UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

Invitational B • 2015

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. **Evaluate:** $4! \div (16)^{\frac{1}{2}} - 4 \times (16)^{-1} + 4 \times 16^{0}$

   (A) $\frac{23}{8}$ **(B)** $\frac{31}{4}$ **(C)** $\frac{25}{4}$ **(D)** $\frac{31}{8}$ **(E)** $\frac{39}{4}$

2. **I. M. Broke borrowed $250.00 from his bank at a simple interest rate of 5%. He paid the loan off in 10 monthly payments. What was his monthly payments? (nearest cent)**

   (A) $26.04 **(B)** $26.15 **(C)** $26.25 **(D)** $28.06 **(E)** $28.82

3. **What is** $8\frac{1}{3}\%$ **of** $(\frac{1}{16} \div (0.1666...)$**?**

   (A) $\frac{1}{32}$ **(B)** $\frac{1}{16}$ **(C)** $\frac{1}{8}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{1}{2}$

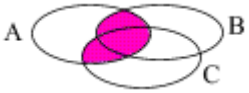
4. **Simplify:** $\left(\frac{16x^2 + 8x - 3}{16x^2 - 1}\right)\left(\frac{16x^2 + 8x + 1}{12x^2 + x - 6}\right)(3x - 2)$

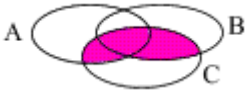
   (A) $12x^2 - 5x - 2$ **(B)** $4x + 1$ **(C)** $\frac{4x-1}{4x+1}$ **(D)** $\frac{3x-2}{4x+1}$ **(E)** $12x^2 - 11x + 2$

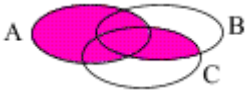
5. **Justin Time is** $\frac{4}{5}$ **as old as Soh Yung. Fours years ago Justin was** $\frac{3}{4}$ **as old as Soh. What will the sum of their ages be in two years.**

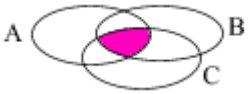
   (A) 28 **(B)** 32 **(C)** 36 **(D)** 38 **(E)** 40

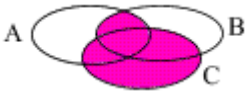
6. **In which of the following Venn diagrams does the shaded regions represent the set** $A \cup (B \cap C)$**?**

   (A) 

   (B) 

   (C) 

   (D) 

   (E) 

7. **Three less than twice a number is the same as one more than twice the difference of four and the number. Find the number.**

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

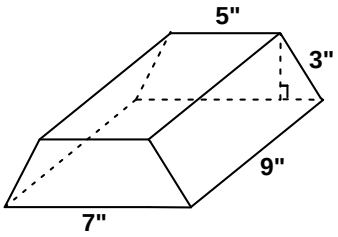
8. **What is the probability that a factor of 160 is a multiple of 10?**

   (A) $71\frac{3}{7}\%$ **(B)** $33\frac{1}{3}\%$ **(C)** $45\frac{5}{11}\%$ **(D)** $41\frac{2}{3}\%$ **(E)** $6\frac{1}{4}\%$

9. **The measure of the interior angle of a regular n-gon is three times the measure of its exterior angle. How many sides does the regular n-gon have?**

   (A) 5 **(B)** 6 **(C)** 8 **(D)** 9 **(E)** 12

10. Find the volume of the isosceles trapezoid prism shown. (nearest cu. in). Drawing is not to scale.



(A) 153 cu. in (B) 158 cu. in (C) 162 cu. in (D) 167 cu. in (E) 171 cu. in

11. Which of the following points of concurrency are on the exterior of an obtuse triangle?
(1) circumcenter (2) centroid (3) orthocenter (4) incenter

(A) none of them (B) 1 & 3 (C) 2 & 4 (D) 1 only (E) all of them

12. The center of the circle, $x^2 + y^2 - 4x - 6y + 9 = 0$, is (h, k) and the radius is r. Find $h + k + r$.

(A) 8 (B) 7 (C) 5 (D) 4 (E) -3

13. Simplify: Find $g(f(2a + 1))$ when $g(x) = x - 4a$ and $f(x) = 3x + a$.

(A) $3 - 5a$ (B) $3 - a$ (C) $3(a + 1)$ (D) $5a + 2$ (E) $3a - 5$

14. How many numbers k exist such that $10 \leq k \leq 99$ and the difference between k and the sum of the digits of k is 18.

