UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

Region • 2013

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YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $\left[(2+4!) \div (8)^{\frac{1}{3}} - \sqrt{16} \times 32^{(-1)}\right] \div 0.125$

(A) 105 **(B)** 103 **(C)** 38 **(D)** $2\frac{43}{64}$ **(E)** 2.25

2. ***Bear Foot*** **Shoes is having a sale. Harry Paw needs to buy 4 pair of shoes. Which of the following is the least expensive deal?**

(A) $64.00 a pair **(B)** buy 2 pair at $75 a pair and 2 pair at $50 a pair

(C) buy 3 pair at $70 a pair and 1 pair at $42 a pair **(D)** $72 a pair and get 15% off

(E) $80 for the 1st pair, $70 for the 2nd, $60 for the 3rd, and $50 for the 4th

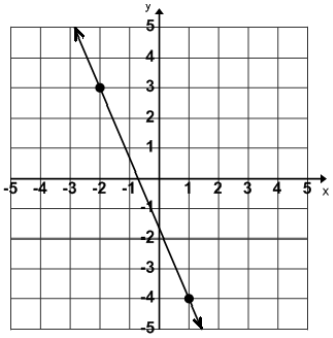
3. Find the arithmetic mean of the median, mode, and range of 2, 1, 3, 4, 7, 1, 1, 2, 3, & 5.

(A) $2\frac{9}{10}$ **(B)** $3\frac{1}{3}$ **(C)** $3\frac{5}{6}$ **(D)** $2\frac{4}{9}$ **(E)** $3\frac{1}{6}$

4. Let N = {n,a,p,i,e,r}, E = {e,u,c,l,i,d}, A = {a,g,n,e,s,i}, and T = {t,h,e,a,n,o}. The number of distinct elements in (A ∩ N) ∪ (T ∩ E) is _____.

(A) 6 **(B)** 5 **(C)** 4 **(D)** 3 **(E)** 2

5. Which of the following points lies on the line shown?



(A) (— 6, 9) **(B)** (4, — 7) **(C)** (— 8, 17) **(D)** (— 3, 7) **(E)** (8, — 17)

6. The set {— 1, 1} is closed under which of these operations:

+ addition — subtraction × multiplication ÷ division $\sqrt{}$ square root

(A) + & × **(B)** × & ÷ **(C)** +, ×, & $\sqrt{}$ **(D)** × only **(E)** none of these

7. Lotta Dough Bakery knows that the amount of flour needed to make bread is directly proportional to the number of loaves of bread needed. Their recipe calls for 5 cups of flour for each loaf. How many pounds of flower will be needed to bake 5 dozen loaves of bread if, according to the Farmer's Almanac, it takes 1.5 cups of flour to equal 1 pound of flour?

(A) 450 lbs **(B)** 312.5 lbs **(C)** 300 lbs **(D)** 200 lbs **(E)** 187.5 lbs

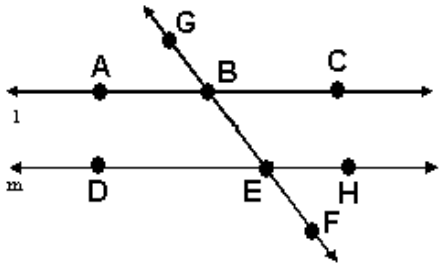
8. $\angle P$ and $\angle R$ are complementary. $\angle P$ and $\angle Q$ are supplementary. If $m\angle R = 37°$ then $m\angle Q$ = ?

(A) 53° (B) 87° (C) 113° (D) 127° (E) 143°

9. If two inscribed angles of a circle intercept the same arc then the angles are _____.

(A) complementary (B) right (C) congruent (D) vertical (E) supplementary

10. The three lines in the figure are coplanar with m // l.. How many of the following are true statements?



1. $\angle DEG$ & $\angle HEB$ are vertical angles 2. $m\angle ABG = \frac{1}{3}m\angle DEF$

3. $\angle HEF \cong \angle CBG$ 4. $\angle BED$ and $\angle BEH$ are supplementary

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

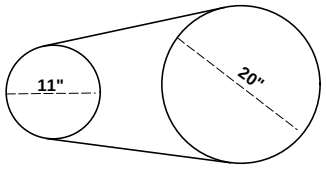
11. Let $f(x) = 2x - 5$ and $g(x) = 3 - 4x$ and $h(x) = x + 5$. Find $h\Big(f\big(g(1-x)\big)\Big)$.

