



UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

State • 2013



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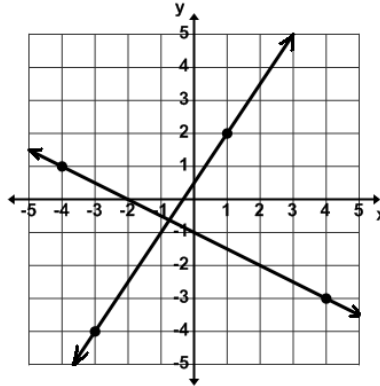
1. $(42 + 30) \div 12 - (18 - 6) \times 12 \div (-18) + (30 - 48)$

- (A) -14 (B) $-10.333\dots$ (C) -6 (D) -4 (E) $-0.777\dots$

2. Robin Banks teaches an accounting class. She has 30 students in the class. 22 students use scientific calculators. 11 use scientific calculators and graphing calculators. 5 don't use scientific or graphing calculators. How many students use just one of the two types of calculators?

- (A) 17 (B) 16 (C) 14 (D) 11 (E) 3

3. The two lines shown intersect at the point (x,y) . Find $x + y$.



- (A) -1.375 (B) -1.5 (C) -1.625 (D) -1.25 (E) -1.125

4. Simplify: $\left(\frac{3 + 4x - 4x^2}{9 - 4x^2}\right) \left(\frac{9 + 3x - 2x^2}{2x^2 - 5x - 3}\right)$

- (A) $x + 3$ (B) $\frac{3+x}{3-x}$ (C) -1 (D) $\frac{-12x}{5}$ (E) $3 - x$

5. If $2x + 3 = 7$ and $7 = 5y - 3$ then $2x + 3 = 5y - 3$ is an example of the _____ property of equality.

- (A) algebraic (B) transitive (C) identity (D) distributive (E) symmetric

6. Simplify: $a^{-2} \times b^{-1} \div a^{-3} \times b^3 \div a \times b^2$

- (A) $a^{-4}b^{-2}$ (B) ab^4 (C) $a^{-5}b^4$ (D) ab^{-6} (E) b^4

7. If y varies directly to $x + 2$, and $y = 14$ when $x = 3$, find x if $y = 21$.

- (A) 1.5 (B) 3 (C) 3.5 (D) 5.5 (E) 7

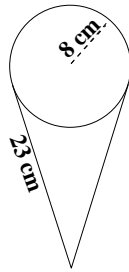
8. If $\frac{7}{4x} - \frac{5y}{6} = \frac{8}{3x}$, then y^{-1} equals _____.

- (A) $-\frac{11}{10x}$ (B) $-4.8x$ (C) $-6\frac{6}{55x}$ (D) $-2.1x$ (E) $-\frac{10x}{11}$

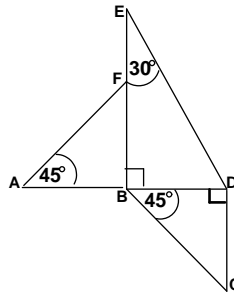
9. If $a_1 = -2$, $a_2 = -1$, $a_3 = 1$ and $a_n = (a_{n-3})^{(a_{n-2})} - (a_{n-1})$ for $n \geq 4$, then a_6 equals:

- (A) $1\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) $-\frac{2}{3}$

10. Given the radius shown find the volume of the right circular cone. (nearest cu. cm.)



- (A) $1,734 \text{ cm}^3$ (B) $1,541 \text{ cm}^3$ (C) $1,445 \text{ cm}^3$ (D) $1,156 \text{ cm}^3$ (E) $1,084 \text{ cm}^3$
11. A rectangular prism water tank is 3 feet high, 2 feet wide, and 4 feet long. It is 75% full. How many gallons of water would it take to fill the tank to the top? (nearest gallon)
- (A) 18 gal (B) 22 gal (C) 36 gal (D) 45 gal (E) 67
12. Let $f(x) = 6x - 5$ and $g(x) = 4x - 3$ and $h(x) = 2x - 1$. Find the constant term of $h(f(g(x + 1)))$.
- (A) -9 (B) -7 (C) -3 (D) -1 (E) 1
13. Find the perimeter of the object shown if $EB = 6\sqrt{3}$ ", and $AF = 4\sqrt{2}$ ". (nearest half inch).