(A) 10 (B) 9 (C) 8 (D) 6 (E) 5

15. The sum of the positive integral divisors of 120 is _____.

(A) 25 (B) 125 (C) 180 (D) 360 (E) 960

16. The fundamental period of the graph of $y = 2 - 3\cos^2(4x + 5)$ is:

(A) 2 (B) 3 (C) $\frac{\pi}{4}$ (D) $\frac{2\pi}{5}$ (E) 5

17. $\cos\left(x + \frac{5\pi}{2}\right)$ = _____.

(A) $-\cos x$ (B) $\sin x$ (C) $-\cos\left(\frac{\pi}{2}x\right)$ (D) $\sin 2x$ (E) $-\sin x$

18. Mack A. Roy drops a golf ball from a height of 5 feet. Each time it hits the ground it rebounds to a height of 80% of the distance it fell. Find the total distance the ball travels when it reaches the ground the third time. (nearest inch)

(A) 22' 0" (B) 19' 5" (C) 18' 8" (D) 16' 2" (E) 14' 5"

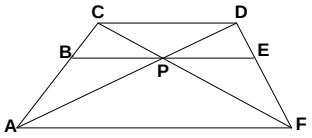
19. Which of the following is a reference angle for $\frac{5\pi}{4}$?

(A) $\frac{\pi}{4}$ (B) $\frac{4\pi}{5}$ (C) $\frac{3\pi}{4}$ (D) $\frac{5\pi}{8}$ (E) $\frac{\pi}{2}$

20. Evaluate: $\sum_{k=1}^{5} (2)^{(2-k)} + k$

(A) 13.625 (B) 16 (C) 18.875 (D) 25.375 (E) 32

21. Given the trapezoid shown where segments AF, BE, and CD are parallel to each other and the three interior segments are concurrent at point P, find the length of segment AF if CD = 20 cm and BE = 24 cm. (nearest tenth)



(A) 26.2 cm (B) 28 cm (C) 28.8 cm (D) 30 cm (E) 32 cm

22. Let $f(x) = x^5 - 2x^4 + 2x^3 - 3x^2 + x - 3$. Find $f''(-1)$.

(A) 20 (B) 12 (C) -1 (D) -15 (E) -62

23. Lotta Latts is building a rectangular parking lot for her *Lotta's Junk* store. The length of the lot will be bordered on one side by the store. She has 1200 feet of fence to enclose the lot. What is the maximum area of her parking lot?

(A) 90,000 ft.^{2} (B) 160,000 ft.^{2} (C) 180,000 ft.^{2} (D) 360,000 ft.^{2} (E) 1,440,000 ft.^{2}

24. Let $f(x) = \frac{1}{\sqrt{x^2 + 3x - 10}}$. At which of these intervals is function f continuous?

(A) $[-5, -2]$ (B) $(-5, 5]$ (C) $[-2, 2)$ (D) $(2, 5]$ (E) $(-2, 5)$

25. The number 15 is considered to be a "polite" number. The "politeness" of 15 is ____.

(A) 1 (B) 2 (C) 3 (D) 5 (E) 15

26. N. A. Hurry enters a convenience store. The probability that she buys bread is 60%, the probability she buys milk is 50%, and the probability she buys both bread and milk is 30%. What is the probability that she will buy either bread or milk or both? (nearest percent)

(A) 100% (B) 80% (C) 70% (D) $53\frac{1}{3}\%$ (E) $46\frac{2}{3}\%$

27. Find the least positive integral sum of d and m if GCD(d , m) = 8 and LCM(d , m) = 320.

(A) 48 (B) 104 (C) 112 (D) 176 (E) 328

28. Let k be a positive integer less than 100 such that k is a multiple of 4 and k is divisible by 3. Find the sum of all such numbers k.

(A) 336 (B) 396 (C) 420 (D) 432 (E) 444

29. Les Cash, Lotta Dough, and Noah Moolah had piggy banks. The average of all 3 banks was $147.00. The average of Lotta's bank and Noah's bank was $141.00. How much money was in Les' bank?

(A) $96.00 (B) $119.50 (C) $144.00 (D) $159.00 (E) $168.50

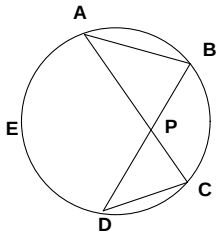
30. A line crosses the x-axis at $x = -2$ and goes through the point (3, 1). Another line crosses the y-axis at $y = 2$ and goes through the point $(-1, -3)$. The lines intersect at (x, y). Find $x + y$.

