



UNIVERSITY INTERSCHOLASTIC LEAGUE
Making a World of Difference

Mathematics

Region • 2014



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YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $128 \div (64)^{\frac{1}{3}} + 16 \times (8)^{-1} - 4! \times 2^0$

- (A) 34 (B) 18 (C) 10 (D) -6 (E) -18

2. If P is 125% more than Q and R is $\frac{3}{4}$ of Q, then R is what percent of P?

- (A) 25% (B) $33\frac{1}{3}\%$ (C) $44\frac{1}{4}\%$ (D) 50% (E) 60%

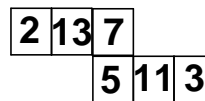
3. Let $F = \{f,r,a,c,t,i,o,n\}$, $R = \{r,a,t,i,o\}$, $P = \{p,a,r,t,o,f\}$, and $W = \{w,h,o,l,e\}$. The number of distinct elements in $(F \cup W) \cap (R \cup P)$ is _____.

- (A) 6 (B) 1 (C) 4 (D) 7 (E) 13

4. Bill Meelator went shopping for a new smart phone. The regular price of the phone he wanted was \$172.80. A sale was going on, so he got one-fourth off of the regular price. When he checked out he was given 10% off of the sale price for being a good customer. How much money did Bill save because of the two discounts? (tax not included)

- (A) \$35.00 (B) \$47.52 (C) \$54.86 (D) \$56.16 (E) \$60.38

5. The net shown is used to fold into a cube. Find the sum of each face and its opposite face. What is the difference between the largest sum and the smallest sum?

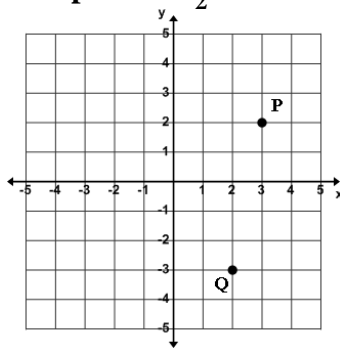


- (A) 17 (B) 16 (C) 15 (D) 9 (E) 7

6. Simplify: $\frac{4x^2 - 4x - 3}{4x^2 + 12x + 9} \div \frac{2x - 3}{8x + 12} \times \frac{4x^2 - 9}{8x^2 - 12x - 8}$

- (A) $2x - 3$ (B) $\frac{x-2}{2x+3}$ (C) $2x^2 - 7x + 6$ (D) $\frac{2x-3}{x-2}$ (E) $2x^2 - x - 6$

7. The coordinates of points P and Q have integral values. A line through point P has a slope of $\frac{2}{3}$. A line through point Q has a slope of $-\frac{3}{2}$. The lines intersect at the point (x, y). Find x + y.



- (A) -5 (B) $-2\frac{1}{6}$ (C) 0 (D) $\frac{5}{6}$ (E) 2

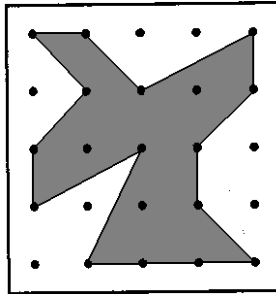
8. Find $f(-6) + f(-4) + f(-2)$ if $f(x) = \begin{cases} 3x + 2 & \text{if } x \leq -5 \\ 4x - 1 & \text{if } -5 < x < -1 \\ 5x & \text{if } x \geq -1 \end{cases}$

- (A) -12 (B) -27 (C) -34 (D) -42 (E) -43

9. $\angle P$ and $\angle R$ are complementary. $\angle P$ and $\angle Q$ are supplementary. If $m\angle Q = 114^\circ$ then $m\angle R = ?$

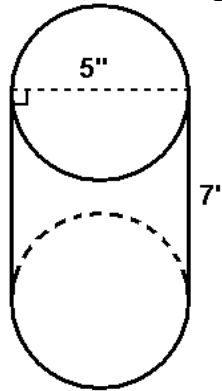
- (A) 44° (B) 66° (C) 34° (D) 56° (E) 24°

10. Dottie Pick determined that the adjacent dots on the grid are 1 cm apart when measured vertically and horizontally. She computed the area of the shaded figure shown to be ____.