(A) $8x - 2$ (B) $1 - 48x$ (C) $8x + 11$ (D) $4 - 3x$ (E) $3x + 4$

12. Find the area of the circle, $2x^2 + 2y^2 + 4x - 5y - 2 = 0$. (nearest tenth)

(A) 14.3 (B) 10.2 (C) 5.9 (D) 5.1 (E) 11.2

13. A belt joins the two pulleys shown with the given diameters. If the smaller pulley is rotating at 75 rpm, then the larger pulley is rotating at _____ rpm .



(A) $1\frac{7}{11}$ (B) $37\frac{1}{2}$ (C) $26\frac{9}{11}$ (D) $41\frac{1}{4}$ (E) $20\frac{5}{8}$

14. If the $\sec\theta = -2\frac{1}{3}$ and θ is in QIII, then $\sin\theta$ is : (nearest tenth)

(A) -0.5 (B) -0.9 (C) -0.4 (D) 0.4 (E) 0.9

15. Let $(-2 + 2i)^4 = a + bi$. Find $a + b$.

(A) -224 (B) -172 (C) -64 (D) -32 (E) 0

16. Mathis Grate placed a tennis ball, a soccer ball, and a golf ball on the football field, each in a different place. He gave his math class the following information. The bearing of the tennis ball from the soccer ball is 45° and they are 24 yards apart. The bearing of the golf ball from the tennis ball is 135° and they are 18 yards apart. What should the students calculate the bearing to be of the soccer ball from the golf ball? (nearest whole degree)

(A) 188° (B) 225° (C) 262° (D) 278° (E) 315°

17. An infinite geometric sequence has a common ratio of 0.75 and a sum of 120. What is third term of the sequence?

(A) 30 (B) 22.5 (C) 16.875 (D) 15 (E) 12.5625

18. Simplify: $\log_b 16x - 2\log_b 8x + \log_b x^{-4}$, if $x > 0$

(A) $-\log_b 4x^5$ (B) $\log_b 3x$ (C) $-\log_b x^4$ (D) $\log_b x$ (E) $-5\log_b x$

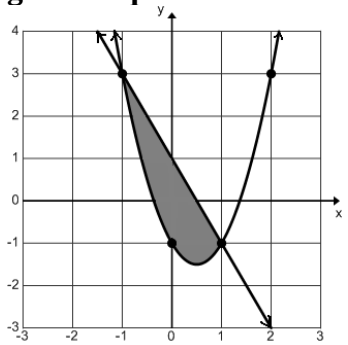
19. Let $f(x) = \frac{x^2 - 3}{2x - 4}$ and s(x) be the slant asymptote of f. Find the value of s(5).

(A) 3.666... (B) 3.5 (C) 3.333... (D) 3.25 (E) 3.1666...

20. Let $f(x) = x^2 - 4x + 1$ and $g(x) = x^2 + 5x - 2$. Find $g(f'(x + 1))$

(A) $4x^2 + 20x + 37$ (B) $2x^2 + 3x$ (C) $4x^2 + 2x - 8$ (D) $4x^2 + 24x + 50$ (E) $x^2 + 7x - 1$

21. Find the area of the shaded region in square units.



(A) $3\frac{1}{2}$ (B) $3\frac{1}{3}$ (C) $3\frac{1}{6}$ (D) 2 (E) $2\frac{2}{3}$

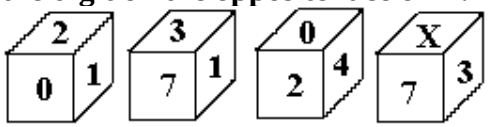
22. How many distinguishable arrangements can be made from the letters "CALCULUS"?