(A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

31. Simplify this expression: $\left(\frac{x^2 y^{-3} z^3}{w^2 x^{-3} y^2}\right)^{-1} \times \left(\frac{x^2 z}{w y^3}\right)^2 \div \frac{1}{(xyz)^2}$

(A) $(xz)^{-1}$ (B) xyz (C) xy^2z^3 (D) $(xyz)^{-1}$ (E) xy^3z

32. $\overline{AB}$, $\overline{AC}$, $\overline{BD}$, and $\overline{CD}$ are chords of circle O and point E lies on circle O. If $m\overset{\frown}{BC} = 46°$ and $m\angle APB = 78°$, then $m\angle ACD = ?$



(A) 101° (B) 62° (C) 75° (D) 56° (E) 79°

33. Betty Drawzette randomly selects a number from the set of all positive 2-digit numbers. What is the probability that the sum of the digits of the number selected is 11? (nearest percent)

(A) 7% (B) 8% (C) 9% (D) 10% (E) 11%

34. Point P(−3, 4) lies on the x-y plane. Point P is rotated 180° counter clockwise about the origin to point Q. Point Q is translated horizontally 5 units to the left to point R. Point R is reflected across the line $y = -x$ to point S. The coordinate of S is (x, y). Find $x + y$.

(A) −7 (B) −2 (C) −1 (D) 5 (E) 6

35. How many integral values of n exist such that $n \leq 1$ and $\frac{(n+1)!}{(n-1)!} \leq 12$

(A) none (B) 2 (C) 4 (D) 6 (E) 7

36. The roots of the equation $x^3 - 13x + 12 = 0$ are 1, 3, and R. Find R.

(A) 9 **(B) 4** **(C) -1** **(D) -3** **(E) -4**

37. The graph of the polar equation $r = 5 + 2\cos(\theta)$ is a _____.

(A) dimpled limacon **(B) convex limacon** **(C) inner loop limacon**

(D) lemniscate **(E) cardioid**

38. Find the smallest positive real number x such that sin x = cos 2x, where x is measured in radians.

(A) $\frac{3\pi}{2}$ **(B) $\frac{5\pi}{3}$** **(C) $\frac{\pi}{6}$** **(D) $\frac{2\pi}{3}$** **(E) $\frac{\pi}{12}$**

39. The harmonic mean of the real roots of $2x^3 + 5x^2 - 4x - 3 = 0$ is _____.

(A) $-0.666...$ **(B) -0.75** **(C) $-1.333...$** **(D) -1.5** **(E) -2.25**

40. Simplify to the form a + bi: $(1 + 2i)(3 + 4i) \div (5i)$

(A) $1 - 2i$ **(B) $2 + i$** **(C) $2 - 2i$** **(D) $2 - i$** **(E) $1 + 2i$**

41. $221_3 + 102_3 + 121_3 =$ _____ 9

(A) 63 **(B) 57** **(C) 48** **(D) 36** **(E) 12**

42. Which of the following statements is a false statement for $f(x) = \begin{cases} \frac{1}{x-2} & \text{if } x \neq 2 \\ 3 & \text{if } x = 2 \end{cases}$?

(A) f is continuous at x = 1 **(B) $f(2)$ exists** **(C) $\lim_{x \to 2^-} f(x)$ exists**

(D) $\lim_{x \to 2^+} f(x)$ exists **(E) f is continuous at 2**

43. Let $f(x) = \frac{3x-1}{2x+5}$. Find $f'(4)$.

(A) $\frac{17}{169}$ **(B) $1\frac{1}{2}$** **(C) $1\frac{2}{13}$** **(D) $2\frac{6}{13}$** **(E) $\frac{11}{13}$**

44. If $\frac{1}{2} - \frac{3}{4x} = \frac{5y}{6}$, then x equals _____.

(A) $\frac{3}{5} - \frac{9}{10y}$ **(B) $-\frac{3}{10y}$** **(C) $-\frac{5y}{3}$** **(D) $-\frac{9}{10y-6}$** **(E) $-\frac{3}{5y-2}$**

45. Which of the following mathematicians is best known for their work in four dimensional geometry and introduced the term "polytope"?

(A) Venn, John **(B) Hypatia** **(C) Smith, Karen** **(D) Zeno of Elea** **(E) Stott, Alicia**

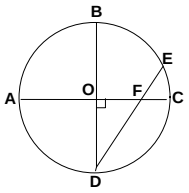
46. Five girls were nominated for homecoming queen and four boys for homecoming king at Royal T high school. How many ways can a king, a queen, and their court be chosen if their court consists of one girl and one boy?

(A) 126 (B) 336 (C) 120 (D) 16 (E) 240

47. Which of the following is not a solution to $2 + |5x + 1| \leq 7$?

(A) — 1.333... (B) — 0.7 (C) — 0.15 (D) 0.4 (E) 0.666...

