $P^2 + 2PQ + Q^2 = 100 - 20R + R^2$ then using substitution, $R^2 + 2PQ = 100 - 20R + R^2$ which simplifies to $2(6) = 100 - 20R$. $R = 4.4$.

35. There are 3 types of pens and the clerk is making packages of 5, so the total number of distinct packages is ${}_{3+5-1}C_5 = 21$.

40. The median will always be the middle number in the row. Using the pattern:

2+4=6
6+8=14
14+12=26…..114+32=146.

49. To find *r*, use the two relationships $a^2 + r^2 = 121$ and $(r+a)(r-a) = 99$. Adding the two together, $2r^2 = 220$ and. Then, because triangle AEC is inscribed in a semi-circle, it has a right angle at E and $20^2 + (AE)^2 = (2r)^2$ and $AE = \sqrt{4r^2 - 400}$ or $AE = \sqrt{2(220) - 400} \approx 6.3$ inches.



52. The constant term will be ${}_8C_2\left(3x^3\right)^2\left(-\frac{1}{x}\right)^6 = 252$.

54. I = # of interior points
P = # of perimeter points
$A = \frac{2I+P}{2} - 1 = \frac{2(5)+8}{2} - 1 = 8$, but each square unit on the graph represents 9 square units because of the scale given, so the actual area is 72

57. $207_9 = 169_{10}$. The square root of 169 is 13 which is written as 14_9.

59. The sum of the first *n* cubes is given by the formula $\left(\frac{n(n+1)}{2}\right)^2$, so the sum of the series given will be $\left(\frac{15(16)}{2}\right)^2 - 1 - 8 - 27 = 14364$.

T M S C A H I G H S C H O O L

M A T H E M A T I C S

T E S T # 4 ©

N O V E M B E R 1 4, 2 0 1 5

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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2015-2016 TMSCA Mathematics Test Four

1. Evaluate: $1\div(1+4)^{-1}\times 10-\frac{5}{8}+17\times(32)^{0}$.

   A. $18\frac{3}{8}$ B. $66\frac{3}{8}$ C. $36\frac{3}{8}$ D. $20\frac{3}{8}$ E. $20\frac{5}{8}$

2. $\angle A$ and $\angle B$ are supplementary. If the $m\angle A : m\angle B$ is 7:8, find the measure of the complement of $\angle A$.

   A. 12° B. 6° C. 15° D. 96° E. 84°

3. What is the least common multiple of 204, 510 and 646?

   A. 29070 B. 58140 C. 329460 D. 9690 E. 19380

4. Leroy walked from his house to the school to pick up his bicycle at an average rate of 5 mph. He rode his bike home at an average rate of 26 mph. The total trip took 33 minutes. How far does Leroy live from the school? (nearest tenth)

   A. 2.6 mi B. 2.8 mi C. 1.6 mi D. 2.9 mi E. 2.3 mi

5. At Hobby Stop the price of a tube of oil paint is $7.85 and the price of a brush is $3.95. Crafty Carl has a 30% off coupon to use for the brushes and the paints are on sale for 15% off. If Carl buys 8 tubes of paint and 4 brushes, what will his cost be after the 8.5% sales tax has been applied?

   A. $69.92 B. $50.27 C. $52.99 D. $57.39 E. $62.27

6. Simplify $a^{-3}\times b^{3}\div a^{-5}\times b^{-5}\div a^{-3}\times b^{5}$.

   A. $a^{5}b^{3}$ B. $\frac{b^{3}}{a^{5}}$ C. $a^{3}b^{5}$ D. $\frac{b^{7}}{a^{11}}$ E. $\frac{a}{b^{7}}$

7. Two parallel lines are cut by a transversal to form two alternate interior angles with measures $(x^{2}-26)°$ and $(3x+2)°$. What is the sum of the two angle measures?

   A. 180° B. 90° C. 92° D. 46° E. 23°

8. What is the distance between the point $(3,7)$ and the line $12x+5y=27$?

   A. $\frac{98}{169}$ B. $\frac{44}{169}$ C. $7\frac{7}{13}$ D. $4\frac{7}{12}$ E. $3\frac{5}{13}$

9. The graph of $f(x)=x^{4}-12x^{3}+48x^{2}-64x$ has critical points when $x=a$ and $x=b$ where $a<b$. What is the value of *b*?

   A. 1 B. 2 C. D. 4 E. 12

10. A right triangular prism has a height of $4\sqrt{2}$ inches and the base is a 30°- 60°- 90° triangle with a hypotenuse of 8 inches. Find the volume.

    A. $16\sqrt{6}$ B. $32\sqrt{6}$ C. 32 D. $16\sqrt{5}$ E. $32\sqrt{3}$

11. Gary gets a job where he earns $300 per week plus a 10% commission on all his sales. The best model for his gross earnings as a function of sales will be

    A. Linear B. Exponential C. Quadratic D. Logistic E. Logarithmic

12. Points *P* and *Q* have coordinates $(-5,6)$ and $(7,-10)$ respectively. Which of the following is an equation of the perpendicular bisector of $\overline{PQ}$?

    A. $3x-4y=5$ B. $4x+3y=-2$ C. $3x-4y=-39$ D. $3x-4y=61$ E. $3x-4y=11$

13. Jay's current age is ¼ of his mother's age. In 10 years, Jay's age will be 2 years less than ½ his mother's age. What is Jay's current age?

A. 7 B. 9 C. 8 D. 15 E. 12

14. The number of integers between 1 and 48 that are relatively prime to 48 is:

A. 14 B. 15 C. 17 D. 18 E. 20

15. The medians of triangle *KLM* intersect at point *N*. If the length of the median from *K* to $\overline{LM}$ is 14.4 cm then $KN =$ _____cm.

A. 9.6 B. 7.2 C. 4.8 D. 10.8 E. 12.0

16. What is the sum of the series $57-19+6.\overline{3}-2.\overline{1}+...$?

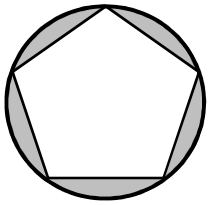
A. $\frac{494}{9}$ B. $\frac{171}{4}$ C. $\frac{172}{3}$ D. $\frac{380}{9}$ E. $\frac{171}{2}$

17. Which of the following has an amplitude of -3, period of $\frac{2}{5}$, phase shift 1 and displacement of -4?

A. $4-3\sin(5\pi x-1)$ C. $\frac{1}{5}-4\sin(3\pi x+1)$ E. $-4-3\sin(5\pi x-5\pi)$

B. $-3-4\sin(5\pi x+5\pi)$ D. $\frac{1}{5}+3\sin(4\pi x-4\pi)$

18. A dart lands randomly on the figure composed of a pentagon inscribed in circle. What are the odds it lands on the shaded region? (nearest hundredth)



A. 0.243 B. 0.321 C. 0.757 D. 0.127 E. 0.282

19. $\frac{\tan^2 t}{\sec t} =$

A. $\sec t+\cos t$ B. $\csc t+\sin t$ C. $\sec t-\cos t$ D. $\sec t-\sin t$ E. $\csc t-\sin t$

20. $\frac{4x^3-4x^2-9x+9}{3x^2-8x+5} \div \frac{2x^2+5x+3}{9x^2-30x+25} =$

A. $\frac{3x-5}{x+1}$ B. $\frac{6x^2-9x-15}{x+1}$ C. $\frac{2x-3}{x-1}$ D. $\frac{6x^2-19x+15}{x+1}$ E. $\frac{3x-5}{x-1}$

21. Find the average rate of change for $f(x)=3x^3-2x^2+5$ on the interval $[2,22]$

A. 8434 B. 4991 C. 1548 D. 3646 E. 480

22. Using Pascal's triangle as shown below, find the sum all the numbers in rows 0 through 10.

				1						(row 0)
			1		1					(row 1)
		1		2		1				(row 2)
	1		3		3		1			(row 3)
			…							…

A. 2047 B. 4095 C. 2048 D. 1023 E. 4096

23. How many distinguishable arrangements can be made from the letters in the word "PROBABILITY"?

A. 39916800 B. 19958400 C. 362880 D. 181440 E. 9979200

24. $867_9 = k_3$. Find the sum of the digits in *k*.

A. 9 B. 8 C. 7 D. 10 E. 6

25. Which of the following quadrant(s) does not contain a solution to $7x-5y\geq -49$?

A. QIV B. QI & QII C. QIII &QIV D. QIII E. None of These

26. A balloon rises at a rate of 4 meters per second from a point on the ground 30 meters from an observer. Find the rate of change of the angle of elevation of the balloon from the observer when the balloon is 30 meters above the ground.

A. $\frac{1}{30}$ rad/sec B. $\frac{1}{60}$ rad/sec C. $\frac{4}{15}$ rad/sec D. $\frac{1}{15}$ rad/sec E. $\frac{2}{15}$ rad/sec

27. Given $a_n=3a_{n-1}-2a_{n-2}$ where $a_1=1$ and $a_0=-1$ find a_4.

A. -82 B. -29 C. 82 D. 29 E. 773

28. Jill can tile a room alone in 12 hours, and her friend Kylie can tile a room alone in 9 hours. How long will it take the two of them to tile a room twice as long and twice as wide if they work together? (nearest minute)

A. 10 hr. 17 min. B. 21 hr. C. 9 hr. 6 min. D. 19 hr. 10 min E. 20 hr. 34 min.

29. If $5^x\bullet 25^{2y}=1$ and $3^{5x}\bullet 9^y=\frac{1}{9}$, then $x+y=$

A. $\frac{1}{3}$ B. $-\frac{1}{3}$ C. $-\frac{4}{9}$ D. $-\frac{5}{9}$ E. $\frac{4}{9}$

30. If $\int_3^8 f(x)\,dx=42$, then $\int_3^8\left[2+5f(x)\right]dx=$

A. 220 B. 212 C. 139 D. 218 E. 232

31. The equation $5x^2-12x+k=0$ has two positive solutions when which of the following is true?

A. $-7.2<k<7.2$ B. $k>7.2$ C. $k<7.2$ D. $-7.2<k<7.8$ E. $0<k<7.2$

32. There are 323 students in the senior class in Texas HS senior class. The ratio of boys to girls is 11:6. If the number of boys remains constant, how many new girls would have to enroll to change the ratio to 1:1?

A. 132 B. 102 C. 95 D. 143 E. 85

33. Find *C* if the remainder of $2x^3-7x^2+3x+C$ divided by $x-5$ is 16.

A. 106 B. 456 C. -74 D. -424 E. 428

34. The chord $\overline{AB}$ has a length of 40 cm and the circle has a diameter of 50 cm. How far is $\overline{AB}$ from the center of the circle?

A. 48 cm B. 30 cm C. $10\sqrt{21}$ cm D. 37 cm E. 15 cm

35. Use the function on the right. Find $f(2)+f(-4)+f(5)$.

A. 19.5 B. 1.5 C. 7.5 D. 9.5 E. 5.5

$$f(x)=\begin{cases} x-5, & x<0 \\ (2x)^2, & 0\leq x\leq 4 \\ \frac{x}{2}, & x>4 \end{cases}$$

36. Solve $\frac{3}{x}+\frac{5}{2x+y}=7$ for *y*.

A. $\frac{14x^2-11x}{3-7x}$ B. $\frac{x^2-2x}{x-5}$ C. $\frac{-14x^2+11x}{7x-1}$ D. $\frac{14x^2-11x}{7x-3}$ E. $\frac{2x^2-11x}{x-3}$

37. A circle is inscribed in a triangle. The center of the circle is the intersection of the _____ of the triangle.

A. Perpendicular Bisectors B. Altitudes C. Medians D. Angle Bisectors E. Sides

38. If y varies directly with x and $y = 340$ when $x = 200$, calculate y when $x = 117$.

A. 68.9 B. 581.1 C. 300.9 D. 198.9 E. 147.3

39. John's Ice Cream carries 12 gourmet flavors of home-made ice cream every day. If they also carry four choices of cones, how many different 2-scoop cone choices are there every day?

A. 264 B. 45 C. 312 D. 180 E. 78

40. P and Q are the roots of $f(x) = 4x^2 + 4x - 15$. Calculate $P^4 - 4P^3Q + 6P^2Q^2 - 4PQ^3 + Q^4$.

A. 625 B. 256 C. 243 D. 81 E. -64

41. The graph of the parametric equations $x = 5\cos t$ and $y = 3\sin t$ is a(n) _____.

A. Circle B. Ellipse C. Hyperbola D. Parabola E. Line

42. $4\log a^3 b - 2\log \frac{a}{b^2} + \frac{1}{2}\log ab =$

A. $\log a^9 b$ B. $\frac{1}{2}\log a^{11} b^9$ C. $\frac{1}{2}\log a^5 b$ D. $\frac{1}{2}\log a^{21} b^{17}$ E. $\log\left(a^{10}\sqrt{ab}\right)$

43. Calculate $4 - \frac{3^2}{2!} - \frac{3^3}{3!} + \frac{3^4}{4!} + \frac{3^5}{5!} - \frac{3^6}{6!} - \frac{3^7}{7!} + \ldots$ to the nearest ten-thousandth.

A. -0.8489 B. -0.8295 C. -1.0464 D. 1.0510 E. -0.1424

44. Let a, b and c be real numbers such that $c = a + b + 12$, $c^2 = a^2 + b^2$ and $ab = 12$. Find the value of $12c$.

A. 5 B. 24 C. 60 D. 7 E. 84

45. Regular hexagon ABCDEF is inscribed in a circle and all the diagonals are drawn. What is $m\angle AEC$?

A. 120° B. 45° C. 60° D. 90° E. 72°

46. Which of the following is an equation of the tangent line to $f(x) = \sin^2(2x)$ when $x = \frac{\pi}{6}$?

A. $y - \frac{1}{4} = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$ C. $y - \frac{\sqrt{3}}{2} = \frac{1}{4}\left(x - \frac{\pi}{6}\right)$ E. $y - \frac{1}{4} = \sqrt{3}\left(x - \frac{\pi}{6}\right)$

B. $y - \frac{3}{4} = \sqrt{3}\left(x - \frac{\pi}{6}\right)$ D. $y - \frac{3}{4} = \frac{1}{4}\left(x - \frac{\pi}{6}\right)$

47. Chance is going to flip a fair coin 6 times. What is the probability that he will get exactly 4 tails in a row?

A. $\frac{1}{16}$ B. $\frac{3}{64}$ C. $\frac{7}{64}$ D. $\frac{1}{8}$ E. $\frac{5}{64}$

48. Find the angle between the vectors $v_1 = \langle -8, 3\rangle$ and $v_2 = \langle 11, 9\rangle$. (nearest degree)

A. 30° B. 23° C. 150° D. 67° E. 120°

49. $\csc\theta < 0$ and $\tan\theta < 0$. Where will θ terminate?

A. QI B. QII C. QIII D. QIV E. x - axis

50. Given triangle ABC such that $m\angle A = 60°$, $AC = 26$ cm and $BC = 25$ cm there are two possible lengths for the side $\overline{AB}$. Find the sum of these two lengths.