(A) 5,040 (B) 120 (C) 13,440 (D) 6,720 (E) 40,320

23. Ura Trechee picks 2 spades, 3 hearts, 4 diamonds, and 5 clubs from a standard deck of cards. She shuffles the cards and deals the top three cards face up on the table. What is the probability that the top card is a heart or a diamond and the next two cards are either a spade or a club? (nearest %)

(A) 51% (B) 27% (C) 13% (D) 11% (E) 7%

24. Each face of a cube is labeled with the digits, 0, 2, 1, 3, 4, 7. Four views of the cube are shown. What is the product of X and the digit on the opposite face of X? Ignore orientation.



(A) 8 (B) 6 (C) 4 (D) 2 (E) 0

25. All of the elements of {125, 343, 729, 1331, 1728, 2197, 4913} are considered to be ____ numbers.

(A) equidigital (B) even (C) extravagant (D) evil (E) economical

26. Simplify: $\left(\frac{x^2+14x+49}{x+7}\right)\left(\frac{x^2-14x+49}{x^2-49}\right)$

(A) $x-7$ **(B)** $x+7$ **(C)** x^2+49 **(D)** x^2-49 **(E)** x^2+7x-7

27. The dihedral angles of a tetrahedron are supplementary to the dihedral angles of a(n):

(A) icosahedron (B) dodecahedron (C) octahedron (D) hexahedron (E) tetrahedron

28. If $a_1 = -1$, $a_2 = 0$, $a_3 = 1$ **and** $a_n = (a_{n-3})(a_{n-2}) + (a_{n-1})$ **for** $n \geq 4$, **then** a_9 **equals:**

(A) 21 (B) 16 (C) 13 (D) 11 (E) 8

29. Using Pascal's triangle, determine the sum of the 3rd term and the 14th term of the 16th row.

(A) 182 (B) 210 (C) 238 (D) 240 (E) 272

30. A(n) _____ number is an even positive integer greater than 2 can be expressed as a sum of two primes, e.g. 8 = 3 + 5, 44 = 13 + 31, 100 = 11 + 89, ...

(A) Archimedean (B) Porter (C) Goldbach (D) Germain (E) Euclidean

31. How many points of intersection occur when $r = 2\cos(\theta) + 1$ **and** $r = 2\sin(\theta)$ **are graphed on a polar coordinate system where** $0 \leq \theta \leq 2\pi$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

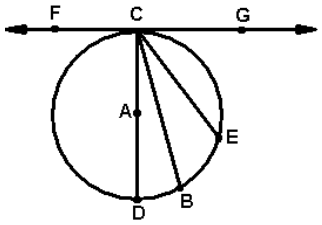
32. Justin Time can stuff 100 envelopes in 30 minutes working at a constant rate. He has to stuff 5000 envelopes in 1 hour. He hires enough people to complete the task on time. Since they all are working at the same constant rate, how many envelopes will each one stuff?

(A) 50 (B) 75 (C) 125 (D) 200 (E) 300

33. P, Q, and R are the real roots of $x^3 + Bx^2 + Cx + D = 0$. **The harmonic mean of P, Q, and R is** $2\frac{10}{13}$ **and PQR =** -24. **Find C.**

(A) -26 **(B)** $-22\frac{2}{13}$ **(C)** $-21\frac{3}{13}$ **(D)** -9 **(E)** $-7\frac{1}{13}$

34. $\overleftrightarrow{FG}$ **is tangent to** $\odot A$ **at point C. Find** $m\angle BCE$ **if** $m\angle DCB = 20°$ **and** $\overset{\frown}{CE} = 80°$**.**



(A) $30°$ **(B)** $35°$ **(C)** $40°$ **(D)** $45°$ **(E)** $50°$

35. Which of the following is a true statement associated with the function $f(x) = \frac{\sin(x)}{x^2}$ **?**

(A) The function does not have a horizontal asymptote. (B) The graph is always concave up.

(C) The graph is symmetric about the axis. (D) The function has a vertical asymptote at 0.

(E) None of the above are true.

36. A tee bag contains 35 colored tees. The probability of randomly selecting a white tee is 60%. What is the least number of white tees needed to be added to the bag to change the probability of randomly selecting a white tee to 75%?

(A) 5 **(B)** 14 **(C)** 15 **(D)** 21 **(E)** 28

37. Find k if $GCF(63, k) = 9$ **and** $LCM(63, k) = 252$**.**

(A) 36 **(B)** 28 **(C)** 22 **(D)** 7 **(E)** 4

38.

