UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

Invitational A • 2015

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YOU ARE INSTRUCTED TO DO SO!

1. **Evaluate:** $1 \div (1+2)^{-1} \times 3 - \frac{5}{8} + 13 \times (21)^0$

   (A) -21.125 **(B)** -11.375 **(C)** 11.375 **(D)** 20.125 **(E)** 21.375

2. **Saul Wood had a two-by-four board that was 12 feet long. He cuts it into 3 pieces such that the ratio of the lengths of the pieces are 2:3:5 with a 8 inch board left over. How long was the longest piece?**

   (A) 2 yds **(B)** 1 yd 2 ft 8 in **(C)** 1 yd 2 ft 5 in **(D)** 1 yd 2 ft 3.5 in **(E)** 1 yd 2 ft 2 in

3. **Find the sum of the arithmetic mean, median, mode, and range of 2, 18, 4, 7, 1, 11, 29, & 3.**

   (A) $38\frac{3}{8}$ **(B)** $40\frac{3}{8}$ **(C)** $41\frac{5}{8}$ **(D)** $42\frac{7}{8}$ **(E)** $43\frac{7}{8}$

4. **Dee Orr rows his boat at 4 mph from his pier to a platform on the lake. A speed boat returns him to his pier at 45 mph. The complete trip took 25 minutes. How far is it from the pier to the platform? (nearest tenth)**

   (A) 0.8 miles **(B)** 1.1 miles **(C)** 1.3 miles **(D)** 1.5 miles **(E)** 1.8 miles

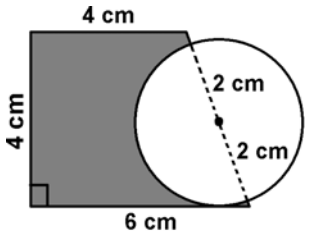
5. **Simplify:** $\left(\frac{x^3 - 21x - 20}{x - 1}\right) \times \left(\frac{x+1}{x^2 - x - 20}\right) \div \left(\frac{1}{x^2 - 1}\right)$

   (A) 1 **(B)** $x+1$ **(C)** $x-1$ **(D)** $x^2 + 2x + 1$ **(E)** $x^3 + 3x^2 + 3x + 1$

6. **Given: $\angle P$ is supplementary to $\angle Q$; $m\angle R = 48°$; and $\angle Q$ is complementary to $\angle R$. Find $m\angle P$.**

   (A) 42° **(B)** 52° **(C)** 128° **(D)** 132° **(E)** 138°

7. **Find the area of the shaded area. (nearest tenth)**



   (A) 7.4 cm^2 **(B)** 16.0 cm^2 **(C)** 17.7 cm^2 **(D)** 13.7 cm^2 **(E)** 22.3 cm^2

8. **The point of intersection of the 3 medians of a triangle is called a _____.**

   (A) center **(B)** centroid **(C)** circumcenter **(D)** incenter **(E)** orthocenter

9. **Lotta Cash, Les Sense, and Noah Dough have a total of $75.00. Noah has five dollars more than twice what Lotta has and Les has ten dollars less than Noah. How much more money does Les have then Lotta?**

   (A) $5.00 **(B)** $10.00 **(C)** $15.00 **(D)** $20.00 **(E)** $40.00

10. If $\frac{2x-3}{3x+2} - \frac{4x+1}{x-4} = \frac{Ax^2+Bx+C}{Px^2+Qx+R}$, then $\frac{A+B+C}{P+Q+R}$ equals:

(A) -2.8 **(B)** $-0.6181818...$ **(C)** $1.4666...$ **(D)** 1.8 **(E)** $2.1333...$

11. The fundamental period of the graph of $y = 1 - 2\sin^2(2x)$ is:

(A) $\frac{\pi}{4}$ **(B)** $\frac{\pi}{3}$ **(C)** $\frac{\pi}{2}$ **(D)** π **(E)** π^2

12. $\sin(\frac{\pi}{2} - \theta)$ equals:

(A) $-\cos(\frac{\pi}{2} - \theta)$ **(B)** $\sin(\theta + \frac{\pi}{2})$ **(C)** $\cos(\frac{\pi+\theta}{2})$ **(D)** $-\sin(\theta + \frac{\pi}{2})$ **(E)** $\sin(\frac{\pi-\theta}{2})$

13. Given the arithmetic sequence 15, a, b, 41.25, c, ..., find $a + b + c$.

(A) 43.75 **(B)** 70.3125 **(C)** 97.5 **(D)** 106.25 **(E)** 123.75

14. Find $m + n$ if $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 7 \\ 11 \end{bmatrix}$

(A) 2 **(B)** 4 **(C)** 8 **(D)** 10 **(E)** 18

15. Find the average rate of change over the interval [2,4] of $f(x) = x^2 + 4x$.

(A) 32 **(B)** 21 **(C)** 12 **(D)** 10 **(E)** 8

16. How many distinguishable arrangements can be made from the letters "TOOTSIEPOPS"?

(A) 720 **(B)** 831,600 **(C)** 15 **(D)** 55,440 **(E)** 1,663,200

17. Lou Cuss labels 8 blank cards with the numbers 2, 1, 3, 4, 7, 11, 18, and 29. One card is randomly drawn. What are the odds that the number is a Fibonacci number?