A. 25 B. 26 C. 51 D. 17 E. 34

51. Solve $e^{2x} - 10e^x + 21 = 0$.

A. $0,\ \log 21$ B. $\log 3, \log 7$ C. $\ln 3, \ln 7$ D. $0,\ \ln 21$ E. $3,\ 7$

52. Find the value of $A+B+C$, where *A*, *B* and *C* are positive integers such that: $\frac{13}{10}=A+\left(\frac{1}{B+\frac{1}{C+1}}\right)$

A. 12 B. 7 C. 15 D. 6 E. 9

53. Let $f_0=0$, $f_1=1$, $f_2=1$, $f_3=2$, $f_4=3$, …be the terms of the Fibonacci sequence. Find $(f_9)^2-(f_8)^2$.

A. 1868 B. 715 C. 272 D. 610 E. 55

54. If $f(x)=ax^5+bx^3+cx+7$ and $f(9)=48$, then $f(-9)=$

A. 41 B. 48 C. -34 D. -48 E. 55

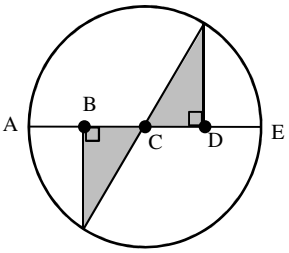
55. Quadrilateral ABCD has vertices $(-7,3)$, $(-4,6)$, $(5,5)$ and $(9,-2)$ respectively. What is the area of ABCD?

A. 77 B. 45 C. 67 D. 71 E. 61

56. Given the set of integers in ascending order $\{a,b,c,d,e\}$ has a median of 14 a mean 16.4, mode 11 and range of 16. Find the value of *d*.

A. 27 B. 21 C. 17 D. 14 E. 19

57. The radius of circle *C* is 9 in, *B* is the midpoint of $\overline{AC}$ and *D* is the midpoint of $\overline{CE}$. Find the area of the shaded region. (nearest in^2)



A. 18 in^2 B. 35 in^2 C. 20 in^2 D. 40 in^2 E. 27 in^2

58. Given $f(x)=2x^2$, $g(x)=\frac{1}{x}$ and $h(x)=\sqrt{x}$ evaluate $h^{-1}(g(f(2)))$.

A. 8 B. $\frac{1}{64}$ C. $2\sqrt{2}$ D. 64 E. $\frac{1}{2\sqrt{2}}$

59. $1331_b=$ _____$_{10}$ when $b>3$.

A. $b^2+3b+3+\frac{1}{b}$ B. $(b^2+1)(b+1)$ C. $b(b+1)(b+2)$ D. $(b+1)^3$ E. $(b^2+1)(b-1)$

60. $\sum_{k=0}^{15}k(k^2+1)=$

A. 11130 B. 14520 C. 74400 D. 1300 E. 3390

2015-2016 TMSCA Mathematics Test Four Answers

1. B	21. C	41. B
2. B	22. A	42. D
3. E	23. E	43. A
4. E	24. A	44. C
5. A	25. E	45. C
6. A	26. D	46. B
7. D	27. D	47. E
8. E	28. E	48. E
9. D	29. B	49. D
10. B	30. A	50. B
11. A	31. E	51. C
12. E	32. C	52. D
13. A	33. C	53. B
14. B	34. E	54. C
15. A	35. D	55. E
16. B	36. A	56. E
17. E	37. D	57. B
18. B	38. D	58. B
19. C	39. C	59. D
20. D	40. B	60. B

2015-2016 TMSCA Mathematics Test Four Selected Solutions

4. $\frac{d}{5}+\frac{d}{26}=\frac{33}{60}$, $d\approx 2.3$ mi

16. This is an infinite geometric series with an initial term 57 and a ratio of $-\frac{1}{3}$. The sum will be $\frac{57}{1-\left(-\frac{1}{3}\right)}=57\div\frac{4}{3}=\frac{171}{4}$

21. The average rate of change for the function will be the slope of the secant line connecting the two endpoints of the function on the given interval or $m=\frac{f(22)-f(2)}{22-2}=1548$.

22. The sum the numbers in each row of Pascal's triangle is 2 to the power of the row number. So the sum of the first 10 rows is going to be the sum of the geometric series $1+2+4+8+....+1024$ or $2^{n+1}-1=2047$.

23. There are 11 letters with "B" and "I" each repeated twice, so the number of distinct arrangements will be $\frac{11!}{(2!)(2!)}=9979200$.

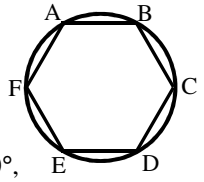
24. $867_9=8(9)^2+6(9)+7=$
$6(3^2)^2+2(3^2)^2+6(3^2)+6+1=$
$2\bullet 3(3^2)^2+2(3^2)^2+2\bullet 3(3^2)+2\bullet 3+1=$
$2\bullet 3^5+2\bullet 3^4+2\bullet 3^3+2\bullet 3+1=222021_3$, so the sum of the digits is 9.

26. Let θ be the angle of elevation and x be the height of the balloon at any particular time. The relationship between the two will be $\tan\theta=\frac{x}{30}$. Taking the derivatives of each side with respect to time $\sec^2\theta\frac{d\theta}{dt}=\frac{1}{30}\bullet\frac{dx}{dt}$. When $x=30$, $\theta=\frac{\pi}{4}$, so $2\frac{d\theta}{dt}=\frac{1}{30}\bullet 4$. $\frac{d\theta}{dt}=\frac{1}{15}$ rad/sec.

39. . If there are 12 types and 2 scoops, the number of distinct ice cream orders will be ${}_{12+2-1}C_2=78$, but that is just the ice cream. With the cone choices, there are $78\bullet 2=156$ order choices.

43. This is the power series for $f(x)=\sin x+\cos x$ when $x=3$.
$\sin 3+\cos 3\approx -0.8489$.

45. The arcs marked by consecutive letters are each 60°. The inscribed angle AEC intercepts an arc with measure 120°, so the measure of the angle is 60°.



48. Using the two definitions of scalar product
$\cos\theta=\frac{(-8)(11)+(3)(9)}{\sqrt{(64+9)(121+81)}}\approx 120^\circ$.

50. Set up a quadratic using the law of cosines: $625=676+x^2-52x\cos 60$ where $x=AB$. Arranged so that there is a 0 on one side, $0=x^2-26x+51$. The roots of this equation are the two possible lengths of AB so the sum of the lengths is 26.

51. $e^{2x}-10e^x+21=(e^x-7)(e^x-3)=0$, so $e^x=7$ or $e^x=3$ then take the natural log of both sides of each equation for $x=\ln 7$ or $x=\ln 3$.

52. $\frac{13}{10}=1+\frac{3}{10}=1+\frac{1}{\frac{10}{3}}=1+\frac{1}{3+\frac{1}{2+1}}$
so $A+B+C=6$.

54. If $f(9)=48=41+7$, then $f(-9)=-41+7=-34$.

57. Each of the small, shaded triangles is a 30°-60°-90° triangle with a hypotenuse length of 9 inches. The shaded region will have an area $A=2\bullet\frac{1}{2}(4.5)(4.5\sqrt{3})$ which is approximately 35 in^2.

59. $1331_b=1b^3+3b^2+3b+1$ which factors to $(b+1)^3$.

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T E S T # 5 ©

N O V E M B E R 2 1, 2 0 1 5

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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2015-2016 TMSCA Mathematics Test Five

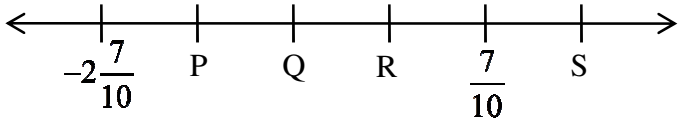
1. Evaluate: $6!+24(3-10)\div 40\times 15$.

   A. 719 B. 656 C. 657 D. 720 E. 678

2. Caroline had a rope that was 30 feet long. She cut off three pieces such that the ratio of lengths of the pieces were 2:5:10 with 20 inches of string left over. How long was the longest piece?

   A. 3 ft. 8 in. B. 16 ft. 8 in. C. 23 ft. D. 17 ft. 8 in. E. 21 ft. 4 in.

3. The distances between the hash marks (|) are equal. Find $P+Q+R+S$.



   A. -1.45 B. -3.32 C. -4 D. -2.3 E. -3.15

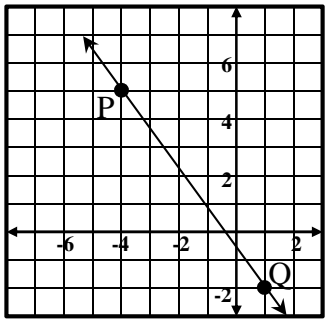
4. What is the harmonic mean of the first three perfect numbers? (nearest tenth)

   A. 43.7 B. 177 C. 30.7 D. 496 E. 14.7

5. Evaluate: $\dfrac{(x+1)!}{(x-2)!}\div\dfrac{x!}{(x-1)!}$.

   A. $x^5+2x^4-x^3-2x^2$ B. x^3+2x^2-x-2 C. x^4-5x^2+4 D. x^6+4x^2 E. x^2-1

6. Which of the following is the standard form of the equation of the perpendicular of $\overline{PQ}$ that passes through the point P?



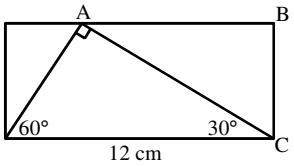
   A. $5x+7y=3$ C. $7x-5y=-10$ E. $7x+5y=15$

   B. $5x-7y=55$ D. $5x-7y=-18$

7. The point of intersection of the three angle bisectors of a triangle is called the_____.

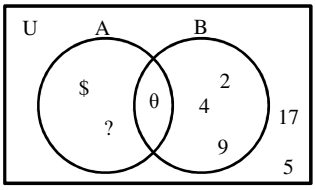
   A. Center B. Circumcenter C. Centroid D. Incenter E. Orthocenter

8. Find the area of the triangle ABC shown.



   A. 18 cm^2 B. $\dfrac{27\sqrt{3}}{2}$ cm^2 C. 12 cm^2 D. $24\sqrt{3}$ cm^2 E. $\dfrac{15\sqrt{3}}{2}$ cm^2

9. Use the Venn diagram to determine the set $A'\cap B$.



   A. $\{2,4,9\}$ C. $\{2,4,5,9,17\}$ E. $\{\$,?,\theta,2,4,9,\}$

   B. $\{\theta\}$ D. $\{\theta,2,4,5,9,17\}$

10. Karolyn invested $1000 for 4 years in a variable interest account. Her annual interest rates are shown in the table below. What was the average interest for the 4 years? (nearest hundredth of a percent)

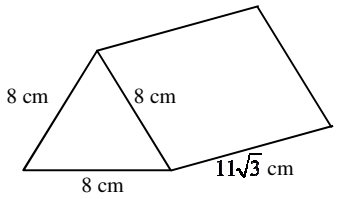
Year	1	2	3	4
Interest	4% gain	2.4% loss	3% loss	2.7% gain

   A. 0.33% B. 0.30% C. 2.97% D. 0.28% E. 3.02%

11. The student council consists of 8 girls and 5 boys. How many different 4-person delegations can the sponsor choose for the state convention if any delegation must have at least two girls?

A. 280 B. 630 C. 560 D. 95 E. 72

12. The volume of the triangular prism shown is _____cm^3.

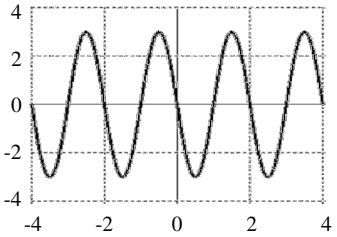


A. $352\sqrt{3}$ B. 704 C. 528 D. $348\sqrt{3}$ E. $328\sqrt{3}$

13. If $x-y=8$, and $xy=-15$, then $x^3-y^3=$

A. -360 B. -320 C. 392 D. 192 E. 152

14. The function $f(x)=$_____ will produce this graph.



A. $3\sin(\pi x+1)$ C. $3\sin(\pi(x-1))$ E. $\sin(\pi(x-1))+3$

B. $3\sin(\pi x-1)$ D. $3\sin(x+\pi)$

15. $A=\begin{bmatrix}3 & 5\\ 1 & -2\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1\\ 0 & 4\end{bmatrix}$. Calculate $\det(2A+B)$.

A. -18 B. -44 C. -11 D. -13 E. -8

16. What are the odds that a factor of 168 is a multiple of 4?

A. 1 to 2 B. 1 to 3 C. 1 to 1 D. 2 to 3 E. 5 to 12

17. $(3+2i)^5=$

A. $-597-122i$ B. $243-32i$ C. $-243-32i$ D. $-597+122i$ E. $-275+211i$

18. If *f* is continuous on the closed interval $[a,b]$ and *k* is any number between $f(a)$ and $f(b)$, then there is at least one number *c* in $[a,b]$ such that $f(c)=k$. This is the _____ .

A. Sandwich Theorem C. Rolle's Theorem E. Fundamental Theorem of Calculus

B. Intermediate Value Theorem D. Fundamental Theorem of Algebra

19. *P* and *Q* are the roots of $f(x)=2x^2+x-15$. Calculate $P^4-4P^3Q+6P^2Q^2-4PQ^3+Q^4$.

A. $\frac{1}{16}$ B. $\frac{28561}{625}$ C. $\frac{83521}{625}$ D. $\frac{5661}{16}$ E. $\frac{14641}{16}$

20. How many 3-digit numbers exist such that the sum of their digits equals 4?

A. 8 B. 9 C. 7 D. 10 E. 11

21. What is the digit in the millionths place for the sum of the sequence $1+3-\frac{3^2}{2!}-\frac{3^3}{3!}+\frac{3^4}{4!}+\frac{3^5}{5!}-\frac{3^6}{6!}-\frac{3^7}{7!}+\ldots$?

A. 2 B. 5 C. 7 D. 8 E. 3

22. How many distinct arrangements can be formed using all of the letters in the word "HOMECOMING"?

A. 40320 B. 907200 C. 151200 D. 20160 E. 3628800

23. A triangle with side lengths 12 cm, 12cm and 20 cm is a(n)_____triangle.

A. scalene acute B. isosceles obtuse C. isosceles acute D. scalene obtuse E. scalene right

24. If $\int_0^a \cos x dx = C$ then $\int_{-a}^a 4\cos x dx =$

A. 0 B. $2C$ C. $4C$ D. $8C$ E. $16C$

25. Given $a_0 = -1$, $a_1 = 2$ and $a_n = 2a_{n-1} + 3a_{n-2}$ find a_5.

A. 62 B. 181 C. 59 D. 122 E. 184

26. The Real value solution set of $|3x+1|+3<11$ is

A. $\left\{x\left|-\frac{7}{3}<x<3\right.\right\}$ B. $\left\{x\left|-3<x<-\frac{7}{3}\right.\right\}$ C. $\left\{x\left|\{x<-3\}\cup\left\{x>\frac{7}{3}\right\}\right.\right\}$ D. $\left\{x\left|-3<x<\frac{7}{3}\right.\right\}$ E. $\left\{x\left|\left\{x<-\frac{7}{3}\right\}\cup\{x>3\}\right.\right\}$

27. A triangle is inscribed in a circle. The center of the circle is the intersection of the _____ of the triangle.

A. Altitudes B. Medians C. Perpendicular Bisectors D. Angle Bisectors E. Sides

28. If $g(x) = x-1$ and $f(x) = x^3 - 1$, find $f(g(x+2))$.

A. x^3+3x^2+3x+2 C. x^3+x^2+x E. x^3+3x^2+3x+1

B. x^3+3x^2+3x D. x^3+3x^2+3x-1

29. The Beanery would like to market a mix of two of its most popular coffees. The Good Morning coffee sells for \$6 per pound, while the Dark Rich coffee sells for \$8 per pound. How much Dark Rich coffee should be used to create one pound of a mix that sells for \$6.50 per pound?

