UNIVERSITY INTERSCHOLASTIC LEAGUE
Making a World of Difference

Mathematics
Invitational A • 2014

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $5 \times 3 - (1 + 2 \div 4) \times 6$
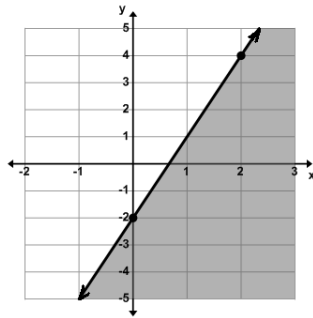
   (A) 6 (B) 7.5 (C) 10.5 (D) 24 (E) 81

2. Len Meecash borrowed $300.00 from the credit union at a simple interest rate of 6% for 9 months. How much interest will he have to pay? (nearest cent)

   (A) $12.00 (B) $13.50 (C) $15.00 (D) $16.20 (E) $18.00

3. What is $3\frac{1}{5}$ of 0.375 minus 11% of 0.090909... ?

   (A) 0.012 (B) 0.099 (C) 1.19 (D) 1.21 (E) 1.1526

4. The shaded area is best represented by which of the following inequalities?

   (A) $3x + 2y \geq 1$ (B) $2 \leq x - 3y$ (C) $3x - y \geq -2$ (D) $2 \leq 3x - y$ (E) $1 \geq 3x + 2y$

5. Simplify: $\dfrac{4x + 1}{3x - 2} - \dfrac{2x - 3}{x + 4}$

   (A) $\dfrac{-2x^2 + 4x - 10}{3x^2 + 5x - 8}$ (B) $\dfrac{10x^2 - 4x + 10}{3x^2 + 10x - 8}$ (C) $\dfrac{-2x^2 - 30x + 2}{3x^2 + 10x - 6}$

   (D) $\dfrac{10x^2 + 30x - 2}{3x^2 + 5x - 8}$ (E) $\dfrac{-2x^2 + 30x - 2}{3x^2 + 10x - 8}$

6. Flo Tilla is rowing her canoe on a lake. She starts rowing back toward the dock which is 500 yards away. She rows at a rate of 3 miles per hour. How long will it take her to reach the dock? (nearest second)

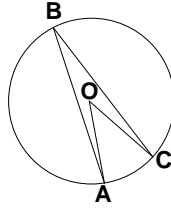   (A) 3 min 31 sec (B) 4 min 19 sec (C) 5 min 41 sec (D) 6 min (E) 6 min 40 sec

7. $\angle P$ and $\angle R$ are complementary. $\angle P$ and $\angle Q$ are supplementary. If $m\angle R = 42°$ then $m\angle Q = ?$

   (A) 122° (B) 132° (C) 58° (D) 48° (E) 138°

8. The numbers in row 9 of Pascal's triangle are 1, 9, k, 84, 126, 126, 84, k, 9, 1. Find k?

   (A) 18 (B) 27 (C) 36 (D) 45 (E) 54

9. Points A, B, and C lie on the circle with center O and $\overset{\frown}{AC} = 54°$. Find $m\angle AOC + m\angle ABC$.

    (A) 40.5°   (B) 54°   (C) 71.5°   (D) 81°   (E) 108°

10. A cylindrical water tank is 12 feet high and has a diameter 8 feet long. The tank contains 480 gallons of water. What per cent of the tanks maximum capacity is water? (nearest %)

    (A) 80%   (B) 53%   (C) 39%   (D) 11%   (E) 3%

11. Find the circumference of the circle, $x^2 + y^2 - 6x - 10y + 30 = 0$. (nearest tenth)

    (A) 17.1   (B) 12.6   (C) 19.3   (D) 14.7   (E) 25.1

12. How many 5 character passwords are possible if the first character must be a positive digit, the second must be a letter, the third must be a vowel (excluding y and w), the fourth must be 0, and the fifth one must be a special character from this set {@, #, $, &,+} ? Repetition is allowed.

    (A) 5,850   (B) 46   (C) 5,625   (D) 55   (E) 6500

13. Find the sum of the coefficients of the quotient: $(4x^3 + 3x^2 + 2x - 9) \div (x - 1)$

    (A) 7   (B) 9   (C) 12   (D) 20   (E) 22

14. The Sue Purr Deer tractor can plow a field in 4 hours 20 minutes. The Anne Teek Deer tractor takes three times as long to plow the same field. How long would it take them if they work together? (nearest minute)

    (A) 2 hrs 10 min   (B) 2 hrs 53 min   (C) 3 hrs   (D) 3 hrs 15 min   (E) 3 hrs 40 min

15. $\triangle ABC$ is an equilateral triangle. Find $m\angle BCD$. (nearest degree)
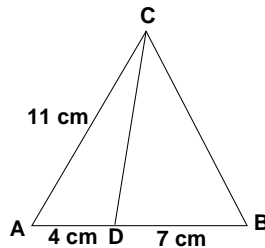