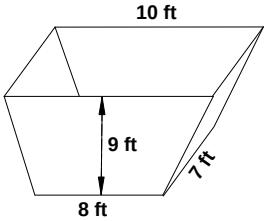
P — 4 Q — 1.125 R

<----------|----------|----------|----------|----------|---------->

The distances between the hash marks (|) are equal. Find $P + Q + R$**.**

(A) -12.8125 **(B)** -8.3125 **(C)** -7.6875 **(D)** -6.25 **(E)** -2.5625

39. Find the volume of this trapezoid bin.



(A) 504 cu. ft. (B) 567 cu. ft. (C) 587 cu. ft. (D) 630 cu. ft. (E) 729 cu. ft.

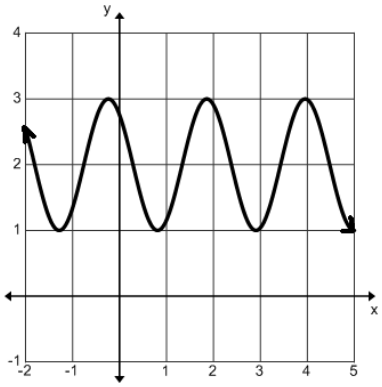
40. Which of the following is not a member of solution set of $|5x - 4| - 3 < 2$**?**

(A) $-\frac{1}{9}$ **(B)** $\frac{2}{3}$ **(C)** $1.444...$ **(D)** $1\frac{8}{9}$ **(E)** $1.777...$

41. The length of the latus rectum of $13x^2 - 3y^2 = 39$ is: (nearest tenth)

(A) 15.0 (B) 8.7 (C) 2.9 (D) 0.9 (E) 0.3

42. The equation y = _____ will produce this graph.



(A) $2 + \cos(3x + 4)$ (B) $2 - \sin(3x + 4)$ (C) $2 - \cos(4x - 3)$

(D) $2 + \cos(4x - 2)$ (E) $2 + \sin(3x - 4)$

43. Let $(2 + i)(3 + 4i) \div (7 + 11i) = a + bi$. Find $a + b$.

(A) $\frac{17}{18}$ (B) $1\frac{2}{17}$ (C) $1\frac{5}{34}$ (D) $1\frac{9}{17}$ (E) $2\frac{1}{36}$

44. The series $\frac{1}{1} + \frac{1}{1} + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \ldots$ converges to _____.

(A) ∞ (B) ln(2) (C) $\frac{\sqrt{5}+1}{2}$ (D) e (E) ψ

45. Three trillion five million two hundred-thousand four hundred one take away six billion forty million five hundred and three thousand seventy-eight equals K. The sum of the digits of K is?

(A) 62 (B) 72 (C) 110 (D) 117 (E) 147

46. If $y = 1 - x$ and $xy = -2$ then $x^3 + y^3 = ?$

(A) 17 (B) 11 (C) 7 (D) 5 (E) 1

47. Les Ismoor had 15 fluid ounces of a 75% solution of Argyrol in water. What is the greatest amount of a 67% solution of Argyrol in water can he add to make a final solution of at least 72% Argyrol?

(A) 3 oz (B) 5 oz (C) 7.25 oz (D) 8 oz (E) 9 oz

48. The sum of the lengths of the edges of regular octahedron is 60 inches. The surface area of this octahedron is: (nearest square inch)

(A) 390 sq. in. (B) 195 sq. in. (C) 173 sq. in. (D) 115 sq. in. (E) 87 sq. in.

49. Let $x^2 - xy + 2y^2 = 4$. Find $D_x y$.

(A) $\frac{y-2x}{4y-x}$ **(B)** $2x+4y$ **(C)** $\frac{4y+x}{y-2x}$ **(D)** $4x-2y$ **(E)** $\frac{4y-x}{2x-y}$

50. The probability that statement P is true is $83\frac{1}{3}\%$, and the probability that statement Q is true is 25%. Determine the probability that P ➞ Q is true. Statements P and Q are independent.