A. 2.5 oz. B. 12 oz. C. 8 oz. D. 4 oz. E. 9.5 oz.

30. Solve $\frac{3}{y}+\frac{5}{x+y}=4$ for x.

A. $\frac{8-4y}{3}$ B. $2-2y$ C. $\frac{2y^2-8y}{3-4y}$ D. $\frac{8y-4}{3}$ E. $\frac{4y^2-8y}{3-4y}$

31. In how many ways can a class of 18 students be split into three groups of 2, 6 and 10 students?

A. 2840292 B. 812323512 C. 6694974 D. 1225224 E. 15148224

32. The chord $\overline{AB}$ has a length of 14 cm and the circle has a diameter of 50 cm. How far is $\overline{AB}$ from the center of the circle?

A. 24 cm B. 48 cm C. 21 cm D. 45 cm E. 37 cm

33. A particle is moving along a straight line with a function $f(t) = 6t^3 - 5t - 11$, where $f(t)$ is the distance in in meters per second. Find the instantaneous rate of change at a time of 2 seconds.

A. 77 m/s B. 19 m/s C. 67 m/s D. 3 m/s E. 31 m/s

34. If $\log 8 = P$, and $\log 5 = Q$, then $\log 6.4 =$

A. $\frac{5P}{3Q}$ B. $\frac{5}{3}P+Q$ C. $\frac{5}{3}P-Q$ D. $\frac{3P}{5Q}$ E. $\frac{3P-Q}{5}$

35. If P, Q and R represent digits in $RPQ_5 + QRP_4 - PQR_3$ has a numeric value in base 10 of:

A. $-3P+20Q+30R$ C. $-3P+14Q+28R$ E. $-3P-12Q+30R$

B. $-3P+20Q+28R$ D. $-3P+14Q+30R$

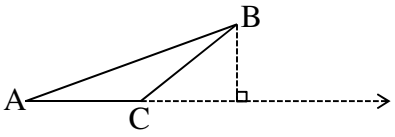
36. If P, Q and R are real numbers such that $P+Q+R=18$, $R^2 = P^2 + Q^2$ and $PQ = 6$, find the value of R.

A. $6\frac{1}{3}$ B. $8\frac{2}{3}$ C. $8\frac{1}{3}$ D. $9\frac{1}{3}$ E. $9\frac{2}{3}$

37. Express the complex number $2\left(\cos\frac{7\pi}{6}+i\sin\frac{7\pi}{6}\right)$ in rectangular form.

A. $-\frac{\sqrt{3}}{2}+\frac{i}{2}$ B. $-\sqrt{3}-i$ C. $-\sqrt{2}+i\sqrt{2}$ D. $-\sqrt{3}+i$ E. $-\sqrt{2}-i\sqrt{2}$

38. On triangle ABC shown, $m\angle BAC=\frac{\pi}{6}$ radians, $AB=12$ in. and $AC=9$ in. Find the area of triangle ABC.



A. $27\sqrt{3}$ in^2 B. 36 in^2 C. 54 in^2 D. $36\sqrt{3}$ in^2 E. 27 in^2

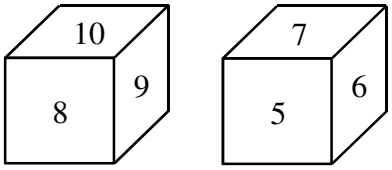
39. If $f(x)=ax^5+bx^3+cx+8$ and $f(7)=48$, then $f(-7)=$

A. -32 B. 41 C. 40 D. -31 E. 56

40. The number of calories in a particular candy varies directly with the mass. If a 10 g serving has 25 calories, how many calories will be in a 0.5 kg bag?

A. 1050 B. 200 C. 1250 D. 250 E. 1500

41. Chauncy created a pair of special dice which have only three numbers on each die. The side opposite each number is the same as the number. When the dice shown are rolled and the top numbers are added, what is the expected value of the sum?

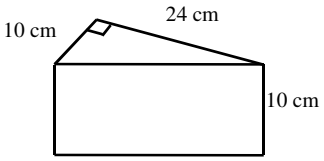


A. 14.5 B. 15 C. $15.\overline{3}$ D. $15.\overline{6}$ E. $16.\overline{3}$

42. $(2x+7)(5x-1)=2x(5x-1)+7(5x-1)$ is an example of _____ property.

A. Transitive B. Associative C. Commutative D. Distributive E. Substitution

43. Calculate the total surface area of the triangular prism shown.



A. 1200 cm^2 B. 644 cm^2 C. 924 cm^2 D. 600 cm^2 E. 840 cm^2

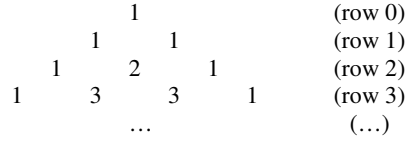
44. $\frac{\cot^2 t}{\csc t}=$

A. $\sec t+\cos t$ B. $\sec t-\cos t$ C. $\csc t+\sin t$ D. $\sec t-\sin t$ E. $\csc t-\sin t$

45. $f''(x)=2$, $f'(2)=5$ and $f(2)=10$. Evaluate $f(7)$.

A. 52 B. 62 C. 54 D. 60 E. 56

46. Given the triangular pattern shown below, find the third number in row 15.



A. 105 C. 136 E. 162

B. 171 D. 120

47. The ratio of length to width of a rectangle is 7:3 and the area is 75.81 in^2. What is the perimeter of the rectangle?

A. 38 in B. 55 in C. 19 in D. 40 in E. 302 in

48. If $\frac{2x-7}{3x-2}-\frac{3x+2}{2x-7}=\frac{Ax^2+Bx+C}{Px^2+Qx+R}$, then $\frac{A+B+C}{P+Q+R}=$

A. $-2\frac{2}{5}$ B. $1\frac{1}{3}$ C. $\frac{2}{5}$ D. -4 E. $-4\frac{4}{5}$

49. Adam is four years younger than Bill. Three years ago, Charles was twice as old as Adam. Charles is five years older than Bill. What is the sum of their current ages?

A. 40 B. 49 C. 38 D. 43 E. 51

50. Calculate the slope of the normal line to $2x^3+3x^2y-y^2=-2$ at the point $(-1,3)$.

A. -4 B. -3 C. $\frac{1}{4}$ D. $\frac{1}{3}$ E. 3

51. The Bears and the Lions are going to play a 3-game tournament. Based on their records, the probability that the Bears will win a particular game is 0.5, while the probability that the Lions will win a particular game is 0.3. What is the probability that the tournament will end in a tie?

A. 0.908 B. 0.008 C. 0.188 D. 0.180 E. 0.900

52. The afternoon train from Seattle, WA to Salem, OR is scheduled to take 5 hours and 12 minutes travelling at an average speed of 43 mph. Due to tracks warping in hot weather, the train must slow to an average of 25 mph for 3 hours of the scheduled trip on a summer day. How late will the arrival be in Salem? (nearest minute)

A. 1 h 46 min B. 1 h 15 min C. 2 h 12 min D. 2 h 23 min E. 2 h 32 min

53. $\int_{-3}^{7} f(x)\,dx=9$. Calculate $\int_{-3}^{7}(6+5f(x)\,dx)$.

A. 105 B. 51 C. 87 D. 110 E. 150

54. Let $f(x)=\frac{2x^2-4x+5}{x-3}$, and $s(x)$ be the slant asymptote of f. Find the value of $s(5)$.

A. 10 B. 0 C. 12 D. 8 E. 15

55. Find the area of the triangle with vertices $(1,2)$, $(6,12)$ and $(4,-1)$.

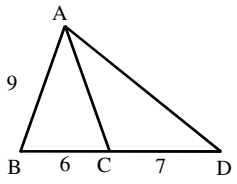
A. 36.5 B. 46.5 C. 15.5 D. 42.5 E. 22.5

56. Meredith set out to row on a lake. She rowed 750 m on a bearing of 75°, then 280 m on a bearing of 338°, then 800 m on a bearing of 210°. How far is she from her original starting point? (nearest meter)

A. 196 m B. 325 m C. 185 m D. 61 m E. 239 m

57. Given that $\overline{AB}\cong\overline{AC}$, find the area of triangle *ABD*.

A. $\frac{117}{2}$ B. $\frac{39\sqrt{5}}{2}$ C. $39\sqrt{2}$ D. $78\sqrt{2}$ E. $\frac{78\sqrt{5}}{2}$



58. Abigail, Bonny and Carla can plant a flower bed for Ms. Daisy in 3, 4 and 8 hours respectively. How long would it take them to plant Ms. Daisy's flower bed working together? (nearest hundredth)

A. 1.41 hr. B. 1.67 hr. C. 1.53 hr. D. 1.61 hr. E. 2.13 hr.

59. What is the constant term in the binomial expansion of $\left(3x^5-\frac{1}{x}\right)^{12}$?

A. 66 B. 792 C. 28 D. 8052 E. 594

60. If $\frac{6x+51}{x^2+3x-10}=\frac{A}{x-2}+\frac{B}{x+5}$, then $AB=$

A. 18 B. -27 C. 12 D. 15 E. -36

2015-2016 TMSCA Mathematics Test Five Answers

1. C	21. A	41. B
2. B	22. B	42. D
3. A	23. B	43. E
4. E	24. D	44. E
5. E	25. A	45. D
6. B	26. D	46. A
7. D	27. C	47. A
8. B	28. B	48. D
9. A	29. D	49. B
10. D	30. E	50. C
11. B	31. D	51. C
12. C	32. A	52. B
13. E	33. C	53. A
14. C	34. C	54. C
15. A	35. C	55. E
16. C	36. B	56. B
17. D	37. B	57. C
18. B	38. E	58. A
19. E	39. A	59. E
20. D	40. C	60. B

2015-2016 TMSCA Mathematics Test Five Select Solutions

10. The average interest will be $\sqrt[4]{(1.04)(0.976)(0.97)(1.027)} \approx 1.0028$ or a 0.28% gain.

11. The number of delegations with at least 2 girls is $({}_8C_2)({}_5C_2)+({}_8C_3)({}_5C_1)+({}_8C_4)({}_5C_0)$ which equals 630.

13. $x^3-y^3=(x-y)\left((x-y)^2+3xy\right)=$ $8\left(64+3(-15)\right)=152$.

20. Listed in order of their first digit:
103, 130, 121, 112,
202, 220, 211
301, 310
400

21. This is the power series for the function $f(x)=\sin x+\cos x$ when $x=3$
$\sin 3+\cos 3 \approx -0.8488724885$. The digit in the millionths place is a 2.

22. There are 10 total letters. "O" and "M" are each appear twice, so the total number of distinct arrangements is $\frac{10!}{(2!)(2!)}=907200$.

31. $({}_{18}C_2)({}_{16}C_6)({}_{10}C_{10})=1225224$.

35. $(25R+5P+Q)+(16Q+4R+P)-(9P+3Q+R)$ equals $-3P+14Q+28R$.

39. $f(7)=40+8=48$,
$f(-7)=-40+8=-32$.

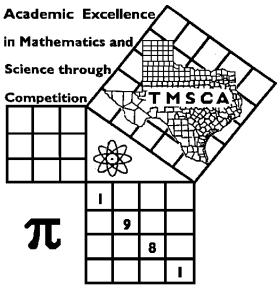
41. $\begin{bmatrix}13 & 14 & 15\\ 14 & 15 & 16\\ 15 & 16 & 17\end{bmatrix}$ This array shows the possible sums with just the faces that are showing. These sums would just be repeated 4 times in a 6x6 array if we wrote out all of the sums, but the mean would be the same, 15.

46. The triangular pattern shown is Pascal's triangle, and the third number on any row beginning with row 2 is always the $(n-1)^{th}$ triangular number. For row 15, $\frac{14(15)}{2}=105$.

50. $6x^2+3\left(x^2\frac{dy}{dx}+y\cdot 2x\right)-2y\frac{dy}{dx}=0$
at $(-1,3)$, $6+3\left(\frac{dy}{dx}-6\right)-6\frac{dy}{dx}=0$.
Solve for $\frac{dy}{dx}=-4$, so the slope of the normal line will be $-\frac{1}{4}$.

51. The tournament could end in a tie if the results are TTT or all 6 arrangements of TLB, so the probability will be $0.2^3+6(0.2)(0.3)(0.5)=0.188$.

59. ${}_{12}C_2\left(3x^5\right)^2\left(-\frac{1}{x}\right)^{10}=594$



TMSCA HIGH SCHOOL MATHEMATICS TEST# 6 © DECEMBER 5, 2015

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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1. Evaluate: $1 - 2 \times 5 + 20 \div (1 - 5) \times 1 - 6$

(A) 0.25 **(B)** 0 **(C)** -9.75 **(D)** -16 **(E)** -20

2. Les Cash had $75 to spend. He spent 40% on clothes. Then he spent 60% of what he had left on shoes. He paid $9.50 for lunch. How much did he have left to spend at the arcade?

(A) $17.50 **(B)** $15.70 **(C)** $14.25 **(D)** $9.50 **(E)** $8.50

3. If F = {f,r,u,g,a,l}, L = {l,u,c,k,y}, and P = {p,o,l,i,t,e}, then $(F \cup L) \cap (L \cup P)$ contains how many distinct elements?

(A) 6 **(B)** 5 **(C)** 4 **(D)** 2 **(E)** 1

4. $(12 + 5) + 15 = (5 + 12) + 15$ and $12 \times (5 \times 16) = 12 \times (16 \times 5)$ are examples of the _____ properties of equality.

(A) associative **(B)** commutative **(C)** distributive **(D)** addition **(E)** subtraction

5. The order pairs $(-3, 5)$, $(3, -7)$, $(7, -15)$, $(9, -19)$ represent which of the following:

(A) a relation only **(B)** a function only **(C)** both a relation and a function

(D) neither a relation nor a function **(E)** a function that is not 1-1 function

6. Simplify: $\left(\frac{x^2 + 2x - 3}{x^2 - 4}\right)\left(\frac{3x - 6}{x^2 - 3x + 2}\right)$

(A) $\frac{3x + 9}{x^2 - 4}$ **(B)** $x + 3$ **(C)** $\frac{x + 3}{3x^2 - 12}$ **(D)** $3x^2 - 12$ **(E)** $\frac{3x + 9}{x + 2}$

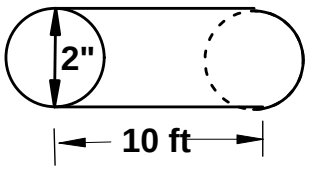
7. Find the greatest common divisor of $3^2 \times 5^3$, $2^3 \times 3 \times 5^2$, and $2^5 \times 5$.

(A) 5 **(B)** 30 **(C)** 15 **(D)** 6 **(E)** 120

8. Line $4x - ky = 5$ is perpendicular to line $2x - 3y = 6$. What is the value of k?

(A) $-\frac{2}{3}$ **(B)** $-1\frac{1}{2}$ **(C)** $-1\frac{2}{3}$ **(D)** $-2\frac{2}{3}$ **(E)** $-3\frac{1}{3}$

9. The diameter and length of the joint of PVC pipe is shown. Find the lateral surface area of the joint of pipe. (nearest in^2)



(A) 377 in^2 **(B)** 480 in^2 **(C)** 503 in^2 **(D)** 628 in^2 **(E)** 754 in^2

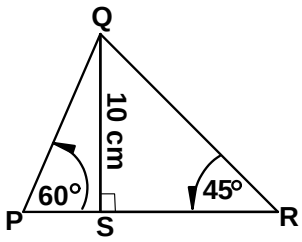
10. Perry Meeter has 200 feet of fencing. He wants to fence in his rectangular garden. What will the area of the garden be if the width is 45 feet and he uses all of the fencing?

