



UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

# Mathematics

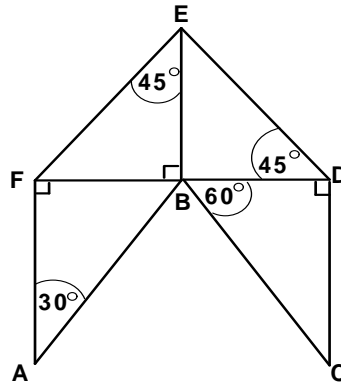
State • 2014



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YOU ARE INSTRUCTED TO DO SO!

1. Evaluate:  $(64)^{\frac{1}{3}} + 32 \div (4! - 2^4) \times 8^{(-1)}$
- (A) 0.5625      (B) 1      (C) 4.5      (D) 6      (E) 21.8333...
2. Rose Thorn went to the *Fruits and Nuts* Nursery to buy some trees. She paid \$14.75 for each fruit tree and \$12.50 for each nut tree. She bought three fruit trees and two nut trees. Since Rose belongs to the *Sav-A-Tree* club she received a 20% discount on fruit trees and a 15% discount on nut trees. How much did the five trees cost her before taxes? (nearest cent)
- (A) \$42.13      (B) \$45.01      (C) \$55.08      (D) \$55.48      (E) \$56.65
3. Find the number of positive integral divisors of 2,400.
- (A) 3      (B) 12      (C) 24      (D) 36      (E) 48
4. If P is 45% of Q and Q is  $\frac{4}{5}$  of R, then P is what percent less than R?
- (A) 80%      (B) 64%      (C) 55%      (D) 36%      (E) 35%
5. Simplify:  $\frac{x^2 - 4x - 5}{x^2 - 2x - 15} \times \frac{x^2 + 2x - 3}{x^3 + x^2 - x - 1}$
- (A)  $\frac{5}{3x^4 + 3x^3}$       (B)  $x - 1$       (C)  $\frac{1}{x + 1}$       (D)  $x + 1$       (E)  $\frac{3}{x - 1}$
6. Line  $m$  passing through (2, 3) has a slope of  $\frac{1}{2}$ . Line  $n$  is perpendicular to line  $m$  and passes through (3, -4). Lines  $m$  and  $n$  intersect at (x, y). Find y.
- (A) -1      (B)  $-\frac{1}{2}$       (C) 2      (D) 4      (E) 5
7. Three years ago, Ima Oldie was one year older than twice Ura Goodie's age. Six years from now Ima's age will be ten years more than half of Ura's age. What is the sum of their ages now?
- (A) 28      (B) 21      (C) 16      (D) 13      (E) 10
8. Let  $p$  and  $q$  be the roots of  $x^2 + 2x - 15 = 0$ . Find  $|p^3 - 3p^2q + 3pq^2 - q^3|$ .
- (A) 8      (B) 64      (C) 90      (D) 169      (E) 512
9. Leon D. Oiler folded a geometry net into a cuboctahedron. It has 24 edges, 12 vertices, 8 triangular faces and some square faces. How many square faces does it have?
- (A) 2      (B) 4      (C) 6      (D) 8      (E) 10
10. If you slice a right circular cone with a plane having the same slope as the slant of the cone, the cross section is a(n) \_\_\_\_\_.
- (A) line      (B) circle      (C) elongated ellipse      (D) parabola      (E) hyperbola

11. Find the perimeter of the hexagon ABCDEF if AB = 10 cm. (nearest tenth).



- (A) 54.5 cm      (B) 51.5 cm      (C) 49.1 cm      (D) 44.4 cm      (E) 39.9 cm

12. Point A  $(-2, -2)$  lying on the x-y plane is rotated  $\frac{3\pi}{2}$  radians counter-clockwise about the origin to point B. Point B is translated horizontally 3 units to the right to point C. Point C is reflected across the line  $y = x$  to point D. Point D is translated vertically down 3 units to point E. Point E is rotated  $\frac{\pi}{2}$  radians clockwise about the origin to point F  $(x, y)$ . Find  $x + y$ .