- (A) 11.5 cm^2 (B) 10.5 cm^2 (C) 10 cm^2 (D) 9.5 cm^2 (E) 9 cm^2

11. Find the total surface area of the figure shown. (nearest sq. in).



- (A) 149 sq. in (B) 175 sq. in (C) 110 sq. in (D) 94 sq. in (E) 165 sq. in

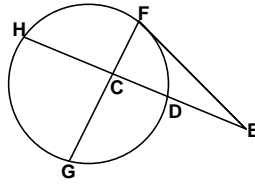
12. A two-digit number exists such that the square of the number is equal to four times the sum of the number and twenty-four. The sum of digits of the two-digit number is:

- (A) 15 (B) 12 (C) 8 (D) 6 (E) 3

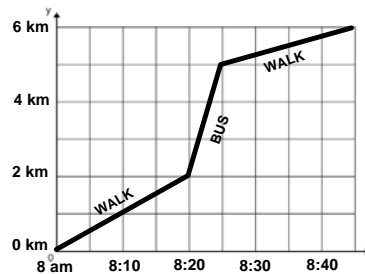
13. Willie Lawkette uses a 4 digit combination to chain his bicycle to the bicycle rack. The first digit has to be a positive even number, the second a prime number, the third a Fibonacci number, and the fourth a triangular number. How many unique sets of 4 combinations can Willie select from if digits can be repeated?

- (A) 625 (B) 375 (C) 300 (D) 256 (E) 240

14. Given the circle with center O, $m\widehat{DF} = 50^\circ$, $m\widehat{HF} = 90^\circ$, and $m\angle HCG = 85^\circ$. Find $m\angle EFG$.
(Drawing not to scale.)

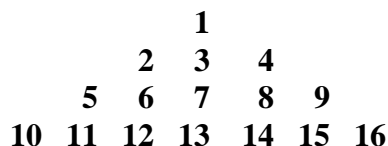


- (A) 70° (B) 75° (C) 80° (D) 85° (E) 90°
15. If $\frac{4x+3}{2x-1} - \frac{x+2}{3x-4} = \frac{Ax^2+Bx+C}{Px^2+Qx+R}$, then $\frac{ABC}{PQR}$ equals:
- (A) $-1.515151\dots$ (B) $-2.121212\dots$ (C) $-3.787878\dots$ (D) $-4.444\dots$ (E) $-5.232323\dots$
16. Ima Nutt sells peanuts for \$1.50 per pound and pecans for \$4.00 per pound. How many pounds of pecans will she need to mix with the 60 pounds of peanuts that will sell for \$2.50 per pound of mixed nuts?
- (A) 140 lbs (B) 100 lbs (C) 60 lbs (D) 40 lbs (E) 20 lbs
17. Willis Fast walks from his house to the bus stop, gets on the school bus, then gets off and walks the rest of the way to school. Using the graph below determine his average walking speeds.



- (A) 4 kmph (B) 4.5 kmph (C) 5 kmph (D) 6.7 kmph (E) 8 kmph
18. Simplify: $\frac{\cos x - \csc x}{\sin x - \sec x}$
- (A) $\sec x$ (B) $\csc x$ (C) $\tan x$ (D) $\cot x$ (E) 1
19. If $\sin B = -\frac{\sqrt{3}}{2}$, where $\frac{3\pi}{2} < B < 2\pi$, and $\cos A = -\frac{1}{2}$, where $\pi < A < \frac{3\pi}{2}$, then $\sin(A+B) - \sin(A-B) = ?$
- (A) $-\frac{1}{2}$ (B) $-\frac{\sqrt{3}}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{\sqrt{3}}{4}$ (E) $\frac{1}{2}$
20. Use the Fibonacci characteristic sequence ... L, -3, U, 5, C, A, S, ... to find A + S.
- (A) 21 (B) 31 (C) 43 (D) 49 (E) 83

21. If the set of numbers $\{1, 2, 3, 4, 5, \dots\}$ continue in the triangular pattern shown below, the arithmetic mean of the numbers in the 13th row would be?