- (A) 2 ft 11 in (B) 3 ft 3.5 in (C) 3 ft 5.5 in (D) 3 ft 6.5 in (E) 4 ft 1.5 in
14. The incenter, circumcenter, orthocenter, and centroid of a triangle are collinear only if the triangle is a(n) _____ triangle.
- (A) isosceles (B) equilateral (C) scalene right (D) scalene obtuse (E) scalene acute
15. Let $f(x) = \frac{3 + 4x}{4x - 3}$. Find $f^{-1}(\frac{1}{2})$.
- (A) -0.375 (B) -2.25 (C) -5 (D) 3.5 (E) 2.666...
16. Two numbers are in the ratio 2:3. If the smaller number is increased by 8 and the larger number is decreased by 4, the resulting numbers are in the ratio 10:7. Find the product of the numbers.
- (A) 120 (B) 132 (C) 192 (D) 216 (E) 252

17. Captain I. C. Delite looks at his navigation chart and determines that lighthouse A is 60 miles from lighthouse B and the bearing of B from A is 100° . He calculates the bearing of lighthouse A from the ship to be 50° and the bearing of lighthouse B from the ship to be 75° . How much further is the ship from lighthouse B than from lighthouse A? (nearest tenth)
- (A) 42.9 mi (B) 48.8 mi (C) 55.0 mi (D) 71.2 mi (E) 77.1 mi
18. Simplify: $\frac{\cos(\theta) - \csc(\theta)}{\sin(\theta) - \sec(\theta)}$
- (A) $\tan(\theta)$ (B) $\sin(2\theta)$ (C) -1 (D) $\sin(\theta)\cos(\theta)$ (E) $\cot(\theta)$
19. Given the polynomial, $8x^6 - 3x^5 - 2x^3 + x^2 - 3x + 8$, how many negative real zeros are possible?
- (A) 4, 2, or 0 (B) 3 or 1 (C) 5, 3, or 1 (D) 2 (E) 0
20. Which of the following functions is neither an even nor an odd function? $f(x) =$
- (A) $2|x|$ (B) $\frac{x^3 - x}{x^5 + x}$ (C) $4x^4 + 2x^2 + x$ (D) $\cos(x)$ (E) $\sqrt{3 - x^2}$
21. If $f'(x) = 6(x^2 - x - 1)$ and $f(1) = -16$, find $f(-1)$.
- (A) -14 (B) -10 (C) -9 (D) -8 (E) -6
22. The series $\frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots$ converges to _____.
- (A) ϕ (B) $\frac{\pi^2}{15}$ (C) $\ln(2)$ (D) ∞ (E) e
23. Let $f(x) = \frac{x^2 - 2x + 1}{x^2 + 2x + 1}$. Find $f'(3)$.
- (A) $\frac{1}{16}$ (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $\frac{3}{16}$ (E) $\frac{3}{8}$
24. Betty Luzes puts nine face cards, four aces, two jokers, and a blank card in a container. She shakes the container then randomly selects three cards placing them face up in a row in the order they were drawn. What is the probability that all three cards are face cards? (nearest %)
- (A) 15% (B) 56% (C) 19% (D) 53% (E) 12%
25. The probability of selecting a prime number from a set containing 2-digit numbers is 16%. What are the odds of selecting a composite number from the set?
- (A) 4 to 25 (B) 21 to 25 (C) 4 to 21 (D) 25 to 21 (E) 21 to 4
26. Using Pascal's triangle, determine which of the following is true about all of the numbers greater than P in row P, where P is a prime number greater than 3? All are _____.
- (A) even (B) odd (C) prime (D) divisible by P (E) triangular numbers

27. A special abacus using an index rod and a set of ten rods corresponding to the digits 0 to 9 was developed for calculating products and quotients. This special abacus is based on lattice multiplication and is known as _____.

