TMSCA HIGH SCHOOL MATHEMATICS STATE MEET © MARCH 18, 2017

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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1. Evaluate: $4! - (4)^4 + 4 \times 4 + (4)^{\frac{1}{2}} \div (4)^{-1}$

(A) — 200 (B) — 204 (C) — 204.5 (D) — 208 (E) — 215.5

2. Find the number of positive integral divisors of 1,488.

(A) 3 (B) 6 (C) 20 (D) 23 (E) 31

3. On a map legend, 2.5 inches represents 125 miles. Booker, Texas is 1 foot 4.75 inches from South Point, Texas on the map. What is the distance from Booker to South Point?

(A) 875 miles (B) 837.5 miles (C) 804.74 miles (D) 750 miles (E) 737.5 miles

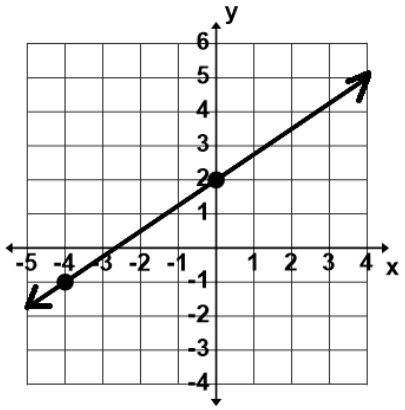
4. Let U (universal set) = {t, m, s, c, a, r, u, l, e}, X = {t, r, u, e}, and Y = {u, t, s, a}. Let $Z = (X \cup Y)^C$. Set Z contains how many distinct elements?

(A) 3 (B) 4 (C) 6 (D) 7 (E) 9

5. I. M. Cheep bought four used DVD movies for $3.25 each. After watching them he sold them at a garage sale. He got $3.75 for one, $5.00 for one, $4.25 for one, and $2.50 for one. What percent profit did he make? (nearest percent)

(A) 8% (B) 13% (C) 16% (D) 19% (E) 25%

6. An equation of a line through (— 6, — 5) and parallel to the line shown is:



(A) 3x — 4y = 2 (B) 4x — 3y = 20 (C) 3x — 4y = — 2 (D) 4x — 3y = 2 (E) 3x — 4y = 38

7. If $ax^2 - 10x - 7 = (bx - 7)(2x + c)$ then $a + b + c =$ _____.

(A) 4 (B) 11 (C) 13 (D) 14 (E) 17

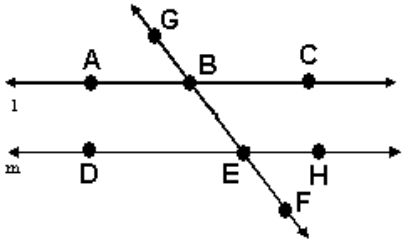
8. Ima Kahnfuzed is trying to find a 2-digit number such that the one's digit is 5 less than the tens digit and the number is equal to 8 times the sum of the digits. Find the product of the digits of Ima's number.

(A) 36 (B) 24 (C) 14 (D) 6 (E) 0

9. Tu Yong is 3 years older than Soh Yong. The sum of three times Tu's age and twice Soh's age is equal to the their father's age. If their father is 44 years old, how old is Tu?

(A) 7 **(B) 13** **(C) 16** **(D) 14** **(E) 10**

10. The three lines in the figure are coplanar with $m \parallel \ell$. Which of the following are true statements?



1. $\angle ABG$ & $\angle EBC$ are vertical angles **2. $m\angle HEB = \frac{2}{3} m\angle BED$**

3. $\angle BED \cong \angle GBA$ **4. $\angle DEF$ & $\angle CBF$ are alternate exterior angles**

(A) 2 & 3 **(B) 1 & 3** **(C) 1 only** **(D) 2 & 4** **(E) all of them**

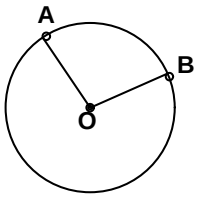
11. Given: $\angle P$ is supplementary to $\angle Q$; $m\angle R = 64°$; and $\angle Q$ is complementary to $\angle R$. Find $m\angle P$.