- (A) 133 (B) 145 (C) 157 (D) 168 (E) 170
22. The sum of the coefficients of the 2nd term in the expansion of $(x + 1)^2$, the 3rd term of $(x + 1)^3$, the 5th term of $(x + 1)^5$, the 2nd term of $(x + 1)^7$ and 11th term of $(x + 1)^{11}$ is:
- (A) 31 (B) 28 (C) 25 (D) 23 (E) 16
23. Which of these mathematicians and his work was important to the development of digital computers?

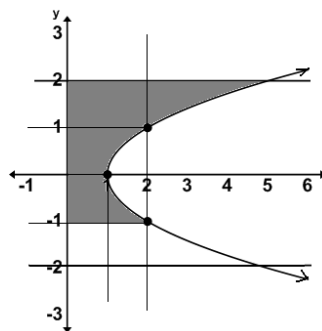
- (A) Eratosthenes (B) John Venn (C) Rene Descartes (D) Agnesi (E) Gottfried Leibniz

24. Which of the following series is/are convergent?

$$1. \sum_{n=1}^{+\infty} e^{-n} \quad 2. \sum_{n=1}^{+\infty} \left(\frac{2}{3}\right)^n \quad 3. \sum_{n=1}^{+\infty} \left(\frac{1}{2n} - \frac{1}{3n}\right)$$

- (A) 1 only (B) 2 only (C) 3 only (D) 1 & 2 (E) 2 & 3

25. Find the area of the shaded region in square units.




- (A) $6\frac{2}{3}$ (B) 6 (C) $5\frac{2}{3}$ (D) $5\frac{1}{3}$ (E) 5
26. Let $f(x) = \frac{5x+9}{5-9x}$. Find $f'(1)$.
- (A) $-11\frac{7}{9}$ (B) $-4\frac{5}{9}$ (C) $3\frac{1}{2}$ (D) $6\frac{5}{8}$ (E) $12\frac{1}{4}$
27. A bucket of balls at the Quicksand golf course driving range contains thirty balls. One-third of them are yellow, two-fifths of them are white, and the rest are pink. What is the probability that a ball randomly selected is pink?

- (A) 40% (B) 25% (C) $26\frac{2}{3}\%$ (D) $37\frac{1}{2}\%$ (E) $33\frac{1}{3}\%$

28. The current United States Supreme Court consists of three women and six men. How many ways can a five member committee be formed consisting of three women and two men or two women and three men?

- (A) 90 (B) 15 (C) 60 (D) 45 (E) 75

29. The Mayan number system consists of three symbols, \bullet — \bigcirc . The total number of these symbols needed to write 5314 base 10 as a Mayan number is:

- (A) 9 (B) 10 (C) 11 (D) 12 (E) 13

30. The three-digit number, 707, is considered to be which of the following type numbers?

- (A) evil (B) unhappy (C) lucky (D) economical (E) harmonic

31. Penni Les had a jar of new pennies. She gave 30% of them to her little sister. Then she gave $\frac{1}{4}$ of what was left to her favorite niece. Then she gave her little brother 21 pennies leaving 42 pennies in her jar. How many were in her jar originally?

- (A) 260 (B) 70 (C) 110 (D) 200 (E) 120

32. Find k when $2 + 3(k - 5) \div 7 = 11 - 3k$

- (A) $2\frac{2}{3}$ (B) 3.25 (C) 2 (D) 3.5 (E) $4\frac{5}{12}$

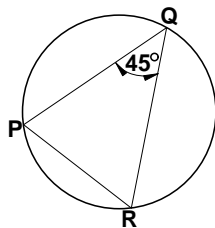
33. Soh Yung is twice as old as her sister, Tu Yung. Soh's age now is 10 years older than Tu was last year. What will the sum of their ages be in 6 years?