- (A) Napier's Bones (B) Eratosthenes' sieve (C) Venn's diagrams
 (D) Descartes' Cartesian System (E) Archimedes' spiral

28. Let $Y = X + X + 1 + X + 2 + \dots + X + N$, where X and N are positive integers. Y is considered to be a _____ number.

- (A) complex (B) Lucas (C) polite (D) lucky (E) Fibonacci

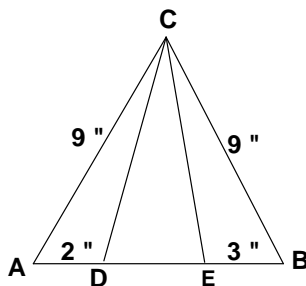
29. The *Lookin Gud* dress shop sold two special prom dresses for \$75.00 each. One dress was sold for a 25% profit and the other sold for a 20% loss. What was the shop's net gain or loss for the sale of the two dresses?

- (A) gained 7.5¢ (B) gained \$1.58 (C) broke even (D) lost \$3.75 (E) lost \$7.50

30. The average age of the contestants and coaches at the state math contest is 25. If the average age of the contestants is 17 and the average age of the coaches is 39, what is the ratio of coaches to students?

- (A) 4:11 (B) 4:3 (C) 7:11 (D) 3:4 (E) 4:7

31. $\triangle ABC$ is an equilateral triangle. Find the perimeter of $\triangle CDE$. (nearest inch)



- (A) 18 " (B) 19 " (C) 20 " (D) 21 " (E) 22 "

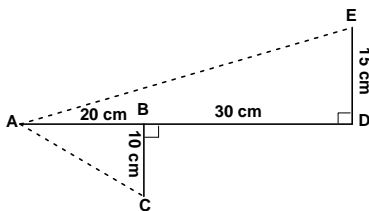
32. Saul T. Bryne has 75 grams of a 4% salt solution. How many grams of salt would he have to add to change his solution to a 10% salt solution?

- (A) 3 grams (B) 3.5 grams (C) 5 grams (D) 6 grams (E) 7.5 grams

33. Which of the following equations will have a graph with an amplitude of 3, a frequency of 3, a phase shift of 3, and a displacement of -3 ?

- (A) $y = 3 - 3\sin(6\pi x - 3)$ (B) $y = 3\sin(\frac{3\pi}{2} - 18) - 3$ (C) $y = 3\sin(6\pi x + 18) + 3$
 (D) $y = 3\sin(3\pi x - 3) - 3$ (E) $y = 3\sin(6\pi x - 18) - 3$

34. Find $m\angle CAE$. (nearest degree).



- (A) 29° (B) 30° (C) 43° (D) 47° (E) 53°

35. Three workers can wash 5 PT Cruisers in 2 hours 15 minutes. If 5 more workers are hired how long would it take the 8 workers to wash 30 PT Cruisers if they all work at the same rate as the original 3 workers? (nearest minute)

- (A) 5 hrs 4 min (B) 5 hrs 21 min (C) 5 hrs 38 min (D) 6 hrs (E) 6 hrs 6 min

36. Let $f(x) = \begin{cases} 2 & \text{if } x < 1 \\ -1 & \text{if } x = 1 \\ -3 & \text{if } 1 < x. \end{cases}$ Which of the following is/are true?

1. $\lim_{x \rightarrow 1^+} f(x) = -3$ 2. $\lim_{x \rightarrow 1^-} f(x) = 2$ 3. $\lim_{x \rightarrow 1} f(x)$ does not exist

- (A) none of these (B) 1 & 2 (C) 3 only (D) 1 & 3 (E) 1, 2, & 3

37. The *I. C. Delite* packaging company has 40-watt bulbs, 60-watt bulbs, 75-watt bulbs and 100-watt bulbs. In how many ways can they package a 12-pack of bulbs if each 12-pack must contain at least one of each size bulb?

- (A) 70 (B) 140 (C) 165 (D) 275 (E) 495

38. The enrollment at Millersview Tech consists of 12 seniors, 11 juniors, 10 sophomores, and 9 freshmen. How many ways can Master Wyte form a Super Nerd committee consisting of 4 seniors, 3 juniors, 2 sophomores and 1 freshman?