- (A)  $-4$       (B)  $-2$       (C)  $0$       (D)  $1$       (E)  $3$

13. These four examples are examples of one of the best-known conjectures of number theory.

- 1)  $8 = 3 + 5$       2)  $20 = 7 + 13$       3)  $66 = 29 + 37$       4)  $100 = 11 + 89$

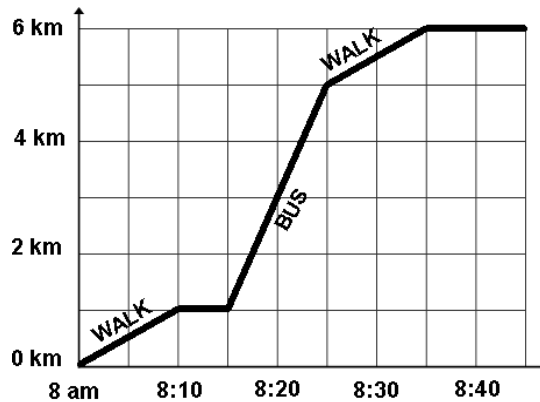
Which of these mathematicians is the conjecture attributed to?

- (A) Zeno of Elea (B) John Napier (C) Theano (D) Lady Lovelace (E) Christian Goldbach

14. The sixth *harmonic number* is:

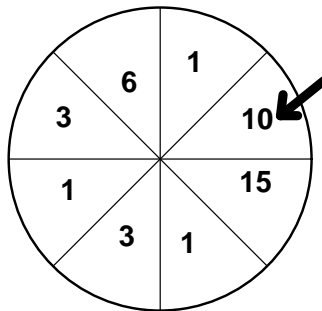
- (A)  $0.0476190$       (B)  $2.5928\dots$       (C)  $1.5$       (D)  $2.45$       (E)  $0.408333\dots$

15. Willie Makette walks to the bus stop, gets on the bus and rides to the next bus stop, then walks the rest of the way to school. Using the graph of Willie's trip to school, determine the difference in time Willie spent walking and the time he spent riding the bus.



- (A) 25 min      (B) 20 min      (C) 15 min      (D) 10 min      (E) 5 min

16. How many positive integral values of  $n$  exist such that  $\frac{(n+3)!}{(n+1)!} < 30$ .
- (A) none      (B) 2      (C) 3      (D) 5      (E) 6
17. Determine the range of  $f(\theta) = 5\sin(3\theta + 1.5\pi) - 2$ .
- (A)  $[-4.5, 1.5]$     (B)  $[-3.5, 3.5]$     (C)  $[3, 7]$     (D)  $[-3, 2]$     (E)  $[-7, 3]$
18. Simplify:  $(1 - \sin^2x)(1 + \tan^2x)$
- (A) 2      (B) 1      (C)  $\cos x$       (D)  $\cos^2x$       (E)  $1 - \cos^2x$
19. Find  $C$  if the remainder of  $8x^4 + 4x^3 + 7x^2 + 5x + C$  divided by  $x + 1$  is 9.
- (A)  $-3$       (B)  $-1$       (C) 3      (D) 2      (E) 1
20. In the binomial expansion of  $(2x + 1)^6$ , the sum of the coefficients of the 2<sup>nd</sup>, 4<sup>th</sup>, and 6<sup>th</sup> terms is:
- (A) 729      (B) 96      (C) 556      (D) 720      (E) 364
21. Betty Luzes spins the spinner shown, where all sectors are the same size. If it lands on a prime number she gets that number of points tripled. If it lands on a unit she gets that number of points doubled. If it lands on any other number, she loses 5 points. Assuming it does not land on a line, what is the mathematical expectation on any one spin?



- (A)  $+2.75$  pts    (B)  $+1.125$  pts    (C)  $-0.375$  pts    (D)  $-0.75$  pts    (E)  $-4.0$  pts
22. How many distinguishable ways can the letters in the word ARRANGEMENT be arranged?
- (A) 2,494,800    (B) 1,260    (C) 226,800    (D) 1,247,800    (E) 5,040
23. A group of 75 northern Aleuts took a survey about ice cream. The results showed the following: 39 liked chocolate; 47 liked vanilla; 38 liked strawberry; 1 did not like ice cream; 11 liked chocolate and strawberry, but not vanilla; 14 liked vanilla and strawberry, but not chocolate; 7 liked chocolate and vanilla, but not strawberry; and 9 liked all three flavors. How many of the Aleuts liked only strawberry?
- (A) 1      (B) 4      (C) 8      (D) 12      (E) 17

24. Let  $\frac{2x+1}{3x-2} - \frac{4x+3}{5x-3} = \frac{Ax^2+Bx+C}{Px^2+Qx+R}$ . Find  $\frac{A+B+C}{P+Q+R}$ .

- (A)  $-5\frac{1}{2}$       (B)  $-\frac{1}{9}$       (C)  $-\frac{1}{20}$       (D)  $-\frac{1}{10}$       (E)  $-\frac{1}{2}$