(A) $\frac{3}{5}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{2}{5}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{3}{8}$

18. Which of the following female mathematicians is known for her work in differential calculus?

(A) Agnesi **(B)** Hypatia **(C)** Germain **(D)** Kovalevsky **(E)** Noether

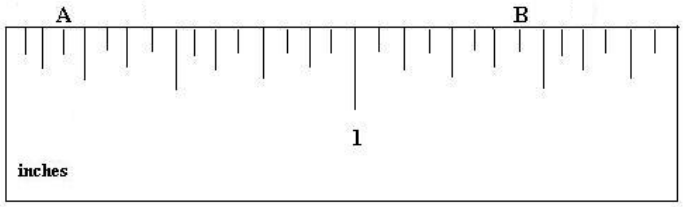
19. Find the arithmetic mean of the first three harmonic numbers.

(A) $2\frac{1}{6}$ **(B)** $1\frac{5}{6}$ **(C)** $1\frac{1}{2}$ **(D)** $1\frac{4}{9}$ **(E)** $\frac{11}{18}$

20. The number 678 in base 9 is equivalent to the number k in base 3. Find the sum of the digits in the number k.

(A) 9 **(B)** 8 **(C)** 6 **(D)** 4 **(E)** 3

21. Using the partial ruler shown below, find the distance from A to B.



(A) $1\frac{3}{8}$" **(B)** $1\frac{3}{16}$" **(C)** $1\frac{5}{8}$" **(D)** $1\frac{1}{4}$" **(E)** $1\frac{7}{16}$"

22. The *Texas Wild Seed* farm mixes 3 pounds of Bluebonnet seeds with 1.5 pounds of Indian Blanket seeds to form a special mixture of wild flower seeds. Find the cost of a half pound of the mixture if Bluebonnet seeds cost $1.25 per pound and Indian Blanket seeds cost 80¢ per pound?

(A) $045 **(B)** $0.55 **(C)** $0.90 **(D)** $1.03 **(E)** $1.10

23. Which of the following sets are closed under addition and/or multiplication?
C = {composite numbers} F = {Fibonacci numbers} M = {multiples of 5}

(A) C & M **(B)** M only **(C)** F only **(D)** C & F **(E)** C, F, & M

24. Which of the following quadrant(s) does not contain a solution to $3x + 4y > 7$ **?**

(A) QIV **(B)** QI & QII **(C)** QIII & QIV **(D)** QIII **(E)** Q1

25. Phil Whitwatter is filling up his empty circular water tank. The diameter of the tank is 12 feet and the height of the tank is 4 feet. What is the least number of whole gallons of water will he need to fill the tank half full?

(A) 1,129 gal **(B)** 1,693 gal **(C)** 1,765 gal **(D)** 1,975 gal **(E)** 2,257 gal

26. A triangle with side lengths of 11 dm, 8 dm, and 15 dm is a(n) _____ triangle.

(A) isosceles acute **(B)** scalene obtuse **(C)** isosceles obtuse **(D)** scalene acute **(E)** scalene right

27. Let $a_1 = 2$, $a_2 = 1$, $a_3 = 3$ **and** $a_n = (a_{n-3}) + [(a_{n-1}) - (a_{n-2})]$ **for** $n \geq 4$. **Find** a_6.

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** 11

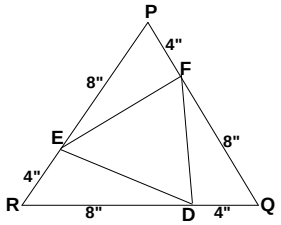
28. Simplify: $\log_3 x - 2\log_3 y + \log_3(0.5)$

(A) $-\log_3(xy^2)$ **(B)** $\frac{1}{2}\log_3(\frac{x}{y^2})$ **(C)** $\log_3(\frac{x}{2y^2})$ **(D)** $\log_3(\frac{x-y^2}{2})$ **(E)** $\log_3 x - y^2 + 0.5$

