



UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

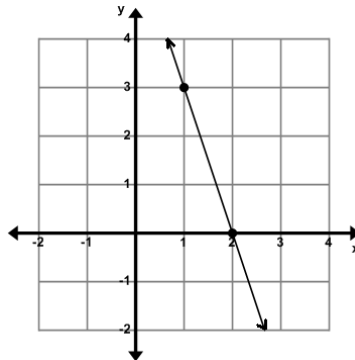
Mathematics

Invitational B • 2013



DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $\frac{1}{3} \div 0.1666... \times \frac{1}{9} + 0.08333... - \frac{1}{15}$
- (A) $\frac{7}{30}$ (B) 0.22777... (C) $\frac{1}{18}$ (D) 0.23888... (E) $\frac{1}{44}$
2. A waitress received a \$4.50 tip from one customer whose bill was \$25.00 and a \$6.00 tip from another customer whose bill was \$30.00. What was her average tip percentage?
- (A) 20% (B) 19% (C) 18% (D) 16% (E) 15%
3. Venn Circle Estates has 25 houses. Three houses have dogs, but not cats. Ten houses have cats and dogs. Eight houses have cats, but no dogs. How many house do not have a cat or a dog?
- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
4. A bag contains 1 white chip, 2 blue chips, and 3 red chips. What is the probability of randomly selecting two chips (without replacement) that are different colors?
- (A) $73\frac{1}{3}\%$ (B) 66% (C) 48% (D) $36\frac{2}{3}\%$ (E) $10\frac{2}{3}\%$
5. Two thousand thirteen plus thirty-one thousand two is subtracted from one million one hundred twenty-three thousand five hundred eight. The sum of the digits in the difference is?
- (A) 32 (B) 27 (C) 26 (D) 20 (E) 13
6. Line k contains point (1, 3) and is perpendicular to the line shown. Which of the following points lies on line k ?



- (A) (11, 7) (B) (10, 6) (C) (9, 3) (D) (13, 8) (E) (−7, 0)
7. The set $\{-1, 0, 1\}$ is closed under how many of these operations:
 + addition − subtraction × multiplication ÷ division $\sqrt{\quad}$ square root
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5
8. Simplify: $\left(\frac{x^2 - 9}{x^2 + 6x + 9}\right) \div \left(\frac{3x^2 + 10x + 3}{3x^2 - 8x - 3}\right)$
- (A) −1 (B) $\frac{x+3}{x-3}$ (C) 1 (D) $\frac{x-3}{x+3}$ (E) $\frac{x^2 - 6x + 9}{x^2 + 6x + 9}$

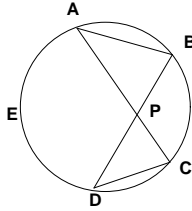
9. If S varies directly as the square of r and $S = \pi$ when $r = 0.5$, find r^2 when $S = 36\pi$.

- (A) 3 (B) 6 (C) 9 (D) $\frac{1}{3}$ (E) $3\sqrt{2}$

10. If a quadrilateral is inscribed in a circle, then its opposite angles are _____ angles.

- (A) right (B) supplementary (C) base (D) congruent (E) complementary

11. \overline{AB} , \overline{AC} , \overline{BD} , and \overline{CD} are chords of circle O . Find $m\angle APB$ if $m\widehat{AED} = 190^\circ$ and $m\widehat{BC} = 30^\circ$.

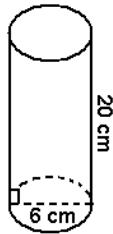


- (A) 70° (B) 80° (C) 90° (D) 100° (E) 110°

12. How many vertices does a Platonic solid have if it has 20 faces and 30 edges?

- (A) 2 (B) 10 (C) 12 (D) 18 (E) 20

13. Given the height and diameter find the lateral surface area of the cylinder. (nearest sq. cm.)



- (A) 754 cm^2 (B) 565 cm^2 (C) 502 cm^2 (D) 377 cm^2 (E) 314 cm^2

14. Willie Mixit has a 15-ounce solution of alcohol and water that is 75% alcohol. He wants to add some 67% alcohol solution to his solution to make a 72% alcohol solution. How many ounces of the 67% solution must he add?

