



UNIVERSITY INTERSCHOLASTIC LEAGUE
Making a World of Difference

Mathematics

Regional • 2015



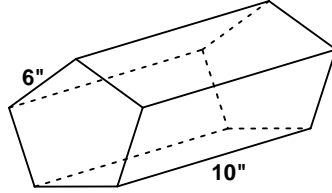
DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. $3! \div (6)^{-1} - 9 \div 3 \times 6 + (9)^{\frac{1}{2}}$
- (A) -17 (B) -15.5 (C) 18 (D) 21 (E) 57
2. Les Sense, Noah Moolah, and Ima Spender went to the movies. Ima bought popcorn and a soda. Les bought a candy bar and a soda. Noah ate some of Ima's popcorn, part of Les' candy bar, and bought his own soda. If movie tickets cost \$6.50 each, popcorn cost \$3.00, candy bars cost \$1.75 and sodas are \$3.50 each, determine the total amount they spent all together?
- (A) \$44.25 (B) \$38.50 (C) \$34.75 (D) \$32.25 (E) \$21.75
3. Let $R = \{r, e, g, i, o, n, a, l\}$, $M = \{m, a, t, h\}$, and $C = \{c, h, a, m, p, i, o, n\}$. How many elements are in $(R \cap C) \cup (M \cap C)$?
- (A) 4 (B) 5 (C) 6 (D) 7 (E) 8
4. $32_6 - 32_8 + 201_5 = \underline{\hspace{2cm}}_7$
- (A) 63 (B) 47 (C) 54 (D) 102 (E) 65
5. Which of the following mathematicians showed that the set of rational numbers is countable and that the infinity of rational numbers is the same size as the infinity of natural numbers?
- (A) Christian Goldbach (B) Georg Cantor (C) George Boole
(D) John Napier (E) Alicia Stott
6. Simplify: $\left(\frac{15 - 13x + 2x^2}{4x^2 - 9}\right) \left(\frac{2x + 1}{1 - 2x}\right) \div \left(\frac{5 - x}{2x - 1}\right)$
- (A) $2x^2 + 8x + 3$ (B) $\frac{2x - 1}{x + 1}$ (C) $2x^2 - 3$ (D) $\frac{x - 1}{2x + 3}$ (E) $\frac{2x + 1}{2x + 3}$
7. If $x - y = -3$ and $xy = -5$ then $x^3 - y^3 = ?$
- (A) 98 (B) 18 (C) 8 (D) -50 (E) -72
8. Which of the following properties is demonstrated from step 5 to step 6?
- | | | |
|--------|---------------------------|--------------------|
| Step 1 | $3x + 4$ | $= 10$ |
| 2 | $(3x + 4) + (-4)$ | $= 10 + (-4)$ |
| 3 | $3x + [4 + (-4)]$ | $= 10 + (-4)$ |
| 4 | $3x + 0$ | $= 10 + (-4)$ |
| 5 | $3x + 0$ | $= 6$ |
| 6 | $3x$ | $= 6$ |
| 7 | $\frac{1}{3}(3x)$ | $= \frac{1}{3}(6)$ |
| 8 | $(\frac{1}{3} \times 3)x$ | $= \frac{1}{3}(6)$ |
| 9 | $(1) \times x$ | $= 2$ |
| 10 | x | $= 2$ |
- (A) reflexive (B) additive inverse (C) associative (D) commutative (E) additive identity

9. How many 8-letter code words can be formed using the letters in the word ABCISSA?

- (A) 10,080 (B) 336 (C) 6,720 (D) 20,160 (E) 3,360

10. Find the lateral surface area of the regular pentagonal prism shown. (nearest sq. in).
Drawing is not to scale.



- (A) 124 sq. in (B) 300 sq. in (C) 308 sq. in (D) 420 sq. in (E) 424 sq. in

11. The ratio of the length to the width of a rectangle is 5:3. If 2 units are subtracted from the width and 2 units are added to the length, then the ratio of the width to the length is now 1:3. What is the difference in the areas of the two rectangles? (square units)

- (A) 12 (B) 28 (C) 4 (D) 30 (E) 26

12. $\triangle PQR$ is inscribed in circle C such that the measure of $\angle PRQ$'s intercepted arc is 70° and $m\angle PQR = 50^\circ$. Find the measure of $\angle QPR$'s intercepted arc.