(A) 26° **(B) 36°** **(C) 116°** **(D) 144°** **(E) 154°**

12. Cal Penn is fencing in a square area of 576 square feet. Cal needs a fence post every 3 feet. What will the cost of the fence and the posts be before taxes if the price of fencing is \$2.50 per foot and the posts cost \$4.99 each?

(A) \$419.64 **(B) \$389.70** **(C) \$424.64** **(D) \$409.66** **(E) \$399.68**

13. Given circle with center O shown, OB = 4", and $m\overset{\frown}{AB} = 120°$. Find the length of $\overset{\frown}{AB}$. (nearest tenth)



(A) 4.2" **(B) 5.7"** **(C) 8.4"** **(D) 12"** **(E) 16.8"**

14. If $\frac{3x+1}{Ax+1} - \frac{4x-1}{2x+B} = \frac{-14x^2}{(Ax+1)(2x+B)}$, where A and B are constants, then $A + B$ equals:

(A) 6 **(B) 5** **(C) 4** **(D) 0** **(E) -1**

15. Let $f(x) = 5x - 4$, $g(x) = 4x - 3$, $h(x) = 3x - 2$, and $h\Big(f\big(g(x)\big)\Big) = 0$. Find $2x - 1$.

(A) 0.9666... **(B) 9.5** **(C) 0.8333...** **(D) 8.25** **(E) 8.1666...**

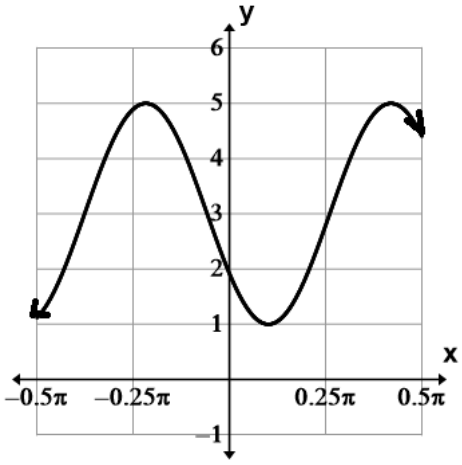
16. Find the sum of the arithmetic sequence: — 4, — 1, 2, 5, ..., 29, 32.

(A) 172 (B) 176 (C) 177 (D) 181 (E) 182

17. The *Grape Nutt* shop sold a bag containing 5 lbs of pecans and 2 lbs of raisins for $18.95. They sold another bag containing 3 lbs of pecans and 4 lbs of raisins for $14.10. What would it cost to buy a bag containing 1 lb of pecans and 6 lbs of raisins?

(A) $10.50 (B) $9.40 (C) $10.53 (D) $9.25 (E) $9.48

18. The equation y = _____ will produce this graph.



(A) $3 - 2\sin(\pi x + 1)$ (B) $3 + 2\cos(\pi x + 1)$ (C) $3 + 2\sin(\pi x - 1)$
(D) $3 - 2\cos(\pi x - 1)$ (E) $3 - 2\cos(\pi x + 1)$

19. $1 - \frac{\sin^3(x) + \cos^3(x)}{\sin(x) + \cos(x)}$ is equivalent to which of the following, where $\cos x \neq -\sin x$.

(A) $\sin^2(x) - \cos^2(x)$ (B) $\sin(x)\cos(x)$ (C) $\sin(2x)$ (D) $\sin(x) - \cos(x)$ (E) $\cos(3x)$

20. The *FRIEND* ship and the *SCHOLAR* ship are moored in the bay waiting their turn to head to Port Abella. The bearing of *FRIEND* from Abella is 225° and the bearing of *SCHOLAR* from Abella is 115°. *FRIEND* is 4 km due west of *SCHOLAR*. How far is the *FRIEND* ship from Port Abella? (nearest tenth)

(A) 1.8 km (B) 2.0 km (C) 2.2 km (D) 4.0 km (E) 5.1 km

21. In the expansion of $(2x + y)^7$, the sum of the coefficients of the 3rd term, the 4th term, and the 5th term is:

(A) 1,288 (B) 1,400 (C) 1,512 (D) 1,616 (E) 1,848

22. Let $x^6 - x^4 - px^3 + qx^2 - x - 1 = 0$, where $p, q > 0$. According to Descartes' Rule of Signs, how many possible negative roots are there?