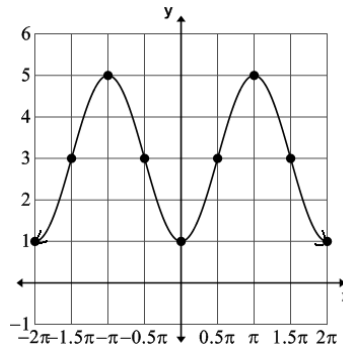
25. A water tank in the shape of a regular hexagonal based prism with side lengths of 4 feet and a height of 5 feet is 80% full of water. How many gallons of water would it take to fill the tank? (nearest gallon)

- (A) 389 gal      (B) 363 gal      (C) 346 gal      (D) 324 gal      (E) 311 gal

26. If  $a_1 = -1$ ,  $a_2 = 3$ ,  $a_3 = -2$  and  $a_n = (a_{n-1} - a_{n-3})(a_{n-2})$ , where  $n \geq 4$ , then  $a_7$  equals:

- (A)  $-468$       (B)  $-42$       (C)  $0$       (D)  $265$       (E)  $12,800$

27. The equation  $y = D + A\sin(Bx - C\pi)$ , where  $-2\pi \leq x \leq 2\pi$ , will produce this graph. If A, B, C, and D > 0, which of the following is the sum of A, B, C, and D?



- (A) 8.5      (B) 7      (C) 6.5      (D) 5      (E) 4.5

28. If  $(5 + 3i)(4 + 2i)(7 - i) = a + bi$  then  $a + b$  is:

- (A) 260      (B) 140      (C) 76      (D)  $-20$       (E)  $-12$

29. How many kinds of symmetry does the graph of the polar equation,  $r = 5\cos(4\theta)$ , have?

- (A) none      (B) 1      (C) 2      (D) 3      (E) 4

30. Find the slope of the line tangent to the curve  $y = 3x^2 + 4x - 5$  at the point  $(-2, -1)$ .

- (A)  $-21$       (B)  $-8$       (C)  $-6$       (D)  $-2$       (E)  $-1$

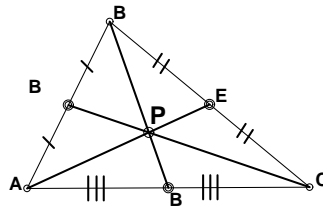
31. A freshman, a sophomore, and a junior are working on this problem. The probability that the freshman will solve this problem is  $\frac{1}{4}$ , that the sophomore will solve it is  $\frac{1}{3}$ , and that the junior will solve it is  $\frac{3}{5}$ . What is the probability that the freshman and the sophomore will solve it, but the junior will not? (nearest tenth)

- (A) 3.3%      (B) 16.7%      (C) 18.3%      (D) 31.7%      (E) 33.3%

32. P varies jointly as Q and R and inversely as the square of Z. If  $P = 150$ ,  $Q = 15$ ,  $R = 18$ , and  $Z = 9$ , find P when  $Q = 21$ ,  $R = 12$ , and  $Z = 6$ .

- (A) 337.5      (B) 315      (C)  $71\frac{3}{7}$       (D) 45      (E) 28.4

33. Find AP if EP = 16 cm.



- (A) 32 cm      (B) 48 cm      (C) 40      (D) 24 cm      (E) 8 cm

34. If  $(-1, 1)$ ,  $(2, -3)$ , and  $(5, k)$  are members of the function  $\{(x, y) \mid y = 4a + bx\}$ , then  $k = ?$

- (A)  $-8$       (B)  $-7$       (C)  $-6$       (D)  $-5$       (E)  $-4$

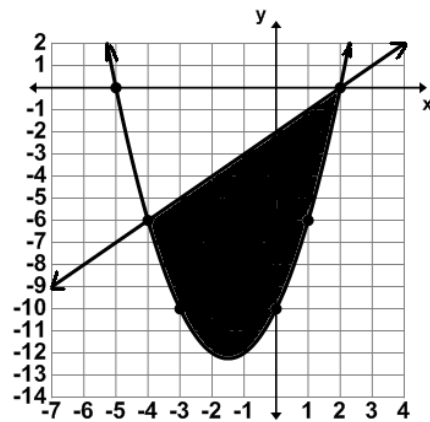
35. Let vector  $u = (-2, 4)$  and vector  $v = (-1, -3)$ . Find the measure of the angle with initial side  $u$  and terminal side  $v$ . (nearest minute)

- (A)  $135^\circ 0'$       (B)  $132^\circ 16'$       (C)  $124^\circ 0'$       (D)  $116^\circ 34'$       (E)  $108^\circ 26'$

36. The directrix of the parabola  $12x + y^2 + 6y = 3$  is:

- (A)  $x = 9$       (B)  $x = 6$       (C)  $x = 4$       (D)  $x = -2$       (E)  $x = -3$

37. Find the area of the shaded region in square units.



- (A)  $34\frac{2}{3}$       (B) 35      (C)  $35\frac{1}{3}$       (D) 36      (E)  $36\frac{2}{3}$

38. The probability that statement P is true is 35%, and the probability that statement Q is true is 70%. Determine the probability that  $P \rightarrow Q$  is false.