- (A) 3 oz (B) 5 oz (C) 9 oz (D) 10 oz (E) 15 oz

15. Let $f(x) = \frac{2-5x}{2x+5}$. Find $f^{-1}(3)$.

- (A) $7\frac{1}{2}$ (B) $\frac{11}{13}$ (C) $\frac{1}{17}$ (D) $-1\frac{2}{11}$ (E) $-3\frac{2}{5}$

16. $ACE_{16} \times 8_{16} = \underline{\hspace{2cm}} 16$

- (A) 5670 (B) 456C (C) 765 (D) 72F (E) 158C

17. If $A + B = 15$ and $A \times B = 27$ then $|B - A| = \underline{\hspace{2cm}}$.

- (A) $3 + \sqrt{13}$ (B) $3\sqrt{13}$ (C) $2 - \sqrt{3}$ (D) $2\sqrt{3}$ (E) 3

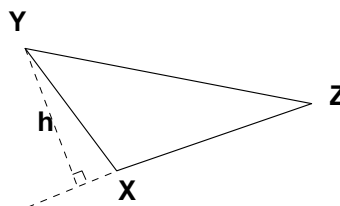
18. Let $f(x) = ax^3 - bx - 11$ where a , b , and c are constants. If $f(7) = -4$ then $f(-7) = ?$

- (A) 15 (B) 7 (C) -7 (D) -18 (E) -26

19. How many degrees are there in $6\frac{3}{4}\pi$ radians?

- (A) $2,250^\circ$ (B) 607.5° (C) $1,080^\circ$ (D) $1,215^\circ$ (E) 911.5°

20. Given the triangle shown, find the height, h , if $m\angle XZY = \frac{\pi}{6}$, $YZ = 22$ cm, and $XZ = 16$ cm.



- (A) 11 cm (B) 12.375 cm (C) 13.125 cm (D) 13.75 cm (E) 14 cm

21. If the $\sin \theta = 0.28$ and θ is in QII, then $\sec \theta$ is :

- (A) -0.68 (B) -0.96 (C) $-1\frac{1}{24}$ (D) $-3\frac{3}{7}$ (E) $-3\frac{4}{7}$

22. In the expansion of $(x - y)^6$, the sum of the coefficients of the 2nd, 4th, and 6th terms is:

- (A) -41 (B) -32 (C) -27 (D) -20 (E) -8

23. Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 2 \\ 1 & -3 \end{bmatrix}$. Find the sum of the elements of AB .

- (A) 24 (B) 18 (C) 12 (D) 6 (E) 0

24. Simplify: $3\log_a b - 2\log_a 2b + \log_a 3b$

- (A) $2\log_a(\frac{3b}{4})$ (B) $\log_a(2b)$ (C) $\log_a(\frac{3b^2}{4})$ (D) $2\log_a(\frac{2b}{3})$ (E) $3\log_a(\frac{b}{2})$

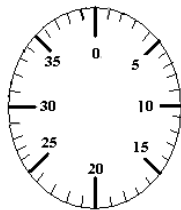
25. Point P has polar coordinates of $(5, \frac{\pi}{6})$ and rectangular coordinates of (x, y) . Where does point P lie on the Cartesian coordinate plane?

- (A) Q1 (B) QII (C) QIII (D) x-axis (E) y-axis

26. If $f'(x) = 2x^3 + 4x - 5$ and $f(2) = 0$, find $f(-1)$.

- (A) 5.5 (B) 1.5 (C) 1 (D) -1 (E) -3.5

27. U. R. SAFE combination locks have three number combinations. The first number of each combination is a prime number, the second is a Fibonacci number, and third digit is a triangular number. How many different combinations are possible?



- (A) 768 (B) 678 (C) 616 (D) 28 (E) 25
28. Evaluate: $\int_{-a}^a (2x^2 - 1) dx$
- (A) $\frac{4a^3}{3}$ (B) $2a^2 - a$ (C) $\frac{4a^3 - 6a}{3}$ (D) $\frac{2a^3 - 2a}{3}$ (E) $-2a$
29. Let $x^2 + y^2 = 16$. Find D_{xy} .
- (A) 4 (B) $x + y$ (C) $-\frac{x}{y}$ (D) $\frac{y}{x}$ (E) $\frac{x+y}{8}$
30. Lotta Scents has 2 quarters, 3 dimes, and 4 nickels. Each of the coins are distinguishable since they all have a different date on them. How many ways can she give her brother Noah Scents \$0.50?
- (A) 144 (B) 120 (C) 50 (D) 26 (E) 25
31. All of the elements of $\{14, 16, 19, 21, 25\}$ are considered to be __ numbers.
- (A) abundant (B) happy (C) lucky (D) odious (E) wasteful
32. The cardinality of the set of real numbers is strictly larger than the cardinality of the set of natural numbers is a major theorem proven by _____ .
- (A) John Napier (B) Georg Cantor (C) Christian Goldbach (D) Freda Porter (E) Euclid
33. The midradius (nearest tenth) of a dodecahedron with edge length of 10 cm is:
- (A) 2.6 cm (B) 6.5 cm (C) 8.1 cm (D) 11.6 cm (E) 13.1 cm
34. Using Blaise Pascal's triangle and letting the 1 at the top be row 0, determine which of the following numbers will be in row 11.
- (A) 136 (B) 222 (C) 330 (D) 451 (E) 505
35. Find the number of positive integral divisors of 1,620.
- (A) 30 (B) 25 (C) 20 (D) 15 (E) 10