(A) 3,025 ft^2 (B) 2,025 ft^2 (C) 3,262.5 ft^2 (D) 2,475 ft^2 (E) 3,487.5 ft^2

11. Let T_n be the nth triangular number, S_n be the nth square number, and P_n be the nth pentagonal number. Then $P_2 + S_2$ has the same value as:

(A) S_3 (B) T_2 (C) P_3 (D) T_4 (E) T_3

12. Find the area of $\triangle PQR$. (nearest tenth).



(A) 64.1 cm^2 (B) 75.0 cm^2 (C) 78.9 cm^2 (D) 95.7 cm^2 (E) 99.6 cm^2

13. The sum of the interior angles of a simple convex hexagon is:

(A) 720° (B) 800° (C) 1080° (D) 1,440° (E) 2,160°

14. Find the odds of randomly selecting a vowel from a box containing the letters from the name POCAHONTAS?

(A) 2:3 (B) 1:2 (C) 2:5 (D) 1:5 (E) 1:3

15. Let $3x + 2y = 1$ and $x - y = -2$. Find $x + y$.

(A) 2 (B) 1.2 (C) -0.6 (D) -1 (E) 0.8

16. If $\frac{5}{(x+2)(x-2)} - \frac{2}{(x-3)(x-2)} = \frac{k}{(x+2)(x-2)(x-3)}$, then k equals:

(A) $3x + 16$ (B) $7x - 11$ (C) $2x + 4$ (D) $3x - 19$ (E) $5x - 15$

17. The formulas that relate the coefficients of a polynomial to the sums and products of its roots is named after which of the following mathematicians?

(A) Sophie Germain (B) Marin Mersenne (C) Zeno of Alea

(D) Franciscus Vieta (E) Freda Porter

18. If $a_1 = 1$ and $a_n = 2(a_{n-1}) - 3$, then a_5 equals:

(A) -55 (B) -29 (C) -26 (D) -21 (E) -15

19. Given the sequence 0, 3, 8, 15, 24, k, 48, 63, ... find k.

(A) 26 (B) 32 (C) 35 (D) 40 (E) 43

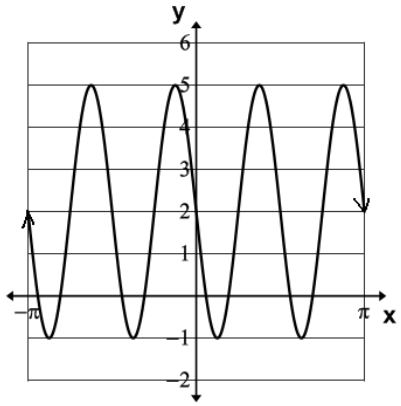
20. Mr. White's height is 6' 1". How long is Mr. White's shadow when the angle of elevation of the sun is 40 ° 15'? (nearest inch)

(A) 6' 0" (B) 7' 2" (C) 7' 6" (D) 8' 6" (E) 9' 0"

21. Determine the range of $f(x) = 2 - 3\cos(5x + 7)$.

(A) $[-1, 1]$ (B) $[-3, 2]$ (C) $[-3, 3]$ (D) $[-7, 5]$ (E) $[-1, 5]$

22. The equation $y =$ _____ will produce this graph.



(A) $2\sin(4x - \pi) + 3$ (B) $3\cos(2x - 2\pi) + 2$ (C) $3\cos(2x + \pi) + 4$

(D) $3\sin(4x + \pi) - 2$ (E) $3\sin(4x - \pi) + 2$

23. In the expansion of $(2x - 3y)^4$, the sum of the coefficients of the 2nd and the 3rd term is:

(A) 120 (B) 48 (C) 312 (D) 625 (E) 60

24. Use the Fibonacci characteristic sequence ..., p, — 2, 3, q, r, ... to Find $p + q + r$.

(A) 11 (B) 10 (C) 8 (D) 5 (E) 1

25. $f(x) = 2x^3 + 3x^2 + kx + 5$ is divided by $x - 1$ the remainder is 8. Find the value of k.

(A) 10 (B) 4 (C) 0 (D) — 1 (E) — 2

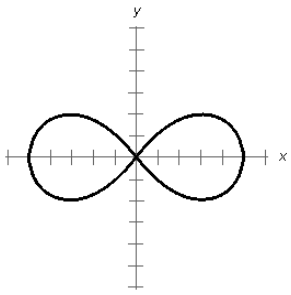
26. Let $f(x) = (2x + 1)^3$ Find $f'(-4)$.

(A) 486 (B) 384 (C) 343 (D) 294 (E) 147

27. What is the probability that a factor of 216 is a multiple of 3?

(A) $33\frac{1}{3}\%$ (B) 50% (C) $66\frac{2}{3}\%$ (D) 70% (E) 75%

28. The polar graph shown using rectangular coordinates is called a(n) _____.



(A) looped limacon (B) cardioid (C) eight rose (D) double circle (E) lemniscate

29. $\left[(1+2+3+4+...+13+14)\times 15\right] \div \left[(16+17+18+19+...+28+29)\times 30\right]$ **= ?**

(A) $\frac{1}{225}$ **(B)** $\frac{1}{900}$ **(C)** $\frac{1}{45}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{2}$

30. Coach Learner has 26 students in his math class class. Fourteen students play sports. Five are in the band. Ten compete in UIL academics. One is in band and UIL academics. Three are in sports and band. Four are in sports and academics. If no one does all three, how many students don't do any of the three things?

(A) 1 (B) 2 (C) 3 (D) 5 (E) 7

31. Let p and q be the roots of $x^2 - 5x + 6 = 0$ **. Find** $p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4$.

(A) 1 (B) 16 (C) 81 (D) 256 (E) 625

32. $\overleftrightarrow{AB}$ **intersects** $\overleftrightarrow{CD}$ **at point E such that m∠AEC = m∠BED. ∠AEC and ∠BED are considered to be _____ angles.**

(A) complementary (B) vertical (C) alternate interior (D) supplementary (E) right

33. Find the sum of the coefficients of the quotient: $(x^4 - 2x^3 - 3x + 6) \div (x - 2)$

(A) -2 **(B)** -1 **(C) 0 (D) 1 (E) 2**

34. Simplify: $(\sec^2\theta - 1)(1 - \sin^2\theta)$.

(A) $-\sec^2\theta$ **(B)** $-\csc^2\theta$ **(C) 1 (D)** $\sin^2\theta$ **(E)** $\cos^2\theta$

35. Which of the following statements about $f(x) = \frac{1}{2}x^3 - 2$ **is/are true?**

I. $f(x)$ **is a function.**
II. $f(x)$ **is a one to one function on its domain.**
III. $f(x)$ **has an inverse function on its domain.**

(A) I & II (B) I only (C) I & III (D) all of them (E) none of them

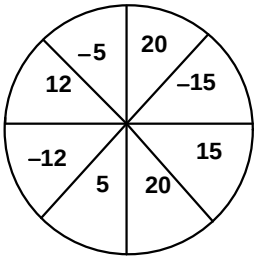
36. How many positive 3-digit numbers exist such that the sum of their digits equals 9?

(A) 27 **(B) 54** **(C) 36** **(D) 63** **(E) 45**

37. Let $f(x) = (x + 3)^2 - 7$ and $g(x) = \sqrt{x + 7} - 3$. Find $(f \circ g)(x + 1)$.

(A) x **(B) $x - 3$** **(C) $x + 1$** **(D) $x + 4$** **(E) $x - 1$**

38. Betty Wensom spins the wheel. The wheel consists of eight congruent sectors as shown. What is the mathematical expectation on any one spin?



(A) 2.5 **(B) 5** **(C) 8.5** **(D) 20** **(E) 400**

39. If the pattern of the sequence 1, 5, 11, 19, 29, 41, ... continues, find the 18th term.

(A) 343 **(B) 341** **(C) 324** **(D) 307** **(E) 305**

40. *U Rent All* rents trucks for \$8.00 an hour plus 30¢ a mile. *I Rent Some* rents the same type of truck for \$7.00 an hour plus 40¢ a mile. A truck needs to be rented for 5 hours that will travel 100 miles. How much would be saved by renting from *U Rent All* instead of *I Rent Some*?

(A) \$1.00 **(B) \$4.00** **(C) \$5.00** **(D) \$7.00** **(E) \$9.00**

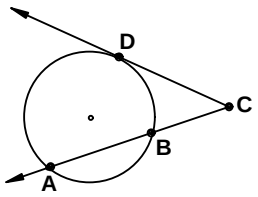
41. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. If $f_{15} = 610$, Find f_{16}.

(A) 976 **(B) 981** **(C) 987** **(D) 993** **(E) 998**

42. Tu Oad is 3 years older than Soh Yung. The sum of three times Tu's age and twice Soh's age is equal to Tan Gram's age. If Tan is 44 years old, what is the sum of Soh's age and Tu's age?

(A) 10 **(B) 13** **(C) 17** **(D) 19** **(E) 22**

43. If CD = 8 cm and CB = 5 cm then AC = ?



(A) $12\frac{4}{5}$ cm **(B) $11\frac{1}{8}$ cm** **(C) 13 cm** **(D) $14\frac{3}{5}$ cm** **(E) $8\frac{1}{8}$ cm**

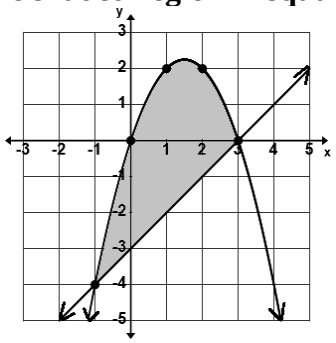
44. Saul T. Water bought some neon tetras and some fantail guppies for his aquarium. Tetras cost $2.10 each and guppies cost $1.80 each. He paid $28.50 for 14 fish, not including tax. How many tetras did he buy?

(A) 13 (B) 11 (C) 10 (D) 7 (E) 3

45. There are 3 judges, 5 policemen, and 9 office workers in the Painted Rock Courthouse. A county planning committee containing 1 judge, 3 policemen, and 4 office workers is to be created. How many different committees can be created?

(A) 68,040 (B) 24,310 (C) 3,780 (D) 473 (E) 136

46. Find the area of the shaded region in square units.



(A) $13\frac{1}{3}$ (B) $10\frac{2}{3}$ (C) 10 (D) $9\frac{2}{3}$ (E) $7\frac{1}{3}$

47. Find the determinant of the 3 x 3 matrix $\begin{bmatrix} 1 & 0 & -3 \\ -2 & 3 & 0 \\ 0 & -1 & 2 \end{bmatrix}$ **.**

(A) 12 (B) 6 (C) 0 (D) — 3 (E) — 6

48. Using the following array, determine the value of the mean of row 8.

2					(row 1)
4	6				(row 2)
8	10	12			(row 3)
14	16	18	20		(row 4)
22	24	26	28	30	(row 5)
	...				(...)

(A) 65 (B) 75 (C) 63 (D) 77 (E) 81

49. $567_8 - 432_8 =$ _____ $_2$

(A) 1101101 (B) 1010101 (C) 1011101 (D) 1110101 (E) 1011011

50. Points (— 4 , 2), (3 , — 5), and (x , y) are collinear. Which of the following could be (x , y).

(A) (— 3 , — 4) (B) (— 5 , 1) (C) (1 , 1) (D) (1, — 3) (E) (— 2, — 3)

51. (2, 3) and (3, 1) are opposite vertices of a parallelogram. If (0, 0) is the third vertex, then the fourth vertex is:

(A) (1, — 1) **(B) (— 1, 2)** **(C) (5, 4)** **(D) (2.5, 2)** **(E) (6, 3)**

52. Solve: $\log_2(8x) - \log_2(x^2 - 1) = \log_2(3)$

(A) 3 **(B) 8** **(C) 2** **(D) $\frac{1}{3}$** **(E) $\frac{1}{8}$**

53. $\triangle XYZ$ exists such that XZ = 20 cm, XY = 12 cm and $m\angle XYZ = 45°$. Find YZ. (nearest tenth)

(A) 16.7 cm **(B) 18.4 cm** **(C) 23.6 cm** **(D) 25.5 cm** **(E) 26.6 cm**

54. If $[(2 + 3i)(4 - 5i)] \div (i) = a + bi$, then $a + b = ?$

(A) — 25 **(B) — 21** **(C) — 1** **(D) 5** **(E) 9**

55. If $f'(x) = 12x^2 - 6x + 2$ and f(1) = 2, find f(0).

(A) — 1 **(B) 0** **(C) 1** **(D) 2** **(E) 8**

56. Rose Thorn creates flower baskets consisting of five flowers in each basket. How many different baskets can Rose make if she has roses, irises, carnations, lilies, geraniums, and snapdragons?

(A) 2,772 **(B) 720** **(C) 462** **(D) 252** **(E) 30**

57. A number M is randomly chosen from the set {3, 6, 8, 11, 13}. A number N is randomly chosen from the set {15, 16, 19, 23}. What is the probability that the product of M and N is an odd number?

(A) 30% **(B) 40%** **(C) 45%** **(D) 50%** **(E) 55%**

58. The length of the sides of $\triangle PQR$ are the roots of $f(x) = x^3 - 11x^2 + 38x - 40$. The perimeter of $\triangle PQR$ is 11. Find the area in square units of $\triangle PQR$. (nearest tenth)

(A) 54.2 **(B) 46.8** **(C) 45.1** **(D) 40.0** **(E) 32.2**

59. Which of the following statements about $f(x) = x^2 -$ x is/are true?

I. $f(x)$ is defined at 1 **II. $\lim_{x \to 1} f(x)$ exists**

III. $f(x)$ is continuous at 1 **IV. $f(x)$ is differentiable at 1**

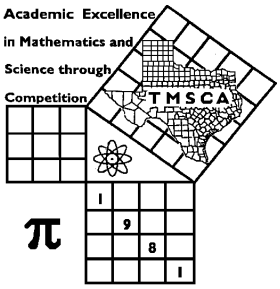
(A) I, II, & III **(B) I & II** **(C) II & III** **(D) all of them** **(E) none of them**

60. The repeating decimal 0.2111... in base 3 can be written as which of the following fractions in base 3 in simplified form?

(A) $\frac{12}{20}_3$ **(B) $\frac{1}{11}_3$** **(C) $\frac{11}{22}_3$** **(D) $\frac{12}{100}_3$** **(E) $\frac{3}{12}_3$**

2015-16 TMSCA HS Math Test #6
Answer Key

1. E	**21. E**	**41. C**
2. E	**22. E**	**42. C**
3. B	**23. A**	**43. A**
4. B	**24. B**	**44. B**
5. C	**25. E**	**45. C**
6. A	**26. D**	**46. B**
7. A	**27. E**	**47. C**
8. D	**28. E**	**48. A**
9. E	**29. D**	**49. C**
10. D	**30. D**	**50. D**
11. A	**31. E**	**51. C**
12. C	**32. B**	**52. A**
13. A	**33. A**	**53. E**
14. A	**34. D**	**54. B**
15. E	**35. D**	**55. A**
16. D	**36. E**	**56. D**
17. D	**37. C**	**57. C**
18. B	**38. B**	**58. B**
19. C	**39. B**	**59. D**
20. B	**40. C**	**60. A**



TMSCA HIGH SCHOOL MATHEMATICS TEST# 7 © JANUARY 16, 2016

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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2015-2016 TMSCA Mathematics Test 7

1. Evaluate: $\left(.\overline{6}\right)\left(6!\right)\div\left(.2\right)\left(7!\right)-10!$.

A. 7257600 B. -3193344 C. 8467200 D. -3628800 E. -3411072

2. Two million, three hundred eighteen thousand, three hundred one plus thirty-six thousand, fifty is subtracted from six million, six thousand, six. What is the sum of the digits in the difference when written out using digits instead of words?