- (A) 190° (B) 170° (C) 120° (D) 100° (E) 70°

13. Joy Ryder hops on a freight train that leaves the station at 9:00 a.m. The train enters a tunnel at 9:45 a.m. at a constant speed of 40 mph. The end of the train exits the tunnel at 9:50 a.m. Find the length of the train if the length of the tunnel is 2 miles.

- (A) $\frac{2}{3}$ miles (B) $\frac{3}{40}$ miles (C) $1\frac{1}{4}$ miles (D) $1\frac{1}{3}$ miles (E) $1\frac{1}{2}$ miles

14. If $\frac{2x-5}{3x+4} - \frac{3x+4}{x-6} = \frac{Ax^2+Bx+C}{Px^2+Qx+R}$, then $\frac{A+B+C}{P+Q+R}$ equals:

- (A) $1\frac{11}{41}$ (B) $1\frac{27}{35}$ (C) $1\frac{23}{41}$ (D) $1\frac{1}{34}$ (E) $\frac{34}{35}$

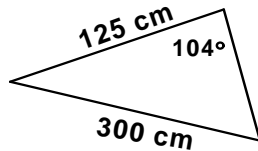
15. Let $A = \begin{bmatrix} 0 & 3 \\ -2 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 1 & -15 \end{bmatrix}$. Find $|A - B^T|$.

- (A) -42 (B) -13 (C) 4 (D) 80 (E) 81

16. Find $m - n$ if $\begin{bmatrix} 3 & -2 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 8 \\ 15 \end{bmatrix}$

- (A) $1\frac{2}{7}$ (B) $1\frac{3}{4}$ (C) $2\frac{7}{8}$ (D) $3\frac{1}{7}$ (E) $4\frac{1}{7}$

17. Find the area of the triangle shown (nearest cm^2).



- (A) $18,193 \text{ cm}^2$ (B) $17,149 \text{ cm}^2$ (C) $15,132 \text{ cm}^2$ (D) $14,806 \text{ cm}^2$ (E) $11,543 \text{ cm}^2$

18. Determine the range of $f(x) = -3\cos(2\pi x + 4\pi) - 1$.

- (A) $[-4, 2]$ (B) $[-2, 4]$ (C) $[-\frac{2\pi}{3}, \frac{4\pi}{3}]$ (D) $[-\frac{3\pi}{2}, \frac{3\pi}{4}]$ (E) $[-1, 1]$

19. The directrix of the parabola $y = 1.25 - 1.5x - 0.25x^2$ is:

- (A) $y = 2$ (B) $y = 2.25$ (C) $y = 3.5$ (D) $y = 4.25$ (E) $y = 4.5$

20. Which of the following is not an even function?

- (A) $y = \cos x$ (B) $y = x^2$ (C) $y = -5$ (D) $y = -1 + 2x^5$ (E) $y = -2x^4$

21. If $f''(x) = 36x + 50$ and $f'(-1) = -30$ and $f(-1) = 9$, then $f(-2) = \underline{\hspace{2cm}}$.

- (A) 144 (B) -26 (C) -21 (D) 25 (E) 40

22. A right triangle has a hypotenuse of length 26". If one of the acute angles is decreasing at the rate of 10° per second, how fast is the area of the triangle decreasing when this acute angle is 13° ? (nearest tenth)

- (A) $58.1 \text{ in}^2/\text{sec}$ (B) $57.4 \text{ in}^2/\text{sec}$ (C) $55.4 \text{ in}^2/\text{sec}$ (D) $53.0 \text{ in}^2/\text{sec}$ (E) $51.7 \text{ in}^2/\text{sec}$

23. The y-intercept of the line that is tangent to $y = 4x^2 - 4x + 1$ at $x = 1$ is (x, y) . Find $x + y$.

- (A) 4 (B) 1 (C) $\frac{1}{4}$ (D) $-\frac{1}{4}$ (E) -3

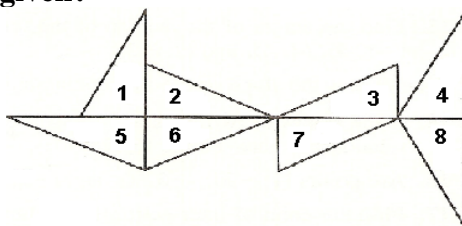
24. Roland Bones created a pair of special dice which have only three numbers on each die. The opposite side of each number is the same number. When the dice are rolled the die with the largest number on top wins. What is the probability that die B will win?