    (A) 28°   (B) 35°   (C) 39°   (D) 40°   (E) 50°

16. For all values of θ, which of the following has the same value as $\cos(\frac{3\pi}{4} - \theta)$?

(A) $\sin(\theta + \frac{\pi}{4})$ (B) $\cos(\theta + \frac{3\pi}{4})$ (C) $\sin(\theta - \frac{5\pi}{4})$ (D) $\cos(\theta + \frac{3\pi}{4})$ (E) $\sin(\theta - \frac{\pi}{4})$

17. Hi Stepper, a drum major, is marking his path on the football field for the band's performance at half time. He walks 200 feet on a bearing of 110°. Then he turns and walks 180 feet on a bearing of 25°. The he turns to return to his starting point. What bearing will he need to take to return directly back to his starting point? (nearest degree)

(A) 250°　　　(B) 245°　　　(C) 230°　　　(D) 225°　　　(E) 205°

18. Use the Fibonacci characteristic sequence ... 1, p, 3, q, r, ... to Find $p + q + r$.

(A) 19　　　(B) 15　　　(C) 13　　　(D) 11　　　(E) 9

19. If the set of numbers {1, 2, 3, 4, 5, ...} continue in the triangular pattern shown below, the arithmetic mean of the numbers in the 8th row would be?

$$1$$
$$2 \quad 3 \quad 4$$
$$5 \quad 6 \quad 7 \quad 8 \quad 9$$
$$10 \quad 11 \quad 12 \quad 13 \quad 14 \quad 15 \quad 16$$
$$...$$

(A) 51　　　(B) 53　　　(C) 57　　　(D) 61　　　(E) 63

20. In the binomial expansion of $(3x - 2)^4$, the coefficient of the 3rd term is:

(A) 216　　　(B) 180　　　(C) 144　　　(D) 125　　　(E) 118

21. Let $f(x) = 5x^2 - 3x - 2$ and $g(x) = 2x - 1$. Find $g(f'(x + 1))$.

(A) $10x - 3$　　(B) $20x + 13$　　(C) $10x - 7$　　(D) $20x - 4$　　(E) $20x + 5$

22. The polar graph of $r^2 = 36\cos(2\theta)$ is symmetric to the: (1) polar axis (2) pole (3) line $\theta = \frac{\pi}{2}$

(A) 1 only　　(B) 2 only　　(C) 3 only　　(D) 1 & 3　　(E) 1, 2, & 3

23. A water tank in the shape of an inverted cone has an altitude of 16 feet and a base diameter of 8 feet. Water is flowing into the tank at a rate of 2 cubic feet per minute. How fast is the water rising when the water is 5 feet deep? (nearest tenth)

(A) 2.4 in/min　(B) 3.7 in/min　(C) 4.9 in/min　(D) 7.4 in/min　(E) 9.8 in/min

24. The current Texas Supreme Court consists of 2 women and seven men. Special committees consist of 4 members. In how many ways can a special committee be formed such that at least one woman is on the committee?

(A) 1,470　　(B) 42　　(C) 70　　(D) 126　　(E) 91

25. **Willie Rollette rolls a pair of standard dice. What is the probability that the sum of the top faces is a Fibonacci number?**

   (A) $11\frac{1}{9}\%$ (B) 25%   (C) 30%   (D) $33\frac{1}{3}\%$ (E) $44\frac{4}{9}\%$

26. **How many 3-digit numbers can be made using the digits 1, 1, 2, 3, and 5?**

   (A) 30   (B) 33   (C) 36   (D) 39   (E) 42

27. **The repeating decimal 0.4222... in base 8 can be written as which of the following fractions in base 8?**

   (A) $\frac{15}{28}_{8}$ (B) $\frac{25}{40}_{8}$ (C) $\frac{17}{34}_{8}$ (D) $\frac{21}{32}_{8}$ (E) $\frac{1}{2}_{8}$

28. **Find the sum of all of the two-digit numbers whose units digit is divisible by 5.**

   (A) 1,050   (B) 945   (C) 900   (D) 855   (E) 765

29. $1001_2 + 301_4 + 53_8 = \underline{\hspace{3cm}}_{10}$

   (A) 1,354   (B) 248   (C) 84   (D) 91   (E) 101

30. **A cash register drawer contains $1 bills, $5 bills, and $10 bills. The ratio of $1 bills to $5 bills is 5 to 6 and the ratio of $5 bills to $10 bills is 3 to 2. The total amount of money in the cash register drawer is $300.00. How many $5 bills are in the drawer?**