A. 32 B. 40 C. 26 D. 29 E. 31

3. Bonnie plans to buy two dozen cupcakes that usually cost $1.25 each. She receives a 15% discount on the first dozen, a 20% discount on the second dozen, and pays 8.25% sales tax on the total. How much will her order cost?

A. $26.79 B. $24.75 C. $30.00 D. $26.85 E. $25.17

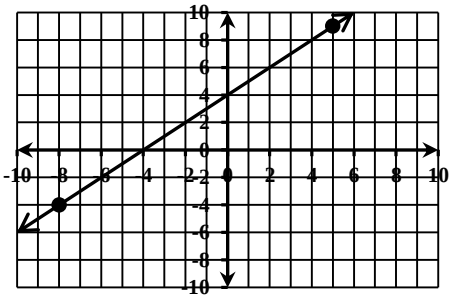
4. If $-3\left(4-x\right)=\left(x-4\right)\left(x-8\right)$, where $x\neq 4$, then $2x+8=$

A. 12 B. 30 C. 21 D. 11 E. 33

5. Simplify $\dfrac{\left(x+2\right)!}{\left(x-3\right)!}\div\dfrac{x!}{\left(x-1\right)!}$.

A. x^4+5x^2+4 B. $\dfrac{x}{x^2-3x+2}$ C. $x^6-5x^4+4x^2$ D. x^4-5x^2+4 E. $\dfrac{x^4-5x^2+4}{x^2}$

6. Find the equation of the line perpendicular to the one shown and through the point $\left(-2,3\right)$.



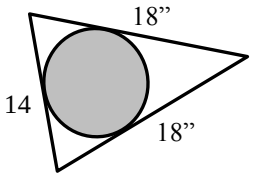
A. $x+y=1$ C. $x-y=1$ E. $x+y=-1$

B. $x-y=-5$ D. $x+y=-5$

7. Carla's mother drove her to an event at an average speed of 55 mph. She rode back with a friend at an average speed of 70 mph. If her total travel time was 2 hours 20 minutes, how far did she ride with her mother? (nearest mile)

A. 73 mi B. 72 mi C. 64 mi D. 68 mi E. 70 mi

8. The circle in the illustration is inscribed in the triangle. Find the area of the circle. (nearest tenth)



A. 67.7 in^2 B. 58.0 in^2 C. 62.8 in^2 D. 78.4 in^2 E. 63.6 in^2

9. The radius of a cone is 16 in. and the lateral surface area is 1106 in^2. Find the volume of the cone. (nearest tenth cubic foot)

A. 3.4 ft^3 B. 2.9 ft^3 C. 2.6 ft^3 D. 2.5 ft^3 E. 2.3 ft^3

10. If $\dfrac{A}{\left(x-8\right)}+\dfrac{B}{\left(3x-1\right)}=\dfrac{17x+2}{3x^2-25x+8}$ then $A+B=$

A. 3.5 B. 7 C. 6 D. 5 E. 5.5

11. The measure of an inscribed angle of a circle is 28°. What is the measure of the intercepted arc?

A. 62° B. 56° C. 14° D. 59° E. 38°

12. Carl has room for 8 travel books on his bookcase, but he has 11 travel books. In how many ways can he organize books on the shelf?

A. 330 B. 2640 C. 6652800 D. 1663200 E. 1980

13. High-speed train A left Paris at 8:00 am travelling at 200 mph due south. Forty-five minutes later, Train B left the same station travelling at 185 mph due east. How far apart were the trains at 12:15 pm? (nearest mile)

A. 1053 mi B. 1362 mi C. 1158 mi D. 1069 mi E. 1468 mi

14. Which of the following statement(s) about function *f* is/are true?

I. Every function *f* has an inverse function f^{-1}.

II. The domain of *f* is the range of f^{-1}.

III. f^{-1} is the same as $\frac{1}{f}$.

A. I, II & III B. I & III C. II & III D. II only E. none of these

15. If the pattern of the sequence 2, 8, 16, 26, 38,…continues, find the 15th term.

A. 268 B. 236 C. 302 D. 284 E. 252

16. Determine the range of $f(x) = -7 + 3\cos(2\pi x - 3)$.

A. $[-9,-5]$ B. $[0,10]$ C. $[-10,-4]$ D. $[2,9]$ E. $[-9,-2]$

17. Find $m+n$ if $\begin{bmatrix} -2 & 3 \\ -1 & 8 \end{bmatrix}\begin{bmatrix} -3 \\ 4 \end{bmatrix} = \begin{bmatrix} m \\ n \end{bmatrix}$.

A. 53 B. 35 C. 22 D. 26 E. 48

18. Leslie started out at her cabin and hiked 2.5 miles on a bearing of 18°, then changed direction and hiked another 3.6 miles on a bearing of 118°. How far would Leslie have to hike to go straight back to the cabin? (nearest tenth)

A. 4.8 mi B. 4.0 mi C. 5.8 mi D. 2.3 mi E. 5.0 mi

19. Find the sum of all the critical points of $f(x) = -3x^3 - 11x^2 + 9x + 8$.

A. $\frac{11}{3}$ B. $\frac{9}{11}$ C. -1 D. $-\frac{22}{9}$ E. $\frac{8}{3}$

20. Use the Fibonacci characteristic sequence …, -2, *p*, 6, *q*, 20, *r*, … to find $p+q+r$.

A. 24 B. 48 C. 28 D. 36 E. 56

21. $\left[(1+2+3+4+...+17)\times 30\right] \div \left[(31+32+33+34+...+46+47)\times 60\right]$=?

A. $\frac{3}{52}$ B. $\frac{90}{13}$ C. $\frac{1}{4}$ D. $\frac{3}{26}$ E. $\frac{1}{26}$

22. Given $x = 7 - y$ and $xy = 14$ find $x^2 + y^2$.

A. 21 B. 35 C. 49 D. 42 E. 56

23. If $f''(x) = 18x - 10$, $f'(1) = 7$ and $f(-1) = -25$ find $f(2)$.

A. 2 B. 11 C. 20 D. 15 E. -9

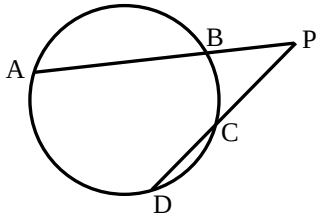
24. How many non-negative proper fractions in lowest terms have a denominator of 36?

A. 13 B. 29 C. 12 D. 10 E. 9

25. The length and width of a rectangle have a ratio of 15:8. The hypotenuse has a length of 3.4 ft. What is the area of the rectangle?

A. 2.4 ft^2 B. 4.8 ft^2 C. 3.6 ft^2 D. 5.4 ft^2 E. 5.1 ft^2

26. Points A, B, C and D lie on the circle shown. Find AB if BP = 4", DC = 9" and CP = 6". (not drawn to scale)



A. 18.5 in B. 13.5 in C. 12 in D. 15.5 in E. 21 in

27. How many distinct arrangements can be made using all of the letters in the word "THANKSGIVING"?

A. 479,001,600 B. 29,937,600 C. 79,833,600 D. 39,916,800 E. 59,875,200

28. Given the functions $f(x)=3x^2+1$, $g(x)=x^3$ and $h(x)=2x-3$ calculate the value of $h(f(g(-1)))$.

A. -8 B. 64 C. -7 D. -6 E. 5

29. Two fair cubic dice, each with the numbers 2, 4, 6, 8, 10 and 12 are rolled and the numbers on top are added. What is the expected value of the sum? (nearest tenth)

A. 13.6 B. 13.7 C. 14.0 D. 14.4 E. 14.7

30. The sum of the coefficients of the second and fifth terms in the expansion of $(2x-3)^8$ is

A. 90720 B. 93792 C. 87648 D. 51456 E. -51456

31. Given that the set of natural numbers continues in the triangular pattern shown below. Find the sum of the means of the fourth row and seventh row.

A. 64 C. 68 E. 39

B. 72 D. 78

			1				(row 0)
		2	3	4			(row 1)
	5	6	7	8	9		(row 2)
10	11	12	13	14	15	16	(row 3)
			…				(…)

32. Given triangle ABC with $AB=15$ cm, $BC=13$ cm and $m\angle A=60^\circ$. AC has two possible values. What is the product of these values?

A. 42 B. 56 C. 55 D. 48 E. 15

33. Bill and his brother paint at the same rate. Together, they can paint a wall of their house in 2.5 hours. How long would it take one of them to paint a wall twice as wide and twice as high?

A. 10 hours B. 5 hours C. 40 hours D. 20 hours E. 25 hours

34. If P, Q and R represent digits then $PRQ_5+RPQ_4-QRP_3$ has a numeric value in base 10 of:

A. $28P+18R-9Q$ B. $28P+24R-5Q$ C. $28P+18R-7Q$ D. $28P+24R-7Q$ E. $28P+18R-5Q$

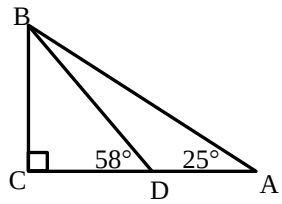
35. How many acute triangles with integral sides exist with sides 8", 14" and x"?

A. 5 B. 4 C. 8 D. 10 E. 12

36. Daily Delight Cupcakery sells 6 different flavors of cupcakes each day. How many different dozen cupcake boxes can they put together?

A. 924 B. 18564 C. 665280 D. 50388 E. 6188

37. Find AB if AD = 4,580 feet. (nearest foot)

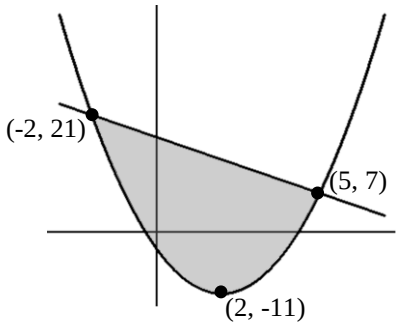


A. 8568 ft B. 9190 ft C. 7131 ft D. 4707 ft E. 9415 ft

38. If $\frac{x-5}{x+15}+\frac{x+15}{x-5}$ is equal to the mixed number $A\frac{B}{(x+15)(x-5)}$, then $B=$

A. 40 B. 400 C. 200 D. 225 E. 150

39. The graphs shown are a parabola and a line. Find the area of the shaded region. (nearest whole number)



A. 220 B. 44 C. 84 D. 99 E. 114

40. Simplify $\left(3-\sqrt{-75}\right)\left(2+\sqrt{-12}\right)$ to the form $a+bi$.

A. $-24+16\sqrt{3}i$ B. $36-4\sqrt{3}i$ C. $24+4\sqrt{3}i$ D. $-36-4\sqrt{3}i$ E. $-36+16\sqrt{3}i$

41. The square root of 331 in base 7 is:

A. 24_7 B. 12_7 C. 21_7 D. 13_7 E. 16_7

42. What is the sum of the solutions to the equation $\sin(2\theta)=\cos\theta$ if $0\le\theta<\frac{3\pi}{2}$.

A. π B. $\frac{2\pi}{3}$ C. $\frac{1}{2}$ D. 2π E. $\frac{3\pi}{2}$

43. Harold mixed 10 ounces of salt with 64 ounces of water. If he then added an additional 16 ounces of water, what percent of the new mixture would be water? (nearest tenth of a percent)

A. 88.9% B. 11.1% C. 12.5% D. 87.5% E. 90.0%

44. If $a_1=-3$, $a_2=-1$, and $a_n=(a_{n-1})(a_{n-2})-a_{n-2}$ for $a>2$, find the value of a_6.

A. 0 B. -36 C. 185 D. 125 E. -115

45. What is the digit in the millionths place in the sum of the series $4+\frac{16}{2!}+\frac{64}{3!}+\frac{256}{4!}+\frac{1024}{5!}+\ldots$?

A. 5 B. 8 C. 0 D. 3 E. 6

46. An archer consistently hits her target 4 out of every 7 shots. If she shoots at her target 9 times, what is the probability that she will make at least 7 shots? (nearest hundredth)

A. 0.82 B. 0.36 C. 0.02 D. 0.18 E. 0.22

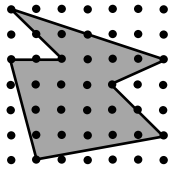
47. Let *a*, *b* and *c* be real numbers such that $c=a+b+9$, $c^2=a^2+b^2$ and $ab=8$. Find the numerical value of $18c$.

A. 65 B. 81 C. -81 D. 63 E. -65

48. $\lim_{h\to 0}\frac{\sin\left[2\left(\frac{\pi}{3}+h\right)\right]-\sin\left[2\left(\frac{\pi}{3}-h\right)\right]}{2h}=$

A. $\sqrt{3}$ B. -1 C. $-\sqrt{3}$ D. 1 E. does not exist

49. The horizontal and vertical distance between the dots is 5 cm. What is the area of the heptagon?



A. 19.5 cm^2 B. 97.5 cm^2 C. 975 cm^2 D. 512.5 cm^2 E. 487.5 cm^2

50. The Bulldogs and the Pirates are going to play a three game series. The probability that the Bulldogs will win any single game is 0.25. The probability that the Pirates will win any single game is 0.65. What is the probability that the 3-game series will be a tie?

A. $\frac{1}{1000}$ B. $\frac{197}{2000}$ C. $\frac{69}{4000}$ D. $\frac{13}{800}$ E. $\frac{39}{400}$

51. If *w* is 10% larger than *x*, *x* is 40% larger than *y*, and *y* is 50% smaller than *z*, by what percentage is *w* smaller than *z*?

A. 70% B. 27% C. 73% D. 23% E. 35%

52. The odds of drawing a pink raffle ticket at random from a bucket 495 tickets are 4:7. How many pink tickets would have to be removed from the bucket to reduce the odds to 1:3?

A. 15 B. 75 C. 65 D. 105 E. 95

53. If $\int_0^k a\cos\theta d\theta = 29$, then $\int_{-k}^k a\cos\theta d\theta =$.

A. 0 B. 58 C. 29 D. 116 E. 841

54. Given $(f \circ g)(x) = 2x+6$ and $g(x) = 2x+1$, calculate $f(9)$.

A. 14 B. 24 C. 23 D. 13 E. 29

55. The slant height of a particular cone is equal to the diameter of the base. What is the volume of the cone in terms of the radius?

A. $V = \pi r^3\sqrt{3}$ B. $V = \frac{\pi r^3\sqrt{2}}{3}$ C. $V = 2\pi r^2$ D. $V = \frac{\pi r^3}{3}$ E. $V = \frac{\pi r^3\sqrt{3}}{3}$

56. The length of a rectangular picture is three times the width. The picture is surrounded by a frame which is 5 inches wide. If the perimeter of the outside of the frame is 96 inches, what is the length of the picture in inches?