- (A) 39 (B) 36 (C) 34 (D) 33 (E) 30

34. If a trip takes 7 hours at 60 miles per hour, then how long would the same trip take at 75 miles per hour? (nearest minute)

- (A) 6 hrs (B) 5 hrs 24 min (C) 5 hrs 15 min (D) 5 hrs 12 min (E) 5 hrs 36 min

35. Points P, Q, and R lie on the circle shown. Find the length of the chord PR if the radius of the circle is 4". (nearest tenth)



- (A) 2.8" (B) 4.8" (C) 5.7" (D) 8.5" (E) 11.3"

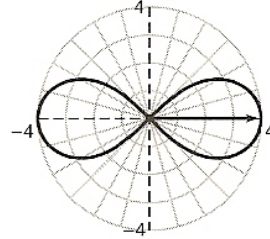
36. The measure of an external angle of an n -sided regular polygon is 30° . What is the sum of the interior angles of this polygon?
- (A) 720° (B) 900° (C) 1080° (D) 1440° (E) 1800°
37. Let $A = \begin{bmatrix} 1 & 4 & 9 \\ -1 & 0 & 1 \\ 2 & 3 & 5 \end{bmatrix}$. Find $|A|$.
- (A) -58 (B) -52 (C) -2 (D) 4 (E) 17
38. Points $(1, 3)$ and $(-1, 7)$ are members of the function $\{(x, y) \mid y = 3ax + b\}$. Find $a + b$.
- (A) $5\frac{2}{3}$ (B) $4\frac{1}{3}$ (C) $3\frac{1}{2}$ (D) $2\frac{1}{3}$ (E) $1\frac{1}{2}$
39. The ranger at station 1 saw smoke in the distance. He used his transit to determine the smoke was on a bearing of 70° . Ranger station 2 is 10 miles from station 1 on a bearing of 120° . The ranger at station 2 used his transit to determine the smoke was on a bearing of 20° . How far from station 1 was the smoke? (nearest tenth)
- (A) 12.9 mi (B) 11.3 mi (C) 10.5 mi (D) 12.2 mi (E) 13.1 mi
40. Let $A = \begin{bmatrix} -2 & 3 \\ 1 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -3 \\ 6 & 10 \end{bmatrix}$. Find $|A^T - B|$.
- (A) -18 (B) -13 (C) -13 (D) 26 (E) 44
41. Tye Guhr drops a golf ball from his balcony which is 12 feet from the ground. If the golf ball rebounds up 75% of the distance it falls each time, how far will the golf ball have traveled when it hits the ground the 4th time?
- (A) $57' 5\frac{1}{16}"$ (B) $53' 7\frac{1}{2}"$ (C) $48' 6\frac{1}{4}"$ (D) $41' 7\frac{1}{2}"$ (E) $32' 9\frac{3}{4}"$
42. Leo Lyan and Elsa Lyaness start at the same time and place running to their designated spots. Leo runs due West at 12 mph and Elsa runs due South at 10 mph. How fast is the distance between them changing after 30 minutes of running? (nearest tenth)
- (A) 15.6 mph (B) 30.0 mph (C) 15.4 mph (D) 10.7 mph (E) 31.2 mph
43. If $2x^2 - 3y = 5$ and $x > 0$, then $\frac{dy}{dx} = \frac{dx}{dy}$ when $x = ?$
- (A) 0.6 (B) 0.666... (C) 0.75 (D) 1.333... (E) 3.333...
44. Find the area of an obtuse scalene triangle whose side lengths are 5", 8" and 12". (nearest sq. inch)
- (A) 13 in^2 (B) 20 in^2 (C) 30 in^2 (D) 23 in^2 (E) 15 in^2