(A) 5, 3, or 1 (B) 2 or 0 (C) 4, 2, or 0 (D) 3 or 1 (E) 0

23. If $\log(30) - \log(\frac{x}{2}) = \log(x - 4)$ then $\log(x) = ?$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 6 **(E)** 10

24. Clara Nett is making music CDs for her friends. She wants to put ten songs on each CD. How many different CDs can she make from the following genre of music: country, rock, rap, classical, easy listening, religious, and big band?

(A) 8,008 **(B)** 70 **(C)** 5,040 **(D)** 720 **(E)** 11,440

25. Given the function $f(x) = x^2 - 1$, find the equation of the secant line between $x = -3$ and $x = 4$.

(A) $x + y = 19$ **(B)** $x - y = -7$ **(C)** $x - y = -11$ **(D)** $x - y = -1$ **(E)** $x + y = 11$

26. $\int \left(\frac{3x}{3x^2 - 1}\right) dx =$ _____ $+ C$, where C is some arbitrary constant.

(A) $\frac{\ln|3x^2 - 1|}{2}$ **(B)** $3\ln|3x^2 - 1|$ **(C)** $\frac{1}{\ln|3x^2 - 1|}$ **(D)** $\ln|3x^2 - 1|$ **(E)** $\frac{\ln|3x^2 - 1|}{3}$

27. Let $f''(x) = 6x - 12$, $f'(1) = 2$, and $f(1) = 0$. Find $f(-1) + f'(-1)$.

(A) -6 **(B)** -3 **(C)** 0 **(D)** 2 **(E)** 49

28. Les Lite is 5 feet tall. He is walking at a rate of 3 feet per second toward a street light that is 12 feet tall. What is the rate of change of the length of Les' shadow? (nearest tenth)

(A) 2.4 ft/sec **(B)** 2.1 ft/sec **(C)** 1.7 ft/sec **(D)** 1.3 ft/sec **(E)** 1.1 ft/sec

29. The Millersview Muffs and the Doole Duffs are two evenly matched teams. They play a three game series. What are the odds that the Muffs sweep the Duffs or the Duffs sweep the Muffs?

(A) $16\frac{2}{3}\%$ **(B)** 25% **(C)** 30% **(D)** $33\frac{1}{3}\%$ **(E)** 50%

30. $120201_3 + 12210_3 + 2201_3 =$ _____ $_9$

(A) 403 **(B)** 901 **(C)** 606 **(D)** 303 **(E)** 805

31. Poly Gawn scored 238 on the TMSCA #6 math test and 290 on the TMSCA #13 math test. What will Poly have to score on this math test to average 266?

(A) 276 **(B)** 253 **(C)** 280 **(D)** 265 **(E)** 270

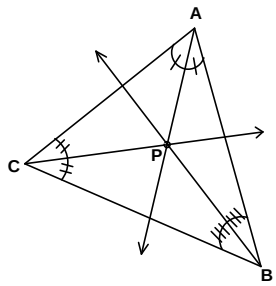
32. Determine the value of k so that $kx^2 - 4x - 3 = 0$ has one real root.

(A) $-1\frac{1}{3}$ **(B)** $-1\frac{7}{16}$ **(C)** $\frac{9}{16}$ **(D)** $\frac{3}{4}$ **(E)** 1

33. If $x - 2y = 5$, $2x - y = 3$ and $kx + y = 4$, then k equals:

(A) $-2\frac{1}{3}$ **(B)** -2 **(C)** 5 **(D)** 19 **(E)** 21

34. The point of concurrency P of the triangle shown is called the:



(A) incenter (B) centroid (C) orthocenter (D) circumcenter (E) line of Euler

35. If $a_1 = 4$, $a_2 = 1$, $a_3 = -2$ and $a_n = (a_{n-2}) + (a_{n-1}) - (a_{n-3})$ where $n > 3$ then $a_6 = ?$

(A) -13 **(B)** -11 **(C)** -8 **(D)** -5 **(E)** -1

36. P and Q are the roots of $2x^2 - 3x - 5 = 0$, where $P > Q$. Find $4P^2 + 12PQ + 9Q^2$.

(A) 30.25 (B) 12.25 (C) 4 (D) 22.5 (E) 9

37. Given: $f(x) = 2\cos[\pi(x + 3)] - 5$. The sum of the amplitude and the vertical displacement minus the sum of the period and the phase shift is ?