   (A) 12   (B) 16   (C) 20   (D) 24   (E) 30

31. **Four years ago Tu Yung was half as old as Soh Yung was. In four years Tu's age will be two-thirds of the age Soh will be. What is the sum of their ages now?**

   (A) 12   (B) 20   (C) 32   (D) 36   (E) 40

32. **Line m passing through $(2, -1)$ and $(1, -3)$ is perpendicular to line n passing through $(2, -3)$. Line n intersects the x-axis at (x, y). Find x.**

   (A) -4 (B) -3 (C) 1   (D) 2   (E) 8

33. **The solution set of $3x + 4y = 25$ is $\left\{(x, y)\mid -5 \leq x \leq 7, y \geq 0, \text{ and } x,y \in \{\text{Integers}\}\right\}$. How many such ordered pairs exist?**

   (A) 2   (B) 3   (C) 4   (D) 5   (E) 6

34. **Which of the following are the side lengths of a scalene acute triangle?**

   (A) 7, 24, 25   (B) 7, 7, 11   (C) 9, 10, 11   (D) 6, 9, 12   (E) 8, 13, 21

35. The four lines in the figure are coplanar with $m \parallel l$. Which of the following are true statements?
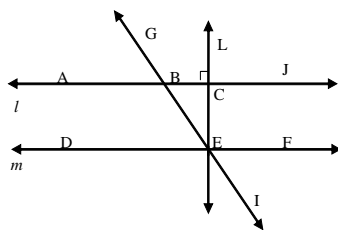
   1. $\angle BEC$ & $\angle GBA$ are congruent   **2.** $m\angle FEI + m\angle CEB = 90°$
   3. $\angle FEI$ & $\angle ABG$ are vertical angles   **4.** $\angle CBE$ & $\angle GEF$ are supplementary angles

  (A) 2 & 4   (B) 1 & 2   (C) 1, 2, 3,& 4   (D) 3 & 4   (E) 1 & 4

36. Point P $(-4, 0)$ is translated 3 units up vertically to point Q. Point Q is reflected across the y-axis to point R. Point R is rotated clockwise $\frac{\pi}{2}$ radians about the origin to point S. Point S is translated 3 units left horizontally to point T (x, y). Find $x + y$.

  (A) -8 (B) -4 (C) -2 (D) 0   (E) 4

37. If $a_1 = 1$, $a_2 = -3$, $a_3 = 2$ and $a_n = a_{n-3} - a_{n-2} + a_{n-1}$, where $n \geq 4$, then a_8 equals:

  (A) -2 (B) -1 (C) 0   (D) 3   (E) 6

38. If $xy - \frac{1}{4} = y - x = 4 - x - y$, what is the value of $x + y$?

  (A) 2.125   (B) 2.75   (C) 3.375   (D) 3.75   (E) 4.25

39. Betty Whens spins the spinner. It will land on a $1.00 bill, a $5.00 bill, or $10.00 bill. The probability of landing on a $1.00 bill is 60%, landing on a $5.00 bill is 25%, and landing on a $10.00 bill is 15%. What is the mathematical expectation on any one spin?

  (A) $1.17   (B) $2.15   (C) $2.75   (D) $3.25   (E) $3.35

40. Let $f(x) = x^2 - 2x - 5$ and $g(x) = 4x + 3$. Find $g^{-1}\Big(g\big(f(-1) \big) \Big)$.

  (A) 2.5   (B) 0.75   (C) -1 (D) -1.25 (E) -2
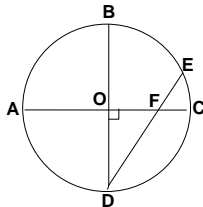
41. The graph of the polar equation $r = 3 + 4\sin(\theta)$ is a:

  (A) dimpled cardioid   (B) looped cardioid   (C) dimpled limacon

  (D) looped limacon   (E) convex limacon

42. Which of the following is a reference angle for $2014°$?

  (A) 214°   (B) 146°   (C) 56°   (D) 34°   (E) 17°

43. Given the circle O with perpendicular diameters and a chord, find the area of circle O if EF = 1 inch and DF = 6 inches. (nearest square inch)

    (A) 66 sq. in.   (B) 77 sq. in.   (C) 44 sq. in.   (D) 57 sq. in.   (E) 79 sq. in.