45. The *Phil Upp* gas station mixes 50 gallons of a special grade gasoline worth \$3.05 per gallon with some regular grade gasoline worth \$2.80 per gallon. The mixture sells for \$2.95 per gallon. How many gallons of the regular grade did the station mix with the special grade? (nearest gallon)
- (A) 25 gal (B) 30 gal (C) 33 gal (D) 38 gal (E) 40 gal
46. A box contains five rods whose lengths are 1", 3", 6", 10" and 15". How many different obtuse triangles can be made using only three rods at a time.
- (A) 1 (B) 3 (C) 6 (D) 10 (E) 15
47. Let $f(x + 1) = 2 - f(x)$. Find $f(2)$ if $f(-1) = 3$.
- (A) -1 (B) 0 (C) 1 (D) 2 (E) 3
48. How many integral values of n exist such that $n > 4$ and $\frac{(n-2)!}{(n-4)!} \leq 20$
- (A) none (B) 2 (C) 3 (D) 4 (E) 7
49. Let $(2\text{cis}(\frac{\pi}{6}))^5 = a + bi$. Find $a + b$. (nearest tenth):
- (A) -11.7 (B) -7.3 (C) -5.9 (D) 3.7 (E) 43.7
50. Convert the polar equation $r = 2\sin \theta$ to a rectangular equation.
- (A) $x^2 + y^2 = 4$ (B) $x^2 + (y - 2)^2 = 4$ (C) $x^2 + y^2 = 2$
(D) $(x - 1)^2 + 2y = 1$ (E) $x^2 + (y - 1)^2 = 1$
51. How many solutions are there for the equation $5x + 3y = 2014$ such that both x and y are non-negative integers?
- (A) 72 (B) 81 (C) 101 (D) 115 (E) 134
52. If the three numbers 425, 356, and 264 are each divided by the number D , each of their quotients will have the same remainder R . Find R where $R > 1$.
- (A) 11 (B) 7 (C) 5 (D) 3 (E) 2
53. Find a real value of x such that $\log_2(\log_2 x) = \log_4(\log_4 x)$?
- (A) 1 (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$ (E) 4
54. The polar graph of $r^2 = 9\sin(2\theta)$ is symmetric to the: (1) polar axis (2) pole (3) line $\theta = \frac{\pi}{2}$
- (A) 2 only (B) 3 only (C) 1 & 2 (D) 2 & 3 (E) 1, 2, & 3

55. The probability that Larry will ask Shemp to go to the movies is 30%, that Moe will ask him is 25%, and that Curley will ask him is 12.5%. What is the probability that at least two of them will ask Shemp to go to the movies? (nearest tenth)

- (A) 23.1% (B) 15.3% (C) 14.4% (D) 12.5% (E) 11.6%

56. Which of the following polar equations will produce the graph shown?



- (A) $r^2 = 16\cos 2\theta$ (B) $r = 4\sin 2\theta$ (C) $r^2 = 2\sin 2\theta$ (D) $r = 4\cos 2\theta$ (E) $r^2 = 16\sin 2\theta$

57. The odds of scoring less than 150 on this test is $\frac{2}{5}$. Based on these odds, if 126 take this test how many would be expected to score less than 150?

- (A) 64 (B) 60 (C) 50 (D) 40 (E) 36

58. The repeating decimal 0.3666... in base 8 can be written as which of the following fractions in base 8?

- (A) $\frac{27}{56}_8$ (B) $\frac{9}{25}_8$ (C) $\frac{33}{70}_8$ (D) $\frac{39}{100}_8$ (E) $\frac{13}{31}_8$

59. The centroidal mean of 20 and 30 is: (nearest tenth)

- (A) 24.8 (B) 25.0 (C) 25.3 (D) 25.5 (E) 26.0

60. How many positive integers less than or equal to 2014 are divisible by neither 3 nor 5?

- (A) 939 (B) 941 (C) 1,007 (D) 1,073 (E) 1,075

**University Interscholastic League
MATHEMATICS CONTEST
HS • Regional • 2014
Answer Key**

- | | | |
|-------|-------|-------|
| 1. C | 21. C | 41. B |
| 2. B | 22. B | 42. A |
| 3. A | 23. E | 43. C |
| 4. D | 24. D | 44. E |
| 5. B | 25. B | 45. C |
| 6. D | 26. D | 46. A |
| 7. C | 27. C | 47. A |
| 8. D | 28. E | 48. C |
| 9. E | 29. D | 49. A |
| 10. D | 30. B | 50. E |
| 11. A | 31. E | 51. E |
| 12. E | 32. B | 52. A |
| 13. E | 33. A | 53. B |
| 14. B | 34. E | 54. A |
| 15. C | 35. C | 55. D |
| 16. D | 36. E | 56. A |
| 17. B | 37. C | 57. E |
| 18. D | 38. B | 58. C |
| 19. C | 39. A | 59. C |
| 20. D | 40. D | 60. E |