29. Which of the following equations in rectangular form can be written as $r - 12\cos\theta = 0$ **in polar form?**

(A) $x^2 - y^2 = 6$ **(B)** $x^2 + y^2 = 12$ **(C)** $x^2 + y^2 = 2\sqrt{3}$
(D) $y^2 - x^2 = 2\sqrt{3}$ **(E)** $(x-6)^2 + y^2 = 36$

30. Find the area of △DEF to the nearest tenth.



(A) 52.0 sq. in. (B) 17.4 sq. in (C) 62.4 sq. in (D) 17.9 sq. in (E) 20.8 sq. in

31. How many distinct solutions exist for $12\cos^2(x) - 5\cos(x) - 2 = 0$, where $-\frac{\pi}{2} < x < \frac{3\pi}{4}$?

(A) 8 (B) 5 (C) 3 (D) 4 (E) 7

32. Find the remainder when $f(x) = 4x^3 + 8x^2 - x - 2$ is divided by $x - 3$.

(A) -41 (B) -35 (C) 31 (D) 155 (E) 175

33. $\int (x^2 + 4x)\,dx$ = _____ + C, where C is some arbitrary constant.

(A) $\frac{x^3}{3} + 2x^2$ (B) $2x + 4$ (C) $x^3 + 2x^2$ (D) $3x + 2x^2$ (E) $\frac{x^3}{3} + 2x$

34. Find the area bounded by $y = 2x^2 + 2x - 3$ and $y = 2x - 1$. (square units).

(A) 3.5 (B) 3.75 (C) 2.666... (D) 4.25 (E) 4.5

35. Betty Kant has a stack of 8 cards consisting of J♠, J♥, J♦, J♣, Q♠, Q♥, Q♦, and Q♣. Betty shuffles the stack then deals out the top 3 cards. What is the probability that two of the cards dealt were Jacks and one was a Queen?

(A) $\frac{1}{56}$ (B) $\frac{1}{7}$ (C) $\frac{3}{28}$ (D) $\frac{3}{7}$ (E) $\frac{3}{8}$

36. Kandy Krunchur had a large bag of Tootsie Pops. She had chocolate ones, cherry ones, lime ones, strawberry ones, and raspberry ones. How many different small bags of 5 Pops can she package to sell?

(A) 126 (B) 25 (C) 120 (D) 24 (E) 1,512

37. Let P be a two-digit prime number less than 100 such that both digits are prime numbers. What is the sum of all such numbers, P?

(A) 348 (B) 253 (C) 221 (D) 186 (E) 113

38. Two of the roots of $f(x) = x^3 + bx^2 + cx + d$ are 3 and $2 + i$. Find $b + c + d$.

(A) -7 (B) -5 (C) 6 (D) 9 (E) 25

39. How many proper fractions in lowest terms have a denominator of 24?

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

40. Line *m* contains point (— 3, 4) and intersects the y-axis at y = — 5. An equation for line *m* is:

(A) 3x — y = 5 (B) 4x — 3y = — 5 (C) 3x + y = — 5 (D) 4x + 3y = 5 (E) —3x + 4y = 5

41. The point (3, 4) lies on a circle whose center is (0, 2). Where does the point (— 1, 5) lie in reference to the circle?

(A) on the circle (B) outside the circle (C) inside the circle

(D) in quadrant III (E) cannot be determined

42. Consider the complex number $0 + i$, find the value of $i^{-1} + i^{-2} + i^{-3} + i^{-4}$.

(A) 0 (B) 1 (C) — 1 (D) i (E) $-i$

43. The Real value solution set for $2 + 3|5x - 7| < 11$ is?

(A) $\{x \mid \{0.2 < x < 2.6\}$ (B) $\{x \mid \{x > 2\} \cup \{x < 0.8\}\}$ (C) $\{x \mid -2 < x < -\frac{4}{5}\}$

(D) $\{x \mid \{x > 2.6\} \cup \{x < -2\}$ (E) $\{x \mid 0.8 < x < 2\}$

44. A baseball groundskeeper uses his line striping machine to create a triangle for a 3-bag baseball game. He marks a line 90 feet on a bearing of 100° from home base to first base. Then he marks a line 100 feet on a bearing of 80° from first base to second base. How long is the line he marked from second base to home base? (nearest foot)

(A) 109 ft (B) 122 ft (C) 136 ft (D) 165 ft (E) 187 ft

45. The graph of the parametric equations x = 3t and y = 4t + 1 is a(n) _____.

(A) circle (B) ellipse (C) hyperbola (D) line (E) parabola

46. The harmonic mean of the real roots of $4x^3 + 8x^2 - x - 2 = 0$ is ?

(A) — 1.5 (B) — 2 (C) — 3.555... (D) — 6 (E) — 9

47. Find the y-intercept of the line tangent to the $3x^2 + 4y^2 = 48$ at the point (2,3).

(A) (0, 4) (B) (0, 8) (C) (0, 9) (D) (0, — 2) (E) (0, — 3)