- (A) 33,078,375 (B) 420 (C) 1,471,442,973 (D) 12,969 (E) 714

39. Find the harmonic mean of the real roots of $x^3 - 16x^2 + 73x - 90 = 0$. (nearest tenth)

- (A) 6.1 (B) 5.3 (C) 4.5 (D) 4.0 (E) 3.7

40. If the roots of $x^3 + bx^2 + cx + d = 0$ are -2 , 3 , and 5 , then $c - b - d$ equals:

- (A) -25 (B) -23 (C) 0 (D) 35 (E) 37

41. A regular octahedron has F faces, E edges, and V vertices. Find $F + E + V$.

- (A) 14 (B) 24 (C) 26 (D) 32 (E) 36

42. The Real value solution set for $|6x + 2| > 4$ is?
- (A) $\{x \mid -\frac{1}{3} < x < 1\}$ (B) $\{x \mid \{x < -1\} \cup \{x > \frac{1}{3}\}\}$ (C) $\{x \mid \{x > \frac{1}{3}\} \cup \{x < 1\}\}$
(D) $\{x \mid -1 < x < \frac{1}{3}\}$ (E) $\{x \mid \{x < -\frac{1}{3}\} \cup \{x > 1\}\}$
43. If $8^{(x+2y)} = 64$ and $9^{(2x-y)} = 81$ then $x + y$ equals _____.
- (A) 1.6 (B) 0.48 (C) 2.0 (D) 2.4 (E) 0.8
44. $(2 + 3i)^2 - (3 + 2i)^2 = a + bi$. Find $(a + b)^2$.
- (A) 100 (B) 121 (C) 125 (D) 144 (E) 169
45. Find the sum of the first three terms of the arithmetic sequence $x^2, 2x + 5, x, \dots$, where $x > 0$.
- (A) 45 (B) 40 (C) 30 (D) 25 (E) 15
46. Use the Fibonacci characteristic sequence ... $-4, p, q, r, 13 \dots$ to Find $p + q + r$.
- (A) 21 (B) 20 (C) 19 (D) 18 (E) 17
47. Find the surface area of an icosahedron with an edge length of 5". (nearest sq. in)
- (A) 87 sq. in (B) 130 sq. in (C) 173 sq. in (D) 217 sq. in (E) 344 sq. in
48. $11_{16} \times (ABC_{16} - 321_{16}) = \underline{\hspace{2cm}}_{16}$
- (A) 8811 (B) 7F4B (C) 814B (D) 8181 (E) 884B
49. Determine the number of non-negative integer solutions to the equation: $p + q + r = 9$.
- (A) 27 (B) 36 (C) 45 (D) 55 (E) 66
50. $F(x) = 2x^2 - x^4$ has an inflection point at (x, y) . Find y .
- (A) $\frac{\sqrt{3}}{3}$ (B) 1 (C) 0 (D) $\frac{5}{9}$ (E) $\sqrt{3}$
51. Find the area of the region bounded between the graphs of $y = 0$ and $y = x(x^2 - 1)$ for $-1 \leq x \leq 0$, and between the graphs of $y = 0$ and $y = x(x^2 - 1)$ for $0 \leq x \leq 1$.
- (A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{8}$ (D) $\frac{1}{4}$ (E) 0
52. The polar graph of $r = 2\cos(2\theta)$ is symmetric to: (1) polar axis (2) pole (3) line $\theta = \frac{\pi}{2}$
- (A) 1 only (B) 2 only (C) 3 only (D) 2 & 3 (E) 1, 2, & 3

53. If R, S, and T represent digits then $TRS_3 - STR_4 + RST_5$ has a numeric value in base 10 of:

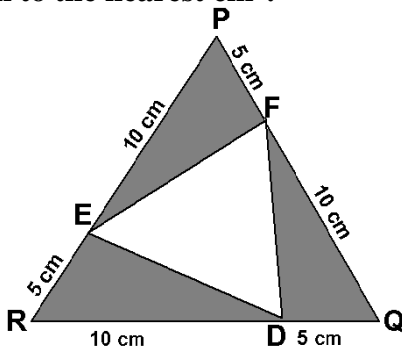
- (A) $27R - 10S + 6T$ (B) $3T - 12R + 2S$ (C) $25R - 4T + 5S$
 (D) $4R + 4S + 4T$ (E) $29R - 10S + 14T$

↔

54. \overline{AB} is tangent to a semicircle at point C. Points P, Q, and R lie on the semicircle such that \overline{CP} is the diameter, $m\angle PCQ = 33^\circ$, $m\angle QCR = 35^\circ$. Find \widehat{CR} .