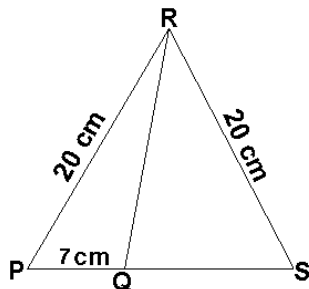
36. If R, S, and T represent digits then $RST_2 - ST_4 + R_8$ has a numeric value in base 10 of:

- (A) $9R - 2S$ (B) $5R - 2S$ (C) $5R + 6S$ (D) $9R - 2S - 2T$ (E) $10R - 2S$

37. Matt E. Mattiks wants to score 194 on this mathematics test. He gets 6 points for each correct answer and loses 2 points for each incorrect answer. Skipped problems are not counted. What is the least number of problems Matt has to work?

- (A) 33 (B) 35 (C) 37 (D) 39 (E) 41

38. Find $m\angle PQR$ of the equilateral triangle shown. (nearest degree)



- (A) 112° (B) 73° (C) 80° (D) 107° (E) 100°

39. Three years ago, Ima Old's age was one year more than twice Ura Young's age. Six years from now, Ima will be ten years more than half Ura's age. What is the difference in their ages now?

- (A) 4 years (B) 10 years (C) 5 years (D) 3 years (E) 6 years

40. Let P and Q be the roots of $3x^2 + 6x - 10 = 0$.
Find $P^5 + 5P^4Q + 10P^3Q^2 + 10P^2Q^3 + 5PQ^4 + Q^5$.

- (A) -12 (B) -15 (C) -19 (D) -21 (E) -32

41. If $\frac{2x}{5} - \frac{1}{6y} = \frac{3x}{4}$, then y equals _____.

- (A) $-\frac{11}{10x}$ (B) $-\frac{10}{21x}$ (C) $-\frac{21}{13x}$ (D) $\frac{13}{21x}$ (E) $\frac{20}{42x}$

42. If $a_1 = 5$, $a_2 = 4$ and $a_n = [(a_{n-1}) - (a_{n-2})]^{(n-3)}$ for $n \geq 3$, then a_5 equals:

- (A) 25 (B) 16 (C) 9 (D) -1 (E) -3

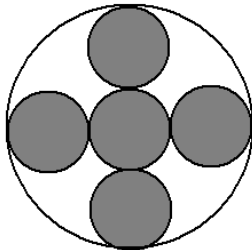
43. The circumcenter, centroid, and orthocenter of a scalene triangle are always:

- (A) concurrent (B) on the interior (C) collinear (D) equidistant (E) on the exterior

44. The Tic Toc Shop's circular clock stopped working at 11:20 pm. What was the measure of the smaller angle between the big hand and the little hand at that time?

- (A) 72° (B) 108° (C) 112° (D) 140° (E) 150°

45. The length of the latus rectum of $9x^2 - 4y^2 = 36$ is:
- (A) $2\frac{2}{3}$ (B) 4 (C) $5\frac{1}{3}$ (D) 9 (E) 18
46. Find the slope of the tangent line to $9x^2 + 4y^2 = 36$ at point $P(1\frac{1}{3}, -\sqrt{5})$. (nearest tenth)
- (A) 1.3 (B) 1.7 (C) 2.3 (D) 3.0 (E) 3.1
47. Captain Saul T. Water leaves port and sails his barge on a bearing of 40° for 20 miles. Then he changes course and sails 12 miles on a bearing of 75° . How far from port is the barge? (nearest tenth)
- (A) 27.3 mi (B) 21.9 mi (C) 32.0 mi (D) 35.0 mi (E) 30.6 mi
48. Find $(2 + 3i)^4$ and express the answer in standard form.
- (A) $61 - 30i$ (B) $-313 - 312i$ (C) $-119 - 120i$ (D) $-1 + 0i$ (E) $0 - 239i$
49. Let $\|V_1\| = 12$, $\|V_2\| = 15$, where the direction angles of V_1 and V_2 are 61° and 331° , respectively. Find the direction angle of $\|V_1 + V_2\|$. (nearest degree)
- (A) 10° (B) 13° (C) 22° (D) 24° (E) 27°
50. Use the Fibonacci characteristic sequence ... p, q, -3, r, 4 ... to Find $p + q + r$.
- (A) -5 (B) -14 (C) -1 (D) 1 (E) 4
51. Let $f(x) = 3x - 4$ and $g(x) = 5x + 2$. Find $g(f(x)) + f(g(x))$.
- (A) $2x + 6$ (B) $30x - 16$ (C) $8x - 2$ (D) $-2x - 6$ (E) -20
52. The outer four small circles are tangent to the big circle and to the inner small circle in the center. All five small circles are congruent. Will Amtel shoots an arrow and hits somewhere in the large circle. What are the odds that the arrow hit the shaded section?