- (A) $55\frac{5}{9}\%$ (B) $44\frac{4}{9}\%$ (C) $66\frac{2}{3}\%$ (D) $33\frac{1}{3}\%$ (E) 20%

25. Betty Luzes is playing the *Pic-A-Pie Digit* game. The game has 12 tiles, each tile containing one of the first 11 digits of pi and a decimal point. She puts the tiles in a bag, shakes them up and randomly selects one tile. If the tile contains a prime digit she gets 5 points. If it is a composite digit she loses 10 points. If it is a 1 or a decimal point she gets 25 points. What is the mathematical expectation of a single game? (nearest tenth)
- (A) $3\frac{2}{11}$ points (B) $4\frac{3}{4}$ points (C) $6\frac{1}{4}$ points (D) $7\frac{1}{2}$ points (E) $8\frac{9}{11}$ points
26. How many 3-digit numbers exist such that the sum of their digits equals 12?
- (A) 36 (B) 43 (C) 66 (D) 78 (E) 91
27. How many proper fractions in lowest terms have a denominator of 54?
- (A) 27 (B) 24 (C) 18 (D) 15 (E) 9
28. Simplify: $(a^{-3} \times b^{-2})^{-6} \div (a^2 \times b^8)^2 \times (a^0 \div b^1)^{-5}$
- (A) $a^{14}b$ (B) $a^{-18}b^{-13}$ (C) $a^{-10}b^7$ (D) $a^{-10}b^{-13}$ (E) $a^{14}b^7$
29. Let k be a positive integer less than or equal to 120 such that k is a multiple of 3 and k is divisible by 4. Find the sum of all such numbers k .
- (A) 792 (B) 660 (C) 540 (D) 600 (E) 936
30. The sum of the first 10 deficient numbers is a(n) _____ number.
- (A) deficient (B) prime (C) abundant (D) harmonic (E) lucky
31. How many pounds of lima beans that cost 90¢ per pound must be mixed with 16 pounds of corn that cost 50¢ per pound to make a mixture of mixed vegetables that cost 65¢ a pound?
- (A) 6.4 lbs (B) 2.6 lbs (C) 9.6 lbs (D) 18.4 lbs (E) 12.8 lbs
32. Tu Yung is two years younger than her sister, Soh Yung. Twelve years ago Soh was twice as old as Tu. What will the sum of their ages be in five years?
- (A) 30 (B) 54 (C) 35 (D) 28 (E) 40
33. Which of the following polar equations has a graph of an inner-loop limaçon?
- (A) $r = 3 + 2\cos(\theta)$ (B) $r = 1 + \sin(\theta)$ (C) $r = 5 + 2\cos(\theta)$
(D) $r = 2 + \cos(\theta)$ (E) $r = 2 + 3\cos(\theta)$
34. If $a_1 = -4$, $a_2 = -1$, $a_3 = 2$, $a_n = (a_{n-2}) \times (a_{n-3}) - (a_{n-1})$, where $n \geq 4$, then a_6 equals:
- (A) -16 (B) -7 (C) 5 (D) 8 (E) 11

35. Mary Goround maps triangle 4 to triangle 5 by using which of the groups of three or four transformations in the order given?



(w) half-turn (x) reflection (y) rotation (z) translation

- (A) w, x, x, & z (B) z, x, & x (C) w, x, y, & z (D) w, y, & z (E) z, w, & x
36. The ratio of the measure of an interior angle of a regular n -gon to the measure of its exterior angle is 3.5 to 1. How many sides does the regular n -gon have?