A. 21 in B. 24 in C. 7 in D. 8 in E. 30 in

57. If p and q are the zeros of the function $f(x) = 28x^2 + 3x - 135$ then $pq^2 + p^2q =$

A. $\frac{405}{784}$ B. $\frac{405}{1568}$ C. $\frac{3}{56}$ D. $\frac{69}{14}$ E. $\frac{1215}{1568}$

58. How many distinct arrangements are there of three letters chosen from the word "CALCULATOR"?

A. 210 B. 228 C. 264 D. 120 E. 90

59. Classify the graph of with the equation: $9x^2 + 6xy + 2y^2 + 2x - 3y + 5 = 0$

A) Circle B) Centroid C) Parabola D) Hyperbola E) Ellipse

60. If $9^{x-y} = 243$ and $27^{x+y} = 243$ then $x^2 - y^2 =$ _____?

A. $\frac{25}{12}$ B. $\frac{25}{6}$ C. $-\frac{25}{12}$ D. $\frac{5}{6}$ E. $-\frac{5}{12}$

2015-2016 TMSCA Mathematics Test Seven Answers

1. C	21. D	41. E
2. E	22. A	42. E
3. A	23. B	43. A
4. B	24. C	44. C
5. D	25. B	45. C
6. A	26. A	46. D
7. B	27. E	47. A
8. A	28. E	48. B
9. E	29. C	49. D
10. D	30. C	50. B
11. B	31. D	51. D
12. C	32. B	52. B
13. D	33. D	53. B
14. E	34. C	54. A
15. A	35. A	55. E
16. C	36. E	56. A
17. A	37. C	57. A
18. B	38. B	58. C
19. D	39. E	59. E
20. E	40. B	60. B

2015-2016 TMSCA Test Seven Select Solutions

8. $r = \frac{b}{2}\sqrt{\frac{2a-b}{2a+b}} = \frac{14}{2}\sqrt{\frac{36-14}{36+14}} \approx 4.64$ and the area is about $\pi(4.64)^2 \approx 67.7$

12. $({}_{11}C_8)(8!) = 6652800$

21. $\frac{\frac{17}{2}(18)(30)}{\frac{17}{2}(78)(60)} = \frac{18}{78(2)} = \frac{3}{26}$

27. There are 12 letters total with 2-Ns, 2-Gs and 2-Is, so the total number of distinct arrangements will be $\frac{12!}{(2!)(2!)(2!)} = 59875200$.

32. Using the Law of Cosines:
$13^2 = 15^2 + x^2 - 2(15x)\cos 60$ or $0 = x^2 - 15x + 56$ and the product of the roots is 56.

34. In base 10,
$25P + 5R + Q + 16R + 4P + Q - 9Q - 3R - P =$
$28P - 7Q + 18R$.

35. All the possible side lengths are 7, 8, 9, 10, 11, **12**, **13**, **14**, **15**, **16**, 17, 18, 19, 20 and 21, but $a^2 + b^2 > c^2$, where *c* is the longest side length for the triangle to be acute which is true of the triangles where the last side length is a bold value.

36. ${}_{6+12-1}C_{12} = 6188$

41. $331_7 = 169_{10}$ and the square root will be $13_{10} = 16_7$.

47. $(c-9)^2 = (a+b)^2$, so
$c^2 - 18c + 81 = a^2 + 2ab + b^2$ and $-18c + 81 = 2ab$ for $-18c + 81 = 16$ and $18c = -65$.

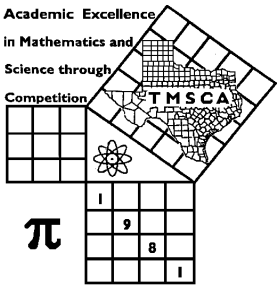
48. This is the definition of the derivative of $y = \sin 2\theta$ when $x = \frac{\pi}{3}$ which is -1.

49. If *P* is the number or points on the perimeter and *I* is the number of points on the interior, then the area without the 5 cm scaling factor is $\frac{2I + P - 2}{2} = \frac{2(16) + 11 - 2}{2} = 20.5$, then multiply by 25 for the area considering the 5 to get 512.5.

50. For a tied series, the teams can either play 3 tie games, or they can each win a game and tie the 3rd game. There are 6 arrangements of the second possibility, so the probability of a tie series is: $(0.1)^3 + 6(0.65)(0.25)(0.1) = \frac{197}{2000}$.

53. Because $f(\theta) = a\cos\theta$ is an even function, it will be symmetric about the *y*-axis and the new integral will have a value of 58.

57. $pq^2 + p^2q = pq(p+q)$ or for a quadratic function $f(x) = ax^2 + bx + c$,
$\left(\frac{c}{a}\right)\left(-\frac{b}{a}\right) = \left(\frac{-135}{28}\right)\left(\frac{-3}{28}\right) = \frac{405}{784}$.



TMSCA HIGH SCHOOL MATHEMATICS TEST# 8 © JANUARY 23, 2016

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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2015-2016 TMSCA Mathematics Test 8

1. Evaluate: $1\div(1+4)\times5-\frac{3}{8}+17\times(18)^0$

A. $\frac{2453}{8}$ B. $\frac{333}{8}$ C. $\frac{141}{8}$ D. $\frac{237}{8}$ E. $\frac{673}{40}$

2. There are 27 students on a bus on their way to a math meet. Eighteen plan on competing in number sense, 14 plan on competing in math and 6 are not competing in either. How many students plan on competing in both number sense and math?

A. 4 B. 13 C. 5 D. 11 E. 14

3. Find the greatest common divisor of 306, 680 and 1190.

A. 17 B. 2 C. 19 D. 34 E. 65

4. Simplify: $\left(a^{-4}\div b^2\right)^{-2}\times a^3\div b^{-2}$

A. $a^{11}b^2$ B. $a^{11}b^{-2}$ C. $a^{-5}b^4$ D. $a^{11}b^6$ E. $a^{-5}b^6$

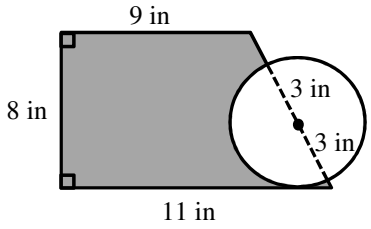
5. It takes 5 typists working 4 hours per day to type a manuscript in 18 days. How many days would it take 6 typists working 5 hours per day?

A. 15 B. 8 C. 9 D. 10 E. 12

6. Brandy walked from home to school at 4 mph and ran back at 10 mph. The entire trip took 32 minutes. How far is Brandy's house from the school? (nearest tenth)

A. 1.2 mi B. 2.1 mi C. 1.8 mi D. 1.5 mi E. 1.6 mi

7. Find the area of the shaded region. (nearest tenth)



A. 70.6 in^2 B. 61.2 in^2 C. 65.9 in^2 D. 46.8 in^2 E. 51.7 in^2

8. The intersection of the three altitudes of a triangle is called a(n)_____.

A. Center B. Orthocenter C. Centroid D. Incenter E. Circumcenter

9. If $\frac{2x-5}{x+2}-\frac{3x+4}{x-3}=\frac{Ax^2+Bx+C}{Px^2+Qx+R}$, then $\frac{A+B+C}{P+Q+R}$ equals:

A. 3.5 B. -3 C. 2.5 D. 2 E. -3.5

10. A transversal intersects two parallel lines such that the measure of one interior angle is $\left(x^2+3x\right)°$ and the measure of the opposite interior angle is $(11x+9)°$. What is the measure of the first angle?

A. 88° B. 108° C. 72° D. 90° E. 92°

11. The fundamental period of the graph of $y=2-5\sin^2(3x)$ is:

A. $\frac{\pi}{3}$ B. $\frac{2\pi}{3}$ C. π D. $\frac{\pi}{2}$ E. $\frac{3\pi}{2}$

12. Given the arithmetic sequence $22,a,b,44.5,c,...$, find the value of $a+b+c$.

A. 144.75 B. 118.5 C. 131.25 D. 66.5 E. 89

13. A farmer has a cylindrical water tank sitting on the circular base. The radius of the tank is 3 feet and the height is 8 feet. If the tank is already one-third full, how much water must he add to fill the tank? (nearest gallon)

A. 1692 gal B. 141 gal C. 94 gal D. 1128 gal E. 539 gal

14. Given $\begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 1 \\ 32 \end{bmatrix}$, find $m+n$.

A. -2 B. 12 C. -6 D. -9 E. -7

15. Find the mean value over the interval $[-2,8]$ of $f(x) = 2x^2 - 3x$.

A. $\frac{770}{3}$ B. 90 C. 9 D. $\frac{77}{3}$ E. $\frac{51}{5}$

16. $\left[(1+2+3+4+...+28+29)\times 30\right] \div \left[(31+32+33+34+...+58+59)\times 60\right] = ?$

A. $\frac{1}{6}$ B. $\frac{1}{3}$ C. 9 D. $\frac{1}{9}$ E. 6

17. Which of the following prime numbers are considered to be Germain Primes?

I. 5 II. 7 III. 23 IV. 29

A. I & II B. III & IV C. I, III & IV D. II, III & IV E. All of these

18. The repeating decimal $0.50505050...$ in base 6 can be written as which of the following fractions in base 6 in simplified form?

A. $\frac{6}{43_6}$ B. $\frac{10}{11_6}$ C. $\frac{10}{111_6}$ D. $\frac{5}{43_6}$ E. $\frac{5}{11_6}$

19. Which of the following is an equation of the perpendicular bisector of the line segment with endpoints $(-7,8)$ and $(3,-6)$.

A. $5x-7y=-17$ B. $5x-7y=-9$ C. $7x+5y=-9$ D. $7x+5y=9$ E. $5x-7y=3$

20. $(2-2i)^5 =$

A. $128-128i$ B. $-128-128i$ C. $-128i$ D. $128+128i$ E. $-128+128i$

21. The point (x, y) is a point of inflection on $f(x) = \frac{\sin x}{1+\cos x}$, where $0 < x \le 3\pi$. Find the value of x.

A. 0 B. π C. $\frac{\pi}{2}$ D. $\frac{\pi}{6}$ E. 2π

22. Mr. Data gives a ten question quiz to his class. When he is done grading, he gives the following frequency table to his class and offers extra credit to the first student to find the mean. What is the mean number of questions the students got right on the quiz? (nearest hundredth)

Questions Right	2	3	4	5	6	7	8	9	10
Number of Students	1	4	2	5	4	8	4	2	1

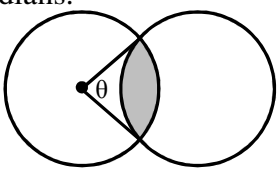
A. 6.00 B. 6.03 C. 5.97 D. 6.28 E. 6.09

23. The binomial $x-2$ is a factor of $6x^3 + Ax^2 + Ax + 30$. Find the value of A.

A. 1 B. -6 C. 11 D. -13 E. 9

24. The illustration shows two congruent circles each with a radius of 23 cm and $\theta = 1.3$ radians. Find the area of the shaded region. (nearest square centimeter)

A. 344 cm^2 B. 178 cm^2 C. 89 cm^2 D. 688 cm^2 E. 676 cm^2



25. If $f(x)=x^4$ and $g(x)=x-2$, then $g(f(2x-1))=$

A. $16x^4-32x^3+24x^2-8x-1$ C. $16x^4-4x^3+6x^2-4x-1$ E. $16x^4+4x^3+6x^2+4x-1$

B. $16x^4+32x^3+24x^2+8x+1$ D. $16x^4-32x^3+24x^2-8x+1$

26. What is the units digit of 2016^{2016}?

A. 2 B. 4 C. 0 D. 6 E. 8

27. James has a drawer full of unmatched socks containing 6 black, 15 white and 11 gray socks. If James reaches into the drawer to pull out 2 socks one at a time without replacement, what is the probability that he will pick two socks of the same color?

A. $\frac{105}{496}$ B. $\frac{175}{496}$ C. $\frac{175}{321}$ D. $\frac{105}{391}$ E. $\frac{10}{31}$

28. Which of the following is NOT a member of the solution set of $3|1-2x|+9<14$?

A. $\frac{7}{6}$ B. $-\frac{1}{5}$ C. $-\frac{1}{3}$ D. $\frac{4}{9}$ E. $\frac{12}{11}$

29. Given $\frac{A}{2x-5}+\frac{B}{x-3}=\frac{x-7}{2x^2-11x+15}$, calculate $A\times B$.

A. $\frac{7}{6}$ B. $-\frac{1}{5}$ C. $-\frac{1}{3}$ D. $\frac{4}{9}$ E. -36

30. Given that *A* and *B* are independent events, $p(A)=0.6$ and $p(A\cup B)=0.8$, calculate $p(A\cap B)$.

A. 0.5 B. 0.2 C. 0.4 D. 0.12 E. 0.3

31. Find $f(-2)+f(1)+f(3)$ if $f(x)=\begin{cases} x-8, & x\le 0 \\ -x^2, & 0<x\le 1.5 \\ \frac{3}{2}x & x>1.5 \end{cases}$.

A. -4.5 B. -7.5 C. -6.5 D. -5.5 E. -8.5

32. The center of the circle $x^2+y^2+6x-2y-6=0$ is _____units from the origin?

A. $\sqrt{10}$ B. 4 C. $2\sqrt{2}$ D. $\sqrt{6}$ E. 3

33. What is the coefficient of the x^2 term in the expansion of $(7x-5)(x-3)^5$?

A. 702 B. 1485 C. 4185 D. 1350 E. 675

34. A 96-ft rope is cut into *n* pieces of increasing lengths that form an arithmetic sequence. The shortest and longest pieces are 2.5 feet and 9.5 feet long respectively. What is the value of the common difference of the sequence?

A. $\frac{4}{5}$ ft B. $\frac{15}{32}$ ft C. $\frac{1}{4}$ ft D. $\frac{7}{15}$ ft E. $\frac{1}{2}$ ft

35. Which of the following is an equation of the normal line of the function $f(x)=\frac{1}{x^2+1}$ when $x=1$

A. $4x-2y=5$ B. $4x-2y=3$ C. $x+2y=2$ D. $x+2y=0$ E. $x-2y=2$

36. A regular pentagon of side length of 4 feet is inscribed in a circle. Find the area of the circle. (nearest tenth)

A. 13.9 ft^2 B. 19.2 ft^2 C. 131.6 ft^2 D. 23.8 ft^2 E. 36.4 ft^2

37. Evaluate: $\lim_{x\to 0}\dfrac{e^x-(1-x)}{x}$.

A. -1 B. 0 C. $\dfrac{1}{2}$ D. 2 E. Does not exist

38. How many distinguishable arrangements can be made using all of the letters in "CELEBRATE"?

A. 60480 B. 120960 C. 362880 D. 10080 E. 30240

39. The Key Club at a particular high school consists of 8 girls and 5 boys. The club has enough money to send a delegation of 2 girls and 2 boys to the state conference. In how many ways can the delegation be chosen?

A. 1120 B. 280 C. 715 D. 918 E. 160

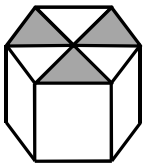
40. $\dfrac{1}{\csc\theta+\cot\theta}=$

A. $\cot\theta-\cos\theta$ B. $\csc\theta-\cot\theta$ C. $\sec\theta-\cot\theta$ D. $\cot\theta-\sin\theta$ E. $\cos\theta-\tan\theta$

41. A prankster removed the labels from all of the cans in Barry's pantry. He has four cans he knows are soup, 2 stews and 2 chowders. If Barry decides to open cans until he opens a chowder then stop, find $E(x)$, the number of cans Barry should expect to open to successfully open his first can of chowder.