- (A) 5:4 (B) 3:2 (C) 5:2 (D) 1:1 (E) 2:1
53. Given the function $f(x) = 2x^4 - 3x^3 - 2x^2 - x + 6$ there is a possibility of how many real zeroes?
- (A) 4, 2, or 0 (B) 3 or 1 (C) 2 or 0 (D) 2 (E) 0

54. Let $f(x) = \frac{1}{\sqrt{3 + 2x - x^2}}$. At which of these intervals is function f continuous?
- (A) $[-1, 3]$ (B) $(-3, 1]$ (C) $[-1, 3)$ (D) $(-1, 3)$ (E) $(-3, 1)$

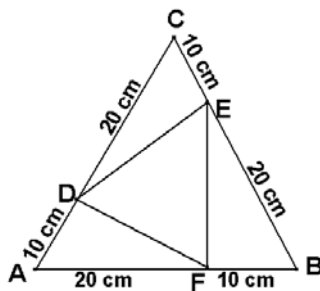
55. The polar graph of $r = 2 + 4\cos \theta$ is symmetric to the:

- (A) polar axis (B) pole (C) line $\theta = \frac{\pi}{2}$ (D) line $\theta = \frac{\pi}{4}$ (E) line $\theta = \frac{3\pi}{4}$

56. Determine the number of non-negative integer solutions to the equation: $p + q + r = 24$.

- (A) 2,024 (B) 1,012 (C) 775 (D) 325 (E) 276

57. Find DE to the nearest tenth.



- (A) 22.4 cm (B) 10 cm (C) 16.4 cm (D) 17.3 cm (E) 14.1 cm

58. The series $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots$ converges to _____.

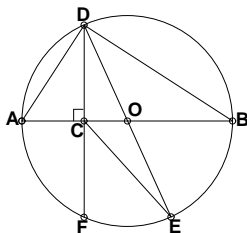
- (A) ∞ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{5}+1}{2}$ (D) 0 (E) $\frac{\pi}{4}$

59. Dee Sprinter ran 4 warm-up laps before practice. The following chart shows his lap speed. What was Dee's average speed? (nearest hundredth)

Lap	1	2	3	4
Speed	22.5 km/hr	20.6 km/hr	20.0 km/hr	24.0 km/hr

- (A) 21.83 km/hr (B) 21.78 km/hr (C) 21.72 km/hr (D) 21.66 km/hr (E) 21.12 km/hr

60. G. I. Amatree drew a circle with center O with $BC = 2 \cdot AC$. After doing some measuring he calculated the area of $\triangle ABD$ to be 48 sq. cm. Based on his calculations what would the area of $\triangle DCO$ be?



- (A) 6 sq. cm. (B) 8 sq. cm. (C) 9.6 sq. cm. (D) 12 sq. cm. (E) 14.4 sq. cm.

**University Interscholastic League
MATHEMATICS CONTEST
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Answer Key**

- | | | |
|-------|-------|-------|
| 1. D | 21. C | 41. B |
| 2. B | 22. B | 42. B |
| 3. E | 23. A | 43. C |
| 4. A | 24. C | 44. D |
| 5. C | 25. A | 45. D |
| 6. B | 26. B | 46. A |
| 7. A | 27. A | 47. E |
| 8. E | 28. C | 48. C |
| 9. C | 29. C | 49. A |
| 10. B | 30. C | 50. E |
| 11. A | 31. D | 51. B |
| 12. C | 32. B | 52. A |
| 13. D | 33. E | 53. A |
| 14. C | 34. C | 54. D |
| 15. D | 35. A | 55. A |
| 16. A | 36. B | 56. D |
| 17. B | 37. B | 57. D |
| 18. D | 38. E | 58. E |
| 19. D | 39. A | 59. D |
| 20. A | 40. E | 60. B |