(A) 5 (B) 7 (C) 9 (D) 11 (E) 14

37. Which of the following is not a member of the solution set for $3|2x - 8| - 20 > 15$?

(A) $-2\frac{1}{3}$ (B) $-1\frac{7}{8}$ (C) 2.666... (D) $10\frac{2}{3}$ (E) $11\frac{1}{11}$

38. The graph of the parametric equations $x = 5\cos(t)$ and $y = 2\sin(t)$, where $0 \leq t \leq 2\pi$ is a(n) ____.

(A) semicircle (B) ellipse (C) line (D) cycloid (E) parabola

39. $(\sqrt{3} + i)^5$ equals:

(A) $16 - 16\sqrt{3}i$ (B) $5\sqrt{3} + 5i$ (C) $-9\sqrt{3} + i$ (D) $-16\sqrt{3} + 16i$ (E) $-32\sqrt{3} + 32i$

40. Given that the set of natural numbers continue in the triangular pattern shown below, find the sum of the 6th number in row 7 and the 9th number in row 10.

| | | | | | | | | | |
|--|----|----|-----|----|----|----|----|--|---------|
| | | | 1 | | | | | | (row 1) |
| | | | 2 | 3 | 4 | | | | (row 2) |
| | | 5 | 6 | 7 | 8 | 9 | | | (row 3) |
| | 10 | 11 | 12 | 13 | 14 | 15 | 16 | | (row 4) |
| | | | ... | | | | | | (...) |

(A) 96 (B) 132 (C) 134 (D) 147 (E) 149

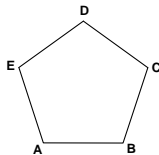
41. Given the geometric sequence 5, p , q , $\frac{5}{8}, \dots$, find the sum of the first 8 terms. (nearest hundredth)

(A) 9.96 (B) 9.92 (C) 9.87 (D) 9.625 (E) 9.375

42. Let $f(x) = \frac{2x-1}{3x+4}$. Find $f'(-5)$.

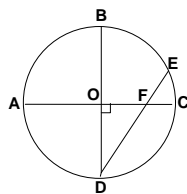
(A) $-\frac{5}{11}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{11}$ (D) $\frac{2}{3}$ (E) 1

43. Which of the following surfaces is generated by $x^2 - \frac{y^2}{4} - z^2 + 2z = 3$?
- (A) elliptic cone (B) elliptic hyperboloids (C) elliptic cylinder
 (D) hyperbolic paraboloid (E) elliptic paraboloid
44. Willie Pikette randomly selects a factor of 144. Betty Wheel selects a factor of 88. What is the probability that they selected the same number? (nearest tenth)
- (A) 17.4% (B) 3.3% (C) 13.3% (D) 6.1% (E) 21.1%
45. Dee Deeler shuffles a standard 52-card deck. She deals out the top two cards. What are the odds that both cards are face cards?
- (A) $\frac{22}{321}$ (B) $\frac{11}{210}$ (C) $\frac{11}{130}$ (D) $\frac{11}{221}$ (E) $\frac{33}{400}$
46. The number of integers between 1 and 328 that are relatively prime to 328 is ?
- (A) 159 (B) 109 (C) 82 (D) 279 (E) 164
47. Find the sum of the lengths of the diagonals if the lengths of the sides of the regular pentagon shown is 6". (nearest tenth)



- (A) 30.0" (B) 58.2" (C) 29.1" (D) 38.8" (E) 48.5"
48. Line m goes through the point $(1, -4)$ and $(-1, -10)$. Line n goes through points $(-11, 0)$ and $(-4, 1)$. Line m intersect line n at (x, y) . Find $x + y$. (nearest tenth)
- (A) 5 (B) 2 (C) 0 (D) -1 (E) -28
49. $\{(x, y) \mid x, y \in \{\text{Integers}\}, -8 \leq x \leq 14, \text{ and } -14 \leq y \leq 8\}$ is the solution set of $5x + 3y = 28$. How many such ordered pairs exist?
- (A) 22 (B) 16 (C) 8 (D) 5 (E) 3
50. How many integral values of n exist such that $n > 3$ and $\frac{(n-1)!}{(n-3)!} \leq 182$
- (A) 20 (B) 16 (C) 15 (D) 12 (E) 10
51. Lynn Kaln has some pennies. Penny Lesse has 4 times as many dimes as the number of pennies Lynn has. Nick Ohl has 16 less nickels than twice the number of dimes Penny has. How many nickels did Nick have if the total amount is they have \$8.92?
- (A) 104 (B) 76 (C) 80 (D) 60 (E) 36