A. $\dfrac{5}{3}$ B. $\dfrac{4}{3}$ C. 2 D. $\dfrac{3}{2}$ E. $\dfrac{7}{6}$

42. A candy company designed a box in the shape of a prism with regular hexagonal bases and square lateral faces as shown. If each edge is 3 cm. long, what is the volume of the box?



A. $\dfrac{27\sqrt{3}}{2}$ cm^3 B. $\dfrac{81\sqrt{3}}{2}$ cm^3 C. $27\sqrt{3}$ cm^3 D. $54\sqrt{3}$ cm^3 E. $\dfrac{27\sqrt{3}}{4}$ cm^3

43. Given $a_1=-1$, $a_2=\dfrac{3}{2}$ and $a_n=\dfrac{a_{n-2}}{a_{n-1}}$ find the value a_6.

A. $\dfrac{81}{16}$ B. $\dfrac{243}{32}$ C. $\dfrac{27}{8}$ D. $-\dfrac{243}{32}$ E. $\dfrac{8}{27}$

44. Let $f(x)=\dfrac{6x^3+5x-8}{2x^2-7}$ and $s(x)$ be the slant asymptote of $f(x)$. Calculate $s(-2)$.

A. -6 B. -4 C. 4.5 D. 6 E. -4.5

45. On the triangle ABC, $m\angle A=35^\circ$, $BC=4$ cm and $AC=6.5$ cm. Find the sum of the two possible values of AB. (nearest hundredth)

A. 14.42 B. 10.25 C. 7.5 D. 10.65 E. 13.55

46. What is the equation of the directrix of a parabola with the equation $x^2+24y-8x=-16$?

A. $x=4$ B. $y=-6$ C. $y=-4$ D. $x=-6$ E. $y=6$

47. Given $f''(x)=18x-10$, $f'(2)=24$ and $f(2)=19$, calculate $f(1)$.

A. -2 B. 6 C. 7 D. 5 E. -1

48. What is the smallest angle formed by the hour and minute hand of a clock at 2:20 pm?

A. 50° B. 45° C. 60° D. 55° E. 65°

49. Let $a=\log x$, $b=\log y$ and $c=\log z$. What is $\log\left(\frac{x^2\sqrt{y^3}}{z^3}\right)$ in terms of *a, b* and *c*?

A. $\frac{3ab}{2}-3c$ B. $2a+\frac{3}{2}b-3c$ C. $\frac{a^2\sqrt{b^3}}{c^3}$ D. $-\frac{3ab}{2c}$ E. $a^2+\sqrt{b^3}-c^3$

50. The line $y=20x-7$ is tangent to the curve $y=5x^3+ax^2+bx+11$ at the point (1, 13). Find the value $a+b$.

A. -7 B. -11 C. -3 D. 7 E. 3

51. The operation € is defined as A€B = A^3+B^3. Compute 3€(1€2).

A. 27 B. 1008 C. 756 D. 42876 E. 36

52. The function f is such that $\int_{-1}^{3} f(x)\,dx=7$. What is the value of $\int_{3}^{-1}(2f(x)+3)\,dx$?

A. 17 B. -26 C. 10 D. -17 E. 26

53. The equation $3x^2-5x+k=0$ always has two positive roots when which of the following is true?

A. $-\frac{25}{12}<x<\frac{25}{12}$ B. $0<k<\frac{25}{12}$ C. $-\frac{25}{12}<x<0$ D. $-\frac{12}{25}<x<0$ E. $-\frac{12}{25}<x<\frac{12}{25}$

54. Find the area in square units of a scalene triangle whose vertices are $(-2,-1)$, $(2,12)$ and $(3,3)$.

A. 27.5 B. 30.5 C. 22.5 D. 25.5 E. 24.5

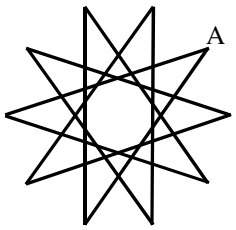
55. Let *a*, *b* and *c* be real numbers such that $c=a+b+16$, $c^2=a^2+b^2$ and $ab=10$. Find the numeric value of $16c$.

A. 118 B. 236 C. 123 D. 246 E. 126

56. $654_7+543_6+432_5=$ _____$_{10}$.

A. 681 B. 1629 C. 657 D. 682 E. 412

57. All ten points on the star shown are congruent. What is the measure of angle A?



A. 45° B. 30° C. 32° D. 36° E. 40°

58. $\ln(x)^2+\ln(x^3)^2+\ln(x^5)^2+.....+\ln(x^{29})^2=$

A. $(\ln x)^{450}$ B. $(\ln x)^{255}$ C. $255(\ln x)$ D. $\ln x^{420}$ E. $\ln x^{450}$

59. If $y=\frac{9}{x}$ and $x+y=12$, then $x^3+y^3=$

A. 1404 B. 1701 C. 1512 D. 1620 E. 1728

60. $\sum_{k=2}^{\infty}\frac{2}{3}\left(-\frac{3}{4}\right)^k=$

A. $\frac{8}{21}$ B. $-\frac{2}{7}$ C. $\frac{8}{3}$ D. $\frac{3}{14}$ E. $\frac{3}{2}$

2015-2016 TMSCA Mathematics Test Eight Answers

1. C	21. E	41. A
2. D	22. B	42. B
3. D	23. D	43. D
4. D	24. B	44. A
5. E	25. A	45. D
6. D	26. D	46. E
7. C	27. B	47. D
8. B	28. C	48. A
9. C	29. E	49. B
10. B	30. E	50. C
11. A	31. C	51. C
12. B	32. A	52. B
13. D	33. C	53. B
14. A	34. D	54. E
15. D	35. B	55. A
16. A	36. E	56. C
17. C	37. D	57. D
18. B	38. A	58. E
19. A	39. B	59. A
20. E	40. B	60. D

2015-2016 TMSCA Mathematics Test Eight Select Solutions

5. (5 typist)(72 hours)(hourly rate of one typist) = 1 job, so (hourly rate) = 1/360. To find the days , (6 typists)(5 hours per day)(1/360)(number of days) = 1 solve for 12 days.

6. Let *t* be the time it took to walk to school. Then $4t = 10\left(\frac{32}{60} - t\right)$ for $t \approx 0.38$ hours and the distance to school will be $4(0.38) \approx 1.5$ miles

16. $\frac{\frac{1}{2}(29)(30)(30)}{\frac{1}{2}(29)(90)(60)} = \frac{1}{6}$

18. $\begin{array}{rcl} 100n_6 & = & 50.\overline{50}_6 \\ \hline 55n_6 & = & 50_6 \end{array}$, so $\frac{50}{55_6} = \frac{30}{35_{10}} = \frac{6}{7_{10}} = \frac{10}{11_6}$

24. Half of the shaded region can be found by using $A = \frac{r^2\theta}{2}$, so the whole shaded region can be found using $r^2\theta = 23^2(1.3) \approx 688$.

29. $A(x-3) + B(2x-5) = x-7$. Let $x = 3$ for $B = -4$ and $x = \frac{5}{2}$ for $A = 9$ and a product of -36.

30. Use $0.8 = 0.6 + p(B) - 0.6p(B)$ for $p(B) = 0.5$ and $p(A \cap B) = (0.6)(0.5) = 0.3$.

32. Complete the squares for $(x+3)^2 + (y-1)^2 = 14$ which is a circle with a center at $(-3,1)$ and a distance from the origin of $\sqrt{(-3)^2 + 1^2} = \sqrt{10}$.

33. There will be two terms of the binomial expansion that will have an exponent of 2 when they are multiplied by terms in $(7x-5)$.

$7x\left(5(x)(-3)^4\right) + (-5)\left(10(x)^2(-3)^3\right) = 4185$

34. Use the sum of the sequence $96 = \frac{n}{2}(12)$ for $n = 16$, then $9.5 = 2.5 + (16-1)d$ and $d = \frac{7}{15}$.

35. Use L'Hopital's rule and evaluate $\lim_{x \to 0} \frac{e^x + 1}{1} = 2$.

41. $E(x) = 1\left(\frac{2}{4}\right) + 2\left(\frac{2}{4} \times \frac{2}{3}\right) + 3\left(\frac{2}{4} \times \frac{1}{3} \times \frac{2}{2}\right) = \frac{5}{3}$

45.Solve the equation $13 = 5 + a + b + 11$ for $a + b = -3$.

55. $(c-16)^2 = (a+b)^2$, so $c^2 - 32c + 256 = a^2 + 2ab + b^2$ and $-32c + 256 = 2ab$ or $-32c + 256 = 20$ and $16c = 118$.

58. This can be rewritten as $2\ln x + 6\ln x + 10\ln x + ... + 58\ln x$, the coefficients form an arithmetic sequence with a sum of 450, so the sum of the sequence is $\ln x^{450}$.

59. $x^3 + y^3 = (x+y)\left(x^2 - xy + y^2\right) = 12\left(144 - 3(9)\right) = 1404$

T M S C A H I G H S C O O L

M A T H E M A T I C S

T E S T # 9 ©

J A N U A R Y 3 0, 2 0 1 6

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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2015-2016 TMSCA Mathematics Test 9

1. Evaluate: $\left(.\overline{3}\right)(6!)\div(.4)(7!)-10!$

A. -3628800 B. -2116800 C. -907200 D. -3628800 E. -604800

2. Three million, three hundred eighteen thousand, three hundred one plus thirty-six thousand, fifty is subtracted from six million, six thousand, six. What is the sum of the digits in the difference when written out using digits instead of words?

A. 28 B. 31 C. 29 D. 27 E. 30

3. Bonnie plans to buy three dozen cupcakes that usually cost $1.25 each. She pays full price for the first dozen, receives a 15% discount on the second dozen, a 20% discount on the third dozen, and pays 8.25% sales tax on the total. How much will her order cost?

A. $43.03 B. $39.75 C. $38.97 D. $38.95 E. $48.71

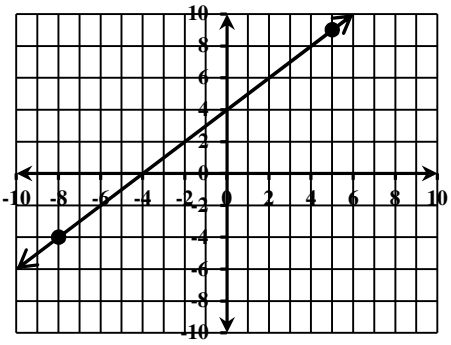
4. If $-3(5-x)=(x-5)(x-8)$, where $x\neq 5$, then $2x+8=$

A. 11 B. 22 C. 30 D. 41 E. 18

5. Simplify: $\dfrac{x(x+2)!}{(x-3)!}\div\dfrac{x!}{(x-1)!}$

A. $x^6-5x^4+4x^2$ B. $\dfrac{x^2}{x^2-3x+2}$ C. x^5-5x^3+4x D. x^5+5x^3+4x E. $\dfrac{x^5-5x^3+4x}{x^2}$

6. Find the equation of the line parallel to the one shown and through the point $(-2,3)$.



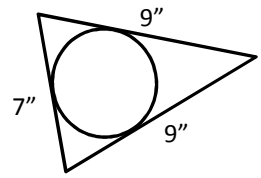
A. $x+y=1$ C. $x-y=1$ E. $x+y=-1$

B. $x+y=-5$ D. $x-y=-5$

7. Carla drove the same route to work 5 days last week. Her mean speeds for each day were: 18 mph, 25 mph, 15 mph, 27 mph, and 16 mph. Calculate her average speed for the whole week. (nearest tenth mph)

A. 19.1 mph B. 20.2 mph C. 19.6 mph D. 20.0 E. 20.1

8. The circle in the illustration is inscribed in the triangle. Find the circumference of the circle. (nearest tenth)



A. 16.9 in B. 14.6 in C. 15.2 in D. 19.6 in E. 15.9 in

9. The radius of a cone is 16 in. and the volume is 1106 in^3. Find the total surface area of the cone. (nearest tenth square foot)

A. 804.2 ft^2 B. 830.6 ft^2 C. 1011.6 ft^2 D. 2465.4 ft^2 E. 1634.8 ft^2

10. If $\dfrac{A}{(x-8)}+\dfrac{B}{(3x-1)}=\dfrac{17x-21}{3x^2-25x+8}$, then $AB=?$

A. -4 B. 10 C. 4 D. 16 E. 24

11. A regular heptagon with vertices *A, B, C, D, E, F* and *G* respectively is inscribed in a circle. What is the measure of $\angle ACE$? (nearest degree)

A. 154° B. 51° C. 103° D. 77° E. 129°

12. Nine council members need to sit at a round table with nine chairs, but there are two members who can not sit next to each other. In how many distinct arrangements can the members sit?

A. 35280 B. 32760 C. 282240 D. 322560 E. 30240

13. A scientist needs 12 liters of 18% salt solution for an experiment. How much 20% solution should she mix with 15% solution to obtain the 18% solution? (nearest milliliter)

A. 1029 ml B. 7200 ml C. 4800 ml D. 10971 ml E. 8000ml

14. The altitude to the base of an isosceles triangle is also a(n)

I. Angle Bisector II. Median III. Perpendicular Bisector

A. I &II B. I &III C. II &III D. II only E. All of these

15. What is the sum of the first 10 terms of the geometric sequence 1, 3, 9, 27, … .

A. 29524 B. 98420 C. 32807 D. 196840 E. 28844

16. Determine the period of $f(x) = -7 + 3\cos(2\pi x - 3)$,

A. 3 B. 1 C. π D. 3π E. $\frac{\pi}{3}$

17. Given that the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & k & 3 \\ 1 & -2 & 5 \end{bmatrix}$ has no inverse, find the value of *k*.

A. $-\frac{5}{3}$ B. 1 C. $\frac{9}{7}$ D. $-\frac{5}{7}$ E. 3

18. The complex number $z = a + bi$, for $a, b \in \mathbb{R}$ satisfies the equation $i(z+2) = 1 - 2z$ Find the value of $a+b$.

A. -1 B. 1 C. $\frac{1}{3}$ D. $-\frac{4}{3}$ E. $\frac{4}{3}$

19. Find the sum of all the critical points of $f(x) = 8x^3 - 33x^2 - 63x + 6$.

A. $-\frac{21}{8}$ B. $\frac{21}{8}$ C. $\frac{11}{4}$ D. $-\frac{11}{4}$ E. $\frac{21}{4}$

20. Use the Fibonacci characteristic sequence $...,6, p, 4, q, 6, r, ...$ to find $p+q+r$.

A. 8 B. 12 C. 10 D. 14 E. 6

21. Solve the equation: $25^{x-1} = \left(\frac{1}{5}\right)^{2x}$.

A. $-\frac{1}{2}$ B. 2 C. 1 D. $\frac{1}{2}$ E. $-\frac{1}{4}$

22. Given $x = 7 + y$ and $xy = 14$, then $x^3 - y^3 =$?

A. 637 B. 539 C. 343 D. 441 E. 392

23. A quadrilateral is inscribed in a circle. The measures of two angles opposite each other are $\left(x^2-12x\right)^\circ$ and $\left(3x-10\right)^\circ$. Find the measure of the larger angle.

A. 160° B. 90° C. 133° D. 20° E. 47°

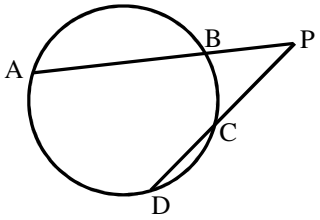
24. How many non-negative proper fractions in lowest terms have a denominator of 28?