48. Given the circle O with perpendicular diameters and a chord, find OF if EF = 4" and DF = 8". (nearest tenth)



(A) 6.9" (B) 5.7" (C) 3.5" (D) 4.0" (E) 5.3"

49. The base of a tree is 20 feet from the base of a flagpole. Both the tree and the flagpole are on level ground. The tree is shorter than the 48-ft flagpole. At some time during the day the shadows of both the tree and the flagpole end at the same point 60 ft from the base of the flagpole. How tall is the tree? (nearest foot)

(A) 25 ft (B) 17 ft (C) 38 ft (D) 20 ft. (E) 32 ft

50. Let x and y exist such that $4 < 8 < x < y$. If 4, 8, x form an arithmetic sequence and 8, x, y for a geometric sequence, then $x + y$ = ?

(A) 32 (B) 30 (C) 26 (D) 24 (E) 18

51. Willie Byette borrowed $12,000.00 for a used car. Part of the loan was at the rate of 8% per year and the rest of the loan was at 18% per year. If the interest was $1000.00, how much of the loan was at 18%?

(A) $666.67 (B) $400.00 (C) $1,500.00 (D) $333.33 (E) $11,600.00

52. Given that the set of natural numbers continue in the triangular pattern shown below, how many numbers will be on row 47?

			1				(row 1)
		2	3	4			(row 2)
	5	6	7	8	9		(row 3)
10	11	12	13	14	15	16	(row 4)
			...				(...)

(A) 91 (B) 92 (C) 93 (D) 94 (E) 95

53. The average monthly high temperature for Anchorage, Alaska in July is 65° F. The average monthly high temperature in January is 22° F. The average monthly high temperature of Anchorage varies sinusoidally with the month. What would be the predicted average high temperature for March? (nearest tenth)

(A) 43.5 ° **(B) 39.5 °** **(C) 38.8 °** **(D) 32.8 °** **(E) 29.2 °**

54. The graph of the parametric equations $x = \frac{2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$ is a(n) _____.

(A) circle **(B) ellipse** **(C) hyperbola** **(D) line** **(E) parabola**

55. Which of the following surfaces is generated by $x^2 = y^2 - z^2$?

(A) elliptic cone **(B) cylinder** **(C) ellipsoid**
(D) hyperbolic paraboloid **(E) elliptic paraboloid**

56. Willie Score throws a dart at the February, 2015 calendar hanging on the wall. Assuming the dart hits one of the dates on the calendar, what are the odds that the date he hit was a Lucas number (2, 1, 3, 4, ...)? Each date has an equal chance of being hit.

(A) $\frac{2}{7}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{7}$ **(E)** $\frac{7}{29}$

57. The *I Scream U Scream* Shoppe make great banana splits using three scoops of ice cream. The flavors of ice cream available are chocolate, vanilla, strawberry, mint swirl, pistachio, blueberry, and raspberry. How many different triple scoop banana splits can they create from the available flavors?

(A) 84 **(B) 72** **(C) 36** **(D) 35** **(E) 21**

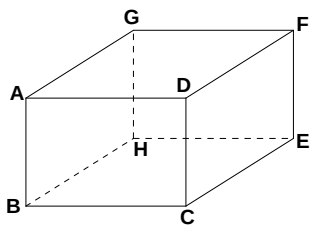
58. How many 3-digit numbers exist such that the sum of their digits equals 4?

(A) 8 **(B) 9** **(C) 10** **(D) 11** **(E) 12**

59. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. Find $(f_4)^2 + (f_5)^2$.

(A) 18 **(B) 20** **(C) 34** **(D) 40** **(E) 55**

60. Given the rectangular solid shown, find BF if AF = 6", FH = 4" and BG = 5". (nearest tenth)



(A) 8.8" **(B) 6.7"** **(C) 6.5"** **(D) 6.2"** **(E) 5.1"**

University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitation B ● 2015
Answer Key

1. E	**21. D**	**41. B**
2. A	**22. E**	**42. C, D, E**
3. A	**23. C**	**43. A**
4. B	**24. D**	**44. D**
5. E	**25. C**	**45. E**
6. C	**26. B**	**46. E**
7. C	**27. B**	**47. A**
8. D	**28. D**	**48. D**
9. C	**29. D**	**49. E**
10. A	**30. C**	**50. B**
11. B	**31. B**	**51. B**
12. B	**32. E**	**52. C**
13. C	**33. C**	**53. D**
14. A	**34. E**	**54. A**
15. D	**35. D**	**55. A**
16. C	**36. E**	**56. B**
17. E	**37. B**	**57. A**
18. B	**38. C**	**58. C**
19. A	**39. E**	**59. C**
20. C	**40. B**	**60. D**