(A) 8 (B) 6 (C) -2 **(D)** -4 **(E)** -8

38. A right triangle, $\triangle ABC$, exists such that segment AC is perpendicular to segment BC, point E lies on segment BC, segment DE is perpendicular to segment BC, and segment AB does not intersect segment DE. If AC = BE = 9 cm and BC = ED = 20 cm then AD = ? (nearest cm)

(A) 28 cm (B) 29 cm (C) 30 cm (D) 31 cm (E) 32` cm

39. Simplify $\left(8 + \sqrt{-320}\right)\left(5 - \sqrt{-125}\right)$ to the form $a + bi$.

(A) $200 - 80\sqrt{5}\,i$ **(B)** $0 - 80\sqrt{5}\,i$ **(C)** $80 + 0i$ **(D)** $281 - 80\sqrt{5}\,i$ **(E)** $240 + 0i$

40. When $f(x) = x^3 + kx^2 + 2x + 5$ is divided by $x + 3$ the remainder is 8. Find the value of k.

(A) $2\frac{2}{9}$ **(B)** $-3\frac{1}{3}$ **(C) 4 (D) 2 (E)** -1

41. The function $f(x) = x^4 + x^3 - 3x^2 + 1$ is concave down on which of the intervals?
I. (— 0.4, 0.4) II. (— 1.1, 1.1) III. (— 0.75, 0.75) IV. (— 0.25, 0.25)

(A) I only (B) II only (C) II & III (D) I & IV (E) I, II, III, & IV

42. Kandy Kruncher has 7 different flavored jawbreakers, 5 different flavored tootsiepops, 4 different flavored peppermint sticks, and 8 different flavored suckers. How many ways can Kandy create gift bags containing 3 jawbreakers, 3 tootsiepops, 2 peppermints and 4 suckers?

(A) 2,704,156 (B) 76 (C) 80,640 (D) 1,352,078 (E) 147,000

43. Which of the following words has only 5,040 unique permutations of its letters?

(A) algebra (B) biology (C) calculus (D) derivative (E) element

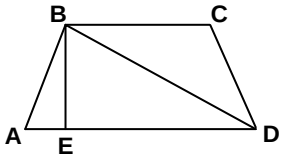
44. Let $f_1 = 2$, $f_2 = 5$, $f_3 = 7$, $f_4 = 12$, ... be the terms of a Fibonacci characteristic sequence. Find f_{11}.

(A) 555 (B) 343 (C) 212 (D) 144 (E) 89

45. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. Which of the following is a member of this sequence?

(A) 46,386 (B) 75,025 (C) 81,321 (D) 121,411 (E) 122,404

46. ABCD is an isosceles trapezoid with altitude BE = 12 cm and diagonal BD = 18 cm. What is the area of ABCD? (nearest cm)



(A) 216 cm^2 (B) 161 cm^2 (C) 121 cm^2 (D) 80 cm^2 (E) not enough information given

47. Simplify: How many ordered pairs (a, b) exist such that the four-digit number, a17b, is divisible by both 4 and 6?

(A) 4 (B) 5 (C) 6 (D) 10 (E) 15

48. The figure shown is rotated 90° clockwise. Then it is reflected over its positive diagonal. Finally it is rotated 90° counter clockwise. Which of the following figures is the result of these three transformations?

(A)  **(B)**  **(C)**  **(D)**  **(E)** 

49. Let P, Q, and R be positive integers. If $P + \frac{1}{Q + \frac{1}{R}} = \frac{57}{15}$, then find $P + Q + R$.

(A) 1 (B) 4 (C) 8 (D) 10 (E) 12

50. Using the following pattern of numbers, determine which of the following is the median number of row 12.

1	(row 0)
1 1	(row 1)
1 2 1	(row 2)
1 3 3 1	(row 3)
1 4 5 4 1	(row 4)
1 5 7 7 5 1	(row 5)
1 6 9 10 9 6 1	(row 6)
...	(row ...)