A. 14 B. 12 C. 11 D. 10 E. 9

25. The length and width of a rectangle have a ratio of 15:8 and the length a diagonal is 3.4 ft. Find the area of the rectangle.

A. 2.4 ft^2 B. 3.6 ft^2 C. 5.4 ft^2 D. 5.1 ft^2 E. 4.8 ft^2

26. Points A, B, C and D lie on the circle shown. Find AB if BP = 8", DC = 18" and CP = 12". (drawing not to scale)



A. 37 in B. 27 in C. 24 in D. 31 in E. 42 in

27. How many distinct arrangements can be made using all of the letters in the words "STATE MEET"?

A. 60480 B. 90720 C. 45360 D. 560 E. 10080

28. A 25-ft. ladder is sliding down a wall at a rate of 2 inches per second. When the top of the ladder is 7 ft from the floor, the angle the ladder makes with the wall is changing at a rate of _____radians per second.

A. 0.0007 B. 0.0833 C. 1.0032 D. 0.0069 E. 0.0803

29. Two fair six-sided dice are rolled. What are the odds that the sum of the top faces is a deficient number?

A. 5:6 B. 5:1 C. 8:3 D. 8:11 E. 31:5

30. The real value solution set of $\left|3x+1\right|+3<11$ is:

A. $\left\{x\left|-\frac{7}{3}<x<3\right.\right\}$ C. $\left\{x\left|\left\{x<-3\right\}\cup\left\{x>\frac{7}{3}\right\}\right.\right\}$ E. $\left\{x\left|-3<x<\frac{7}{3}\right.\right\}$

B. $\left\{x\left|-3<x<-\frac{7}{3}\right.\right\}$ D. $\left\{x\left|\left\{x<-\frac{7}{3}\right\}\cup\left\{x>3\right\}\right.\right\}$

31. What is the constant term in the binomial expansion of $\left(2x^2-\frac{5}{x}\right)^6$?

A. 15 B. 2500 C. 20000 D. 37500 E. 1000

32. Let $a_1=7$, $a_2=-2$ and $a_n=a_{n-2}-2a_{n-1}$. Find a_6.

A. 59 B. -142 C. -24 D. -66 E. 25

33. Given that the multiples of three continue in the triangular pattern shown below. Find the sum of the means of the fourth row and seventh row.

A. 210 C. 168 E. 117

B. 234 D. 264

			3				(row 0)
		6	9	12			(row 1)
	15	18	21	24	27		(row 2)
30	33	36	39	42	45	48	(row 3)
			…				(…)

34. Two workers can paint a fence in three hours. How long would it take three workers to paint a fence twice as long and twice as high if they each paint at the same rate?

A. 4 hours B. 6 hours C. 12 hours D. 9 hours E. 8 hours

35. Find the area of the ellipse with the equation $9x^2+25y^2-36x+150y=-36$.

A. 25π B. $\frac{25\pi}{9}$ C. 9π D. $\frac{9\pi}{25}$ E. 15π

36. If *P*, *Q* and *R* represent digits then $PRQ_6+RPQ_5-QRP_4$ has a numeric value in base 10 of:

A. $40P+27R-14Q$ B. $40P+27R+18Q$ C. $42P+35R-14Q$ D. $40P+27R-18Q$ E. $42P+35R+18Q$

37. Find the acute angle that the line $3x+4y=18$ forms with the *x*-axis. (nearest hundredth)

A. 41.41° B. 48.59° C. 33.69° D. 36.87° E. 12.52°

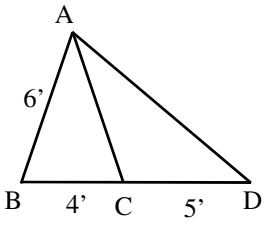
38. Myrtle has eight different colors of markers. In how many ways can she package four markers to sell if she can repeat colors?

A. 495 B. 220 C. 210 D. 70 E. 330

39. How many scalene acute triangles with integral sides exist with sides 9", 14" and x"?

A. 7 B. 4 C. 6 D. 5 E. 14

40. If $\overline{AB}\cong\overline{AC}$, then the area of triangle ABD is _____ft^2.



A. $36\sqrt{2}$ B. $18\sqrt{2}$ C. 27 D. 54 E. $9\sqrt{2}$

41. If $\frac{x-6}{x+22}+\frac{x+22}{x-6}$ is equal to the mixed number $A\frac{B}{(x+22)(x-6)}$, then $B=$

A. 784 B. 256 C. 132 D. 264 E. 324

42. The square root of 2011 in base 5 is:

A. 140_5 B. 14_5 C. 31_5 D. 34_5 E. 41_5

43. If $\int_{-2}^{4} f(x)\,dx=10.5$ then $\int_{-2}^{4}\left(f(x)+3\right)dx=$

A. 21 B. 13.5 C. 22.5 D. 28.5 E. 16.5

44. Solve $\log_3 x+\log_3(x-6)=3$.

A. 9 B. 6 C. -3 D. 3 E. -6

45. What is the area of a hexagon with an apothem length of 1 cm?

A. $\frac{\sqrt{3}}{2}$ cm^2 B. $\frac{8\sqrt{2}}{3}$ cm^2 C. $\frac{\sqrt{2}}{3}$ cm^2 D. $2\sqrt{3}$ cm^2 E. $\frac{8\sqrt{3}}{2}$ cm^2

46. Let the region *R* be in the first quadrant bounded by the *x*-axis, *y*-axis and the graph of $f(x)=\cos(x^2)$. Find the volume of the solid generated by the complete revolution of *R* around the *y*-axis.

A. $\sqrt{\pi}$ B. $\frac{\sqrt{2\pi}}{2}$ C. π D. 2π E. $\sqrt{2\pi}$

47. Ray consistently makes 3 out of every 5 free-throws he shoots. If he shoots 8 free-throws during a Friday night game, what is the probability that he makes exactly 7 of them?

A. $\frac{4374}{390625}$ B. $\frac{2187}{78125}$ C. $\frac{4374}{78125}$ D. $\frac{6561}{390625}$ E. $\frac{34992}{390625}$

48. Let *a*, *b* and *c* be real numbers such that $c=a+b+11$, $c^2=a^2+b^2$ and $ab=22$. Find the numerical value of $18c$.

A. 3.5 B. 126 C. -126 D. 63 E. -63

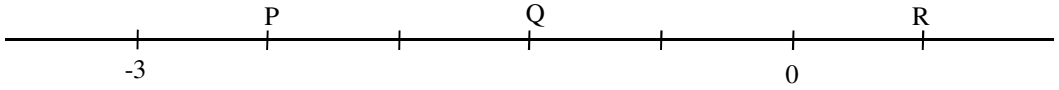
49. Evaluate $\lim_{h\to 0}\frac{\tan(\pi+h)-\tan(\pi)}{h}$.

A. 0 B. -1 C. 1 D. $\sqrt{3}$ E. undefined

50. $\frac{\cot^2 t}{\csc t} =$

A. $\sec t+\cos t$ B. $\sec t-\cos t$ C. $\csc t-\sin t$ D. $\csc t+\sin t$ E. $\sec t-\sin t$

51. The distances between the hash marks (|) are equal. Find P + Q + R.



A. -2.4 B. -1.2 C. 1.2 D. -3 E. 0.6

52. Find the sum of the series to the nearest ten thousandth: $7-\frac{7^3}{6}+\frac{7^5}{120}-\frac{7^7}{5040}+\ldots$.

A. 0.7539 B. 0.6570 C. 1.9459 D. 0.8451 E. 0.1219

53. If $g(x)=x-1$ and $f(x)=x^3$, find $g(f(x+2))$.

A. $x^3+6x^2+12x+8$ B. x^3+2x^2+4x+7 C. x^3+7 D. $x^3+6x^2+12x+7$ E. x^3+3x^2+3x+7

54. If p and q are the zeros of the function $f(x)=28x^2+x-15$ then $pq^2+p^2q=$?

A. $-\frac{15}{784}$ B. $-\frac{15}{28}$ C. $\frac{225}{784}$ D. $-\frac{225}{28}$ E. $\frac{15}{784}$

55. $5^3+6^3+7^3+\ldots.+11^3=$

A. 4256 B. 4356 C. 14641 D. 14541 E. 11655

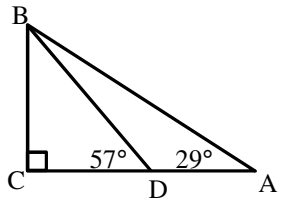
56. If $\frac{7x+13}{x^2+2x-3}=\frac{A}{x+3}+\frac{B}{x-1}$ then $AB=?$

A. 7 B. -6 C. 10 D. -3 E. 6

57. Elisa is going to make a short playlist. She is going to choose 9 songs from a group of 11 songs. In how many ways can she organize her playlist?

A. 362,880 B. 4,435,200 C. 19,958,400 D. 2,195,424,000 E. 199,584,000

58. Find AB if AD = 2,348 feet. (nearest foot)



A. 4067 ft B. 4062 ft C. 4696 ft D. 5250 ft E. 4973 ft

59. If the pattern of the sequence 2, 10, 20, 32, 46, …continues, find the 15th term.

A. 332 B. 262 C. 256 D. 276 E. 296

60. The odds of drawing a pink raffle ticket at random from a bucket of 308 tickets are 4:7. How many pink tickets would have to be removed from the bucket to reduce the odds to 1:4?

A. 49 B. 77 C. 63 D. 47 E. 70

2015-2016 TMSCA Mathematics Test Nine Answers

1. E	21. D	41. A
2. E	22. A	42. C
3. A	23. C	43. D
4. C	24. B	44. A
5. C	25. E	45. D
6. D	26. A	46. C
7. A	27. E	47. E
8. B	28. D	48. D
9. E	29. B	49. C
10. B	30. E	50. C
11. D	31. D	51. D
12. E	32. B	52. B
13. B	33. B	53. D
14. E	34. E	54. E
15. A	35. E	55. A
16. B	36. A	56. C
17. A	37. D	57. C
18. A	38. E	58. B
19. C	39. D	59. E
20. A	40. B	60. C

2015-2016 TMSCA Mathematics Test Nine Select Solutions

7. $\dfrac{5}{\frac{1}{18}+\frac{1}{25}+\frac{1}{15}+\frac{1}{27}+\frac{1}{16}} \approx 19.1$

8. $r = \dfrac{b}{2}\sqrt{\dfrac{2a-b}{2a+b}} = \dfrac{7}{2}\sqrt{\dfrac{18-7}{18+7}} \approx 2.32$ and $C = 2\pi r \approx 2\pi\pi(2.32) \approx 14.6$

10. $A(3x-1)+B(x-8) = 17x-21$. Let $x = 8$ for $24A - A = 136 - 21$ and $A = 5$. Then Let $x = \dfrac{1}{3}$ for $-\dfrac{23B}{3} = -\dfrac{46}{3}$ and $B = 2$, so $AB = 10$.

11. $m\widehat{AE} = 3\left(\dfrac{360}{7}\right) \approx 154.3$, so the inscribed angle that intersects the arc is $\dfrac{1}{2}(154.3) \approx 77$

12. There are 8! total arrangements. If the two **are** together, there are $2(7!)$ arrangements, so the number of arrangements without them together is $8! - 2(7!) \approx 30240$.

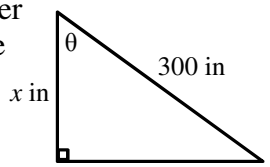
17. If the matrix has no inverse, then the determinant of the matrix is 0, so solve the equation $1(5k+6)+1(10-3)+2(-4-k) = 0$ for $k = -\dfrac{5}{3}$.

22. $x^3 - y^3 = (x-y)\left(x^2+xy+y^2\right) =$ $7(49+3(14)) = 637$

23. The opposite angles of a quadrilateral inscribed in a circle are supplementary, so $x^2 - 12x + 3x - 10 = 180$ for $x = 19$ for a larger angle of 133°.

27. There are 9 letters total including 3-Es and 3-Ts, so the number of distinct arrangements will be $\dfrac{9!}{(3!)(3!)} = 10080$.

28. First, change all units to either inches or to feet. Then set-up the relationship $\cos\theta = \dfrac{x}{300}$.



Taking the derivative yields: $-\sin\theta\left(\dfrac{d\theta}{dt}\right) = \dfrac{1}{300}\left(\dfrac{dx}{dt}\right)$. Find the base of the triangle when $x = 84$ inches to be 288 inches, so $-\dfrac{288}{300}\left(\dfrac{d\theta}{dt}\right) = \dfrac{1}{300}(-2)$ and $\dfrac{d\theta}{dt} = \dfrac{2}{288} \approx 0.0069$ radians per second.

31. The constant term will be the term when the exponents of variables in the numerator and denominator are equal: $\left({}_6C_2\right)\left(2x^2\right)^2\left(-\dfrac{5}{x}\right)^4 = 37500$.

33. The center term of each row is also the mean of the row, so continue the pattern of center numbers for 3, 9, 21, 39, **63**, 93, 129, **171**… with a mean of the 4th and 7th terms $\dfrac{63+171}{2} = 117$.

34. Each worker can paint $\dfrac{1}{6}$ of a fence each hour and the new fence has four times as much area as the old fence, so the total time for the bigger fence can be found using $3\left(\dfrac{1}{6}\right)t = 4$ for a time of 8 hours.

35. Completing each square $9(x-2)^2 + 25(y+3)^2 = 225$ then $\dfrac{(x-2)^2}{25} + \dfrac{(y+3)^2}{9} = 1$. For an ellipse in the form $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)}{b^2} = 1$, the area of the ellipse is $\pi ab = \pi(5)(3) = 15\pi$.

36. The number in base 10 can be written as $36P + 6R + Q + 25R + 5P + Q - 16Q - 4R - P$ or $40P - 14Q + 27R$.

38. The number of distinct arrangements will be ${}_{8+4-1}C_4 = 330$.

39. All the possible lengths for the third side are: 6, 7, 8, 10, **11**, **12**, **13**, **15**, **16**, 17, 18, 19, 20, 21 and 22. But in order for the triangles to be acute, $a^2 + b^2 > c^2$, where *c* is the length of the longest side. The bold numbers in the list meet these requirements.

42. $2011_5 = 256_{10}$ which has a square root of $16_{10} = 31_5$.

43. Split the integration into $\int_{-2}^{4} f(x)\,dx + \int_{-2}^{4} 3dx = 10.5 + 18 = 28.5$

46. The smallest positive zero of the function is at $\sqrt{\frac{\pi}{2}}$, so use shell method and evaluate the integral $2\pi\int_0^{\sqrt{\pi/2}} x\cos\left(x^2\right)dx = \pi$.

47. Use binomial theorem for ${}_8C_7\left(\frac{3}{5}\right)^7\left(\frac{2}{5}\right)^1 = \frac{34992}{390625}$

48. $(c-11)^2 = (a+b)^2$, so $c^2 - 22c + 121 = a^2 + 2ab + b^2$ and $-22c + 121 = 2ab$ for $-22c + 121 = 44$ $c = 3.5$ and $18c = 63$.

49. This is the definition of the derivative for $y = \tan x$ when $x = \pi$ which has a value of 1.

52. This is the McClaurin Series for $\sin 7 \approx 0.6570$

54. $pq^2 + p^2q = pq(p+q)$ or for a quadratic function $f(x) = ax^2 + bx + c$, $\left(\frac{c}{a}\right)\left(-\frac{b}{a}\right) = \left(\frac{-15}{28}\right)\left(\frac{-1}{28}\right) = \frac{15}{784}$.

57. ${}_{11}C_9(9!) = 19958400$.