- (A) 10.5%      (B) 24.5%      (C) 27%      (D) 50%      (E) 52.5%

39. Eratosthenes numbered 20 ping pong balls from 1 to 20 and mixed them up in a bag. He drew out three balls, without replacement, and recorded their numbers. What is the probability all three of them were prime numbers? (nearest tenth)

- (A) 3.1 %      (B) 4.9 %      (C) 6.7 %      (D) 25.5%      (E) 40.0 %

40. Chef Sal Add wants to make a salad dressing using two types of vegetable oil. One type of oil contains 8% saturated fats and a second oil contains 25% saturated fats. How many ounces of the second oil will Chef Sal have to add to 10 ounces of the first oil if the per cent of saturated fats is to be 15%?

- (A) 17 oz      (B) 15 oz      (C) 10 oz      (D) 8 oz      (E) 7 oz

41. Ray Diaz waters his crops using a long pipe with multiple sprinkler heads on it. One end of the pipe is connected to a pivot on his well. The well is 400 feet from the corner of his field on a bearing of  $55^\circ$ . The other end of the irrigation pipe is 300 feet from the same corner of his field on a bearing of  $100^\circ$ . The pipe rotates in a circular motion about the well when it is turned on. How many acres of land will his sprinkler system water when it completes one revolution if 1 acre equals 43,560 square feet. (nearest tenth)

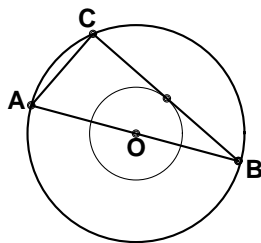
- (A) 3.9 acres      (B) 4.5 acres      (C) 5.8 acres      (D) 6.7 acres      (E) 8.1 acres

42. If the set of numbers  $\{1, 2, 3, 4, 5, \dots\}$  continue in the triangular pattern shown below, in what row would you find the number 14,878?

			1						(row 1)
			2	3	4				(row 2)
		5	6	7	8	9			(row 3)
	10	11	12	13	14	15	16		(row 4)
			...						( ... )

- (A) 148      (B) 122      (C) 130      (D) 154      (E) 112

43. Given the two concentric circles with center O: the ratio of the radii is 1 to 3; segment AB is the diameter of the larger circle; chord BC is tangent to the smaller circle; and  $AC = 12$ ". Find the radius of the larger circle.



- (A) 8 "      (B) 15 "      (C) 16 "      (D) 18 "      (E) 24 "

44. Let  $f(x - 1) = 1 + 2f(x)$ . Find  $f(-2)$  if  $f(1) = -1$ .

- (A) 2      (B) 1      (C) 0      (D) -1      (E) -2

45. Find  $\lim_{x \rightarrow -1} f(x)$ , if it exists. Let  $f(x) = \begin{cases} 2x + 1 & \text{if } x < -1, \\ 3 & \text{if } -1 < x < 1, \\ 2x + 1 & \text{if } x > 1. \end{cases}$

- (A)  $-1$       (B)  $1$       (C)  $2$       (D)  $3$       (E) does not exist

46. Phil Whitwater is putting water in his small cylindrical shaped pool. The diameter of the pool is 12 feet and the height of the pool is 4 feet. The water is flowing into the tank at a rate of 3 cubic feet per minute. How fast is the water rising when the water is 2 feet deep? (nearest tenth)

- (A) 0.7 ft/hr      (B) 1.1 ft/hr      (C) 1.6 ft/hr      (D) 2.1 ft/hr      (E) 2.4 ft/hr

47. Which of the following is NOT a member of the solution set for  $\frac{|4x + 1|}{3} - 2 \geq 5$  ?

- (A) 4.875      (B)  $\frac{21}{4}$       (C) 8      (D)  $-\frac{11}{2}$       (E)  $-6.125$

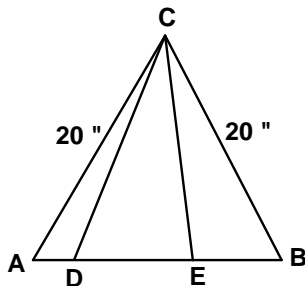
48. Find the total surface area of a right cone with a height of 7 inches and a slant height of 9 inches. (nearest sq. in)