52. Find the shortest distance from the point (3, 2) and the line $5x + 12y = 13$.
- (A) 2 (B) $2\sqrt{3}$ (C) $3\sqrt{2}$ (D) 5 (E) $4\sqrt{2}$
53. Sir Vayer used his theodolite to find the angle measures of two points at the other end of a field. Point A was on a bearing of 290° and point B was on a bearing of 45° from where he stood. He was 150 yards from point A and 120 yards from point B. What was the distance from Point A to point B? (nearest yard)
- (A) 240 yds (B) 228 yds (C) 222 yds (D) 157 yds (E) 151 yds
54. Phil Upp, Doug Upp, and Stan Upp are filling up a large hole. Phil can do the job by himself in 4 hours, Doug in 6 hours, and Stan in 8 hours. How long would it take them working together? (nearest minute)
- (A) 2 hrs (B) 1 hr 51 min (C) 1 hr 40 min (D) 1 hr 33 min (E) 1 hr 18 min
55. Find C if the remainder of $x^3 - 3x^2 - 10x + C$ divided by $x - 4$ is 3.
- (A) 75 (B) 21 (C) 27 (D) 53 (E) 34
56. Let $f(x) = \frac{1}{x^2 + 1}$. The concavity of the curve is upward at which of the following values of x?
- I. $-\frac{2}{3}$ II. 0 III. $\frac{3}{4}$
- (A) II only (B) I, II, & III (C) I & II (D) I & III (E) II & III
57. Cookie Baykur packages cookies 3 to a pack. The types of cookies she can choose from include chocolate chip, oatmeal, sugar-coated, sugar-free, peanut butter, and hazel-nut. How many different packs of 3 cookies can she package?
- (A) 20 (B) 120 (C) 28 (D) 60 (E) 56
58. $0.5323232\dots$ in base 7 can be written as which of the following fractions in base 10?
- (A) $\frac{131}{165}$ (B) $\frac{268}{343}$ (C) $\frac{263}{336}$ (D) $\frac{49}{76}$ (E) $\frac{524}{660}$
59. Let $f_0 = 0, f_1 = 1, f_2 = 1, f_3 = 2, f_4 = 3, \dots$ be the terms of the Fibonacci sequence. Find $(f_6)^2 + (f_7)^2$.
- (A) f_{26} (B) f_{14} (C) f_{13} (D) f_{12} (E) f_8
60. Given the circle O with perpendicular diameters and a chord, find the area of the circle if $EF = 8''$ and $DE = 20''$ inches. (nearest tenth)



- (A) 503 sq. in (B) 377 sq. in (C) 323 sq. in (D) 176 sq. in (E) 151 sq. in

**University Interscholastic League
MATHEMATICS CONTEST
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Answer Key**

- | | | |
|-------|-------|-------|
| 1. D | 21. E | 41. A |
| 2. C | 22. D | 42. C |
| 3. C | 23. E | 43. B |
| 4. A | 24. A | 44. B |
| 5. B | 25. C | 45. B |
| 6. E | 26. C | 46. ■ |
| 7. B | 27. C | 47. E |
| 8. E | 28. A | 48. A |
| 9. E | 29. B | 49. D |
| 10. B | 30. C | 50. D |
| 11. A | 31. C | 51. C |
| 12. A | 32. E | 52. A |
| 13. D | 33. E | 53. B |
| 14. E | 34. D | 54. B |
| 15. A | 35. D | 55. C |
| 16. A | 36. C | 56. D |
| 17. D | 37. C | 57. E |
| 18. A | 38. B | 58. C |
| 19. E | 39. D | 59. C |
| 20. D | 40. B | 60. B |