(A) 24 (B) 27 (C) 36 (D) 37 (E) 39

51. What is the digit in the tens place of $(7)^{77}$?

(A) 0 (B) 1 (C) 3 (D) 4 (E) 9

52. 0.5444... base 6 can be written as which of the following simplified fractions in base 6?

(A) $\frac{2}{9}_6$ (B) $\frac{45}{50}_6$ (C) $\frac{2}{9}_6$ (D) $\frac{53}{50}_6$ (E) $\frac{13}{10}_6$

53. Find the value of $\frac{3}{(1)(1)} - \frac{3}{(1)(2)} + \frac{3}{(2)(3)} - \frac{3}{(3)(5)} + \frac{3}{(5)(8)} - ...$ to the nearest thousandths place.

(A) 0.539 (B) 0.618 (C) 1.000 (D) 1618 (E) 1.854

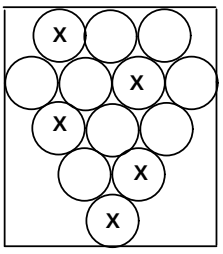
54. How many positive 3-digit numbers exist such that the sum of their digits equals 14?

(A) 83 (B) 77 (C) 70 (D) 69 (E) 56

55. Four playing cards, the 2 of spades, the 3 of clubs, the 4 of diamonds, and the 5 of hearts are put in a box. Two cards are drawn at random, without replacement from the box. What is the probability that the sum of the pips (the small symbols on the card face) of the two cards drawn is a prime number?

(A) 25% (B) $33\frac{1}{3}\%$ (C) 50% (D) $66\frac{2}{3}\%$ (E) 75%

56. A box contains styrofoam cups as shown below. The cups are 8" high with a 4" diameter and are tangent to each other and six of them are tangent to the box. Lynn Koln tosses a penny into the box. What is the probability that the penny lands in one of the cups marked with an X? (nearest whole percent)



(A) 38% (B) 51% (C) 20% (D) 57% (E) 22%

57. Which of the following mathematicians is known for their work and research in algebraic geometry?

(A) Emmy Noether **(B) Karen E. Smith** **(C) Grace Williams**
(D) Freda Porter **(E) Sophie Germain**

58. Which of the following numbers are unhappy, evil, and deficient number(s)?
I. 16 II. 34 III. 50 IV. 71

(A) II & IV **(B) II & III** **(C) III & IV** **(D) none of them** **(E) all of them**

59. Given the sequence, $\frac{1}{(1\times1+1)} - \frac{1}{(2\times2-1)} + \frac{1}{(3\times3+1)} - \frac{1}{(5\times5-1)} + \frac{1}{(8\times8+1)} - \ldots$, **find the digit in the thousandths place.**

(A) 5 **(B) 6** **(C) 7** **(D) 8** **(E) 9**

60. Let f(x) = $(2^x - 1) \div (2^x + 1)$**. Which of the following is not in the range of f(f(x))?**

(A) $\frac{2}{5}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{8}$ **(D)** $\frac{2}{7}$ **(E)** $\frac{1}{6}$

2016-17 TMSCA State Meet HS Math Test
Answer Key

1. D	**21. C**	**41. D**
2. C	**22. D**	**42. E**
3. B	**23. B**	**43. C**
4. A	**24. A**	**44. B**
5. D	**25. C**	**45. B**
6. A	**26. A**	**46. B**
7. C	**27. D**	**47. C**
8. C	**28. B**	**48. D**
9. E	**29. D**	**49. C**
10. B	**30. E**	**50. D**
11. E	**31. E**	**51. A**
12. E	**32. A**	**52. B**
13. C	**33. D**	**53. E**
14. C	**34. A**	**54. C**
15. A	**35. B**	**55. C**
16. E	**36. C**	**56. E**
17. D	**37. C**	**57. B**
18. D	**38. D**	**58. A**
19. B	**39. E**	**59. B**
20. A	**40. C**	**60. A**