(A) $12\frac{1}{2}\%$ **(B)** $20\frac{5}{6}\%$ **(C)** $37\frac{1}{2}\%$ **(D)** $41\frac{2}{3}\%$ **(E)** $62\frac{1}{2}\%$

51. Point P(2, 2) lies in the x-y plane. Point P is reflected across the origin to point Q. Point Q is translated vertically up 2 units and horizontally 3 units to the right to point R. Point R is rotated 270° clockwise about the origin to point S (x,y). Find x + y.

(A) 0 **(B) 1** **(C) 2** **(D) 3** **(E) 4**

52. Let P = 82, Q = 75, and R = 100. Find the arithmetic mean of the geometric mean and the harmonic mean of P, Q, and R. (nearest hundredth)

(A) 84.27 **(B) 84.44** **(C) 84.74** **(D) 85.04** **(E) 85.67**

53. $789B410_{16} \div 5_{16} =$ _____ **16**

(A) 1573010 **(B) 181F0D0** **(C) 157F082** **(D) 181B082** **(E) 1578280**

54. Let $f(x) = ax^5 - bx^3 + cx - 4$ where a, b, and c are constants. If f(— 4) = 2 then f(4) = ?

(A) 0 **(B) — 2** **(C) — 4** **(D) — 6** **(E) — 10**

55. Determine the concavity of the graph of $f(x) = 3\sin(x) - 2\cos(x)$ at $x = \frac{5\pi}{4}$.

(A) $-\frac{\sqrt{2}}{2}$ **(B)** $-\frac{5\sqrt{2}}{2}$ **(C)** $-\sqrt{2}$ **(D)** $\frac{\sqrt{2}}{2}$ **(E)** $\frac{3\sqrt{2}}{2}$

56. Shirley Knott gave the same pop quiz to her three algebra classes. The first class averaged 80%, the second 85%, and the third 89%. Together the grades of the first two classes averaged 82%, while the grades of the second and third class averaged 87%. What was the average of the grades from all three classes?

(A) 83% **(B) 84%** **(C) 85%** **(D) 86%** **(E) 87%**

57. Which of the following is equivalent to $\frac{1}{\sin(\theta)} - \frac{1}{\tan(\theta)}$

(A) $\tan(\frac{\theta}{2})$ **(B)** $\cos(\frac{\theta}{2})$ **(C)** $\cot(\frac{\theta}{2})$ **(D)** $2\sin(\theta)\tan(\theta)$ **(E)** $\cos(\theta)\cot(\theta)$

58. Find the slope of the tangent line to $13x^2 - 3y^2 = 39$ at $x = 3$ and $y > 0$. (nearest hundreth)

(A) 0.50 **(B) 0.67** **(C) 1.18** **(D) 1.50** **(E) 2.55**

59. In the expansion of $(2x + 1)^6$, the sum of the coefficients of the 2^{nd}, 4^{th}, and 6^{th} term is:

(A) 382 **(B) 729** **(C) 225** **(D) 198** **(E) 364**

60. Sam D. Seenyor and Willis A. Fressmann are taking a number sense test, a math test, and a calculator test at the TMSCA State meet. Sam is three times as likely to score higher on each test than Willis. What are the odds that Willis will score higher on at least one of the tests?

(A) $\frac{2}{19}$ **(B)** $\frac{7}{57}$ **(C)** $\frac{27}{64}$ **(D)** $\frac{37}{27}$ **(E)** $\frac{16}{7}$

University Interscholastic League
MATHEMATICS CONTEST
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Answer Key

1. B	**21. E**	**41. A**
2. D	**22. A**	**42. E**
3. E	**23. C**	**43. B**
4. C	**24. C**	**44. D**
5. C	**25. E**	**45. B**
6. B	**26. A**	**46. C**
7. D	**27. C**	**47. E**
8. D	**28. D**	**48. E**
9. C	**29. B**	**49. A**
10. B	**30. C**	**50. C**
11. A	**31. D**	**51. B**
12. E	**32. D**	**52. C**
13. D	**33. A**	**53. B**
14. B	**34. A**	**54. E**
15. C	**35. D**	**55. D**
16. C	**36. D**	**56. B**
17. C	**37. A**	**57. A**
18. A	**38. C**	**58. D**
19. B	**39. B**	**59. E**
20. C	**40. D**	**60. D**