44. If $\begin{bmatrix} 2 & -1 \\ 3 & -4 \end{bmatrix} \times \begin{bmatrix} 4 & -3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $a \times d - b \times c = ?$

    (A) 11   (B) 22   (C) 25   (D) 55   (E) 121

45. The harmonic mean of the real roots of $x^3 - 7\frac{1}{2}x^2 + 15\frac{1}{2}x + k = 0$ is $1\frac{5}{31}$. Find k.

    (A) $-1\frac{14}{31}$ (B) -2 (C) $-2\frac{28}{31}$ (D) -6 (E) -8

46. Solve for x: $\log_2\left[\log_3\left(\log_4\left(x^{3x}\right)\right)\right] = 0$

    (A) 4   (B) 3   (C) 2   (D) 1   (E) 0

47. Find the area, in square units, of the figure bounded by $y = x^2 - 4$ and $y = 5$.

    (A) 40   (B) 36   (C) 32   (D) 30   (E) 27

48. If $3x^2 - 4y = 5$ and $x > 0$, then $\frac{dy}{dx} = \frac{dx}{dy}$ when x = ?

    (A) 0.75   (B) 0.6   (C) 0.8   (D) 0.666...   (E) 0.444...

49. Les Scents flipped a penny four times and recorded the results. What are the odds of only two consecutive heads or only two consecutive tails?

    (A) 6   (B) $\frac{1}{2}$ (C) $\frac{3}{8}$ (D) $\frac{3}{5}$ (E) 1

50. A package of 25 DVD discs contains some good discs and some bad discs. The probability of selecting a good disc is 92%. How many bad discs can be expected to be in the package?

    (A) 1   (B) 2   (C) 3   (D) 5   (E) 8

51. The *Achilles* is one of the four famous paradoxes that is credited to?

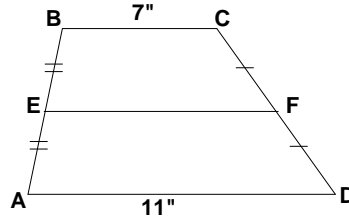   (A) Parmenidas (B) Archimedes (C) Diophantus (D) Theano   (E) Zeno

52. The fourth *harmonic number* is:

  (A) 0.25 (B) 1.5 (C) 2.08333... (D) 2.28333... (E) 5.8333...

53. Let $f(x) = x^3 - 6x^2 + 9x + 5.$ Find the sum of the x-values of the critical points of f(x).

  (A) 4 (B) 3 (C) 2 (D) 0 (E) -5

54. Given the trapezoid shown where AD∥EF∥BC, find EF. (nearest hundredth)

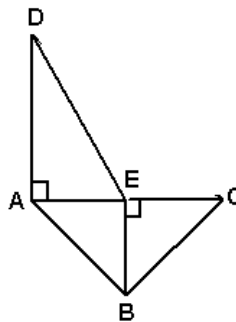 (A) 9.22 " (B) 9.00 " (C) 8.92 " (D) 8.77 " (E) 8.56 "

55. The altitude of △ABC forms two right triangles, △ABD and △CBD. Find AD if CD = 8 cm, AB = 12 cm, and m∠CBD = 50°. (nearest tenth)

  (A) 8.9 cm (B) 7.3 cm (C) 9.9 cm (D) 6.7 cm (E) 11.8 cm

56. The roots of the equation $x^3 + kx^2 - 18x + 40 = 0$ are 2, 5, and R. Find k.

  (A) 7 (B) 3 (C) -3 (D) -4 (E) -11

57. Given: m∠DAB = 135°, m∠BED = 150°, m∠ECB = 45°and BE = 4 cm. Find the perimeter of the figure shown. (nearest tenth).

  (A) 25.7 cm (B) 26.9 cm (C) 27.3 cm (D) 29.2 cm (E) 30.2 cm

58. A coordinate is plotted on the Cartesian plane. It's abscissa is less than zero and its ordinate is greater than zero. Where is the coordinate located on the plane?

  (A) Quadrant I (B) Quadrant II (C) Quadrant III (D) Quadrant IV (E) Origin

59. Let A = {t,r,i,p,o,d}, B = {t,r,a,p,e,z,o,i,d}, and C = {t,r,o,p,h,y}. The number of distinct elements in A ∪ (B ∩ C) is _____.

   (A) 3   (B) 4   (C) 5   (D) 6   (E) 7

60. The Mayan number system consists of three symbols, ● ▬ ⬭ . The symbols have a value of 1, 5, and 0 respectively. They use base 20 instead of base 10 and write their numbers vertically instead of horizontally. What base 10 number would this Mayan number represent?

   (A) 6   (B) 15   (C) 25   (D) 51   (E) 101

University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitation A ● 2014
Answer Key

1. A	21. B	41. D
2. B	22. E	42. D
3. C	23. C	43. A
4. D	24. E	44. E
5. E	25. D	45. D
6. C	26. B	46. C
7. B	27. C	47. B
8. C	28. B	48. D
9. D	29. E	49. E
10. D	30. D	50. B
11. B	31. C	51. E
12. A	32. A	52. C
13. D	33. C	53. A
14. D	34. C	54. B
15. C	35. A	55. C
16. E	36. C	56. C
17. A	37. E	57. E
18. B	38. B	58. B
19. C	39. E	59. D
20. A	40. E	60. C