- (A) 11° (B) 22° (C) 32° (D) 38° (E) 44°

55. Find the area of the shaded region to the nearest cm^2 .

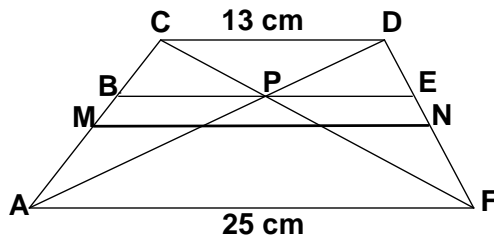


- (A) 96 cm^2 (B) 84 cm^2 (C) 75 cm^2 (D) 65 cm^2 (E) 32 cm^2

56. Let $A = \begin{bmatrix} 5 & -2 \\ 6 & 3 \end{bmatrix}$. Find $\det(A + A^T)$.

- (A) 6 (B) 9 (C) 27 (D) 44 (E) 54

57. Drew Tumeens sketched the trapezoid shown where segments AF, MN, BE, and CD are parallel to each other, segments AD, CF, and BP are concurrent at point P, and M and N are midpoints of segments AC and DF, respectively. Find $MN - BE$. (nearest tenth)

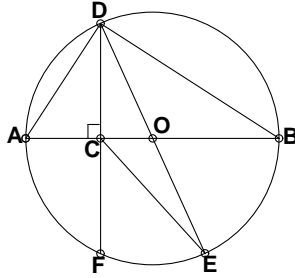


- (A) 1.3 cm (B) 1.5 (C) 1.7 cm (D) 1.9 cm (E) 2.1 cm

58. The curve $y = 4x^3 - 13x^2 + 4x - 3$ has two horizontal tangents. Find the shortest distance between the two horizontal tangents.

- (A) $12\frac{35}{108}$ (B) 13 (C) $14\frac{145}{216}$ (D) 15 (E) $17\frac{73}{108}$

59. I. M. Daboss drew a circle with center O with $BC = 2AC$, $CD = 8\sqrt{2}$ cm, and $AC = 8$ cm. Based on the information and the drawing what is the area of $\triangle CEO$? (nearest tenth)



- (A) 11.3 sq. cm (B) 22.6 sq. cm (C) 32.0 sq. cm (D) 33.9 sq. cm (E) 45.3 sq. cm
60. The coordinates of the vertices of $\triangle DEF$ are $(-1, 1)$, $(2, -2)$, and $(3, 3)$. The coordinates of the incenter is (x, y) . Find $x + y$. (nearest tenth)
- (A) 2.5 (B) 2.0 (C) 1.8 (D) 1.5 (E) 1.2

**University Interscholastic League
MATHEMATICS CONTEST
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Answer Key**

- | | | |
|-------|-------|-------|
| 1. D | 21. D | 41. C |
| 2. C | 22. C | 42. B |
| 3. A | 23. B | 43. A |
| 4. C | 24. A | 44. A |
| 5. B | 25. E | 45. A |
| 6. E | 26. D | 46. B |
| 7. D | 27. A | 47. D |
| 8. E | 28. C | 48. C |
| 9. B | 29. D | 49. D |
| 10. C | 30. E | 50. D |
| 11. D | 31. C | 51. B |
| 12. E | 32. C | 52. E |
| 13. D | 33. E | 53. A |
| 14. A | 34. C | 54. E |
| 15. B | 35. A | 55. D |
| 16. D | 36. E | 56. D |
| 17. B | 37. C | 57. D |
| 18. E | 38. A | 58. A |
| 19. E | 39. E | 59. B |
| 20. C | 40. A | 60. C |