48. If $f''(x) = 24x + 16$ and $f'(0) = -1$ and $f(1) = 9$, then $f(-1) =$ _____.

(A) 6 (B) 3 (C) — 4 (D) — 5 (E) — 9

49. Willie Luze plays a dice game that costs 50¢ to play. He rolls two dice and sums up the top faces. He wins $1.00 if the sum is 7 or 11 and loses 25¢ if the sum is not 7 or 11. What is the mathematical expectation of a single roll? (nearest cent)

(A) 53¢ loss (B) 47¢ loss (C) 4¢ loss (D) 3¢ gain (E) 46¢ gain

50. Let $f_0=0$, $f_1=1$, $f_2=1$, $f_3=2$, $f_4=3$, ... be the terms of the Fibonacci sequence. Find GCD(f_{15}, f_9).

(A) 8 (B) 6 (C) 3 (D) 2 (E) 1

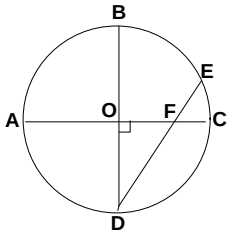
51. Ima Lost walks 1 foot north, then 2 feet west, then 3 feet south, then 4 feet east, then 5 feet north, then 6 feet west and so on, at 1 foot per second. What direction is Ima facing after walking 1 minute?

(A) north (B) west (C) south (D) east (E) northeast

52. The sum of all of the real values of x such that $\sqrt{x-6} = x\sqrt{x-6}$ is:

(A) 13 (B) 7 (C) 6 (D) 1 (E) 0

53. Given the circle O with perpendicular diameters and a chord, find BE if EF = 3" and DF = 7". (nearest tenth)



(A) 5.2" (B) 6.1" (C) 6.3" (D) 7.1" (E) 7.6"

54. Let $p^3 + q^3 = 4$ and $pq = 0.666...$. Find $p + q$.

(A) 2.666... (B) 2 (C) 1 (D) 1.333... (E) 0.1666...

55. Given that the set of natural numbers continue in the triangular pattern shown below, find the median of the numbers in row 12.

			1					(row 1)
		2	3	4				(row 2)
	5	6	7	8	9			(row 3)
10	11	12	13	14	15	16		(row 4)
			...					(...)

(A) 133 (B) 123 (C) 127 (D) 137 (E) 143

56. For which of the following values of θ is it true that $2^{\sin\theta} > 1$ and $3^{\cos\theta} < 1$?

(A) 35 ° **(B) 70 °** **(C) 140 °** **(D) 280 °** **(E) 560 °**

57. Which of the following surfaces is generated by $9x^2 - 72y + 16z^2 = 0$?

(A) cone **(B) cylinder** **(C) ellipsoid** **(D) hyperboloid** **(E) paraboloid**

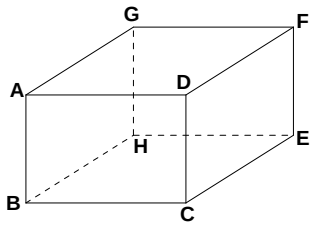
58. Yu-Noh randomly selects a positive integer less than 60 that is a multiple of 7. Yu-Dont randomly selects a positive integer less than 60 that is a multiple of 9. What is the probability that they selected the same number? (nearest percent)

(A) 78% **(B) 22%** **(C) 15%** **(D) 12%** **(E) 0%**

59. The square root of 1134 in base 5 is:

(A) 13_5 **(B) 113_5** **(C) 23_5** **(D) 114_5** **(E) 33_5**

60. Given the rectangular solid shown, find AE if AB = 3", BC = 5" and CE = 7". (nearest tenth)



(A) 15.0" **(B) 6.4"** **(C) 7.5"** **(D) 8.5"** **(E) 9.1"**

University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitation A ● 2015
Answer Key

1. E	**21. D**	**41. C**
2. B	**22. B**	**42. A**
3. D	**23. A**	**43. E**
4. D	**24. D**	**44. E**
5. E	**25. B**	**45. D**
6. E	**26. B**	**46. D**
7. D	**27. C**	**47. A**
8. B	**28. C**	**48. B**
9. B	**29. E**	**49. B**
10. C	**30. E**	**50. D**
11. C	**31. C**	**51. C**
12. B	**32. E**	**52. B**
13. D	**33. A**	**53. C**
14. A	**34. C**	**54. B**
15. D	**35. D**	**55. A**
16. B	**36. A**	**56. C**
17. A	**37. D**	**57. E**
18. A	**38. B**	**58. E**
19. D	**39. A**	**59. C**
20. A	**40. C**	**60. E**