- (A) 187 sq. in      (B) 243 sq. in      (C) 260 sq. in      (D) 278 sq. in      (E) 335 sq. in

49. Find the sum of the coefficients of the quotient:  $(10x^3 + 13x^2 + 10x + 3) \div (2x + 1)$

- (A) 29      (B) 17      (C) 14      (D) 12      (E) 6

50.  $\triangle ABC$  is an equilateral triangle. Find the sum of the perimeters of  $\triangle CAD$  and  $\triangle BEC$  if  $AD = 2''$  and  $BE = 8''$  (nearest tenth)



- (A) 74.6''      (B) 78.0''      (C) 82.7''      (D) 84.4''      (E) 86.5''

51.  $\sum_{k=-2}^{-1} k^k(x+k) + \sum_{k=1}^2 k^k(x-k) = ?$

- (A)  $\frac{17(x-2)}{4}$       (B)  $\frac{19x-34}{4}$       (C)  $\frac{19x}{4}$       (D) 0      (E)  $\frac{17x-42}{4}$

52. If  $f''(x) = 20x^3 - 24x^2 + 18x - 8$  and  $f'(1) = 3$  and  $f(-1) = -21$ , then  $f(1) =$  \_\_\_\_\_.

- (A) 21      (B) 6      (C)  $-3$       (D)  $-7$       (E)  $-18$



53. Penni Tass flipped a penny five times and recorded the results. The odds of only three consecutive heads happening is A:B. Find  $A \div B$  to the nearest hundredths.
- (A) 0.25      (B) 0.14      (C) 0.28      (D) 0.16      (E) 0.19
54. Let K be a two-digit number such that the units digit is divisible by 4. What is the sum of these numbers?
- (A) 1,458      (B) 1,378      (C) 1,368      (D) 1,098      (E) 888
55. McDonald's mc nuggets comes in a 4-pack, a 6-pack, a 9-pack and a 20-pack. They do not sell them in any other amounts and will not break up a pack. What is the largest number of mc nuggets less than 40 can you NOT get?
- (A) 39      (B) 32      (C) 19      (D) 14      (E) 11
56.  $(62_8 - 47_8 + 13_8) \times 3_8 = \underline{\hspace{2cm}}_8$
- (A) 102      (B) 52      (C) 132      (D) 24      (E) 104
57. Mei Yhun wants to change 5212014 from base 10 to base 20. Which of the following is the correct base 20 number for 5212014.
- (A) 1CBA0E      (B) 13B107      (C) 578ABF      (D) 52C014      (E) A4040A
58. Change the base 10 proper fraction  $\frac{6}{7}$  to a repeating decimal in base 4.
- (A) 0.213213213...<sub>4</sub>      (B) 0.323232...<sub>4</sub>      (C) 0.312312312...<sub>4</sub>      (D) 0.232323...<sub>4</sub>      (E) 0.123123123...<sub>4</sub>
59. Let P and Q be positive integers such that the harmonic mean of P and Q is 6.875. Find the contraharmonic mean of P and Q.
- (A) 7.0      (B) 7.75      (C) 8.375      (D) 8.625      (E) 9.125
60. Solve the system of equation for x:  $(\log_7 16)x + (\log_5 3)y = \log_{49} 8$   
 $(\log_3 25)x - (\log_2 7)y = \log_9 125$
- (A) 3      (B) 1.5      (C) 1      (D) 0.5      (E) 0.25

**University Interscholastic League  
MATHEMATICS CONTEST  
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Answer Key**

- |       |       |       |
|-------|-------|-------|
| 1. C  | 21. B | 41. C |
| 2. E  | 22. A | 42. B |
| 3. D  | 23. B | 43. D |
| 4. B  | 24. E | 44. D |
| 5. C  | 25. E | 45. E |
| 6. C  | 26. A | 46. C |
| 7. C  | 27. C | 47. A |
| 8. E  | 28. A | 48. C |
| 9. C  | 29. D | 49. D |
| 10. D | 30. B | 50. E |
| 11. B | 31. A | 51. A |
| 12. A | 32. B | 52. C |
| 13. E | 33. A | 53. E |
| 14. D | 34. B | 54. A |
| 15. D | 35. A | 55. E |
| 16. B | 36. C | 56. A |
| 17. E | 37. D | 57. A |
| 18. B | 38. A | 58. C |
| 19. C | 39. B | 59. E |
| 20. E | 40. E | 60. D |