



UNIVERSITY INTERSCHOLASTIC LEAGUE
Making a World of Difference

Mathematics

District 1 • 2015

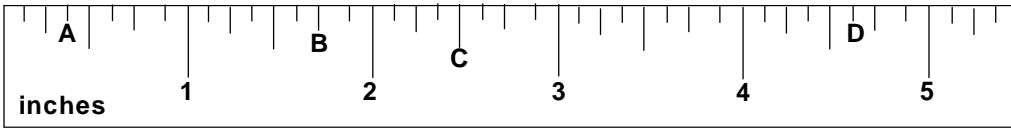


DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. $3 + 2 \times 3 - 3 \div 2 \times 8 + (20 - 15)$

- (A) 2 (B) 5.1875 (C) 5.375 (D) 13.8125 (E) 53

2. Using the partial ruler shown below, find the difference in the lengths of AB and CD.



- (A) $\frac{3}{8}$ " (B) $\frac{3}{16}$ " (C) $\frac{1}{2}$ " (D) $\frac{3}{4}$ " (E) $\frac{1}{4}$ "

3. Three million two hundred eighty-two thousand fifteen plus five million one hundred two thousand three hundred twenty three is subtracted from one billion. What is the digit that appears the most number of times in the difference?

- (A) 0 (B) 2 (C) 5 (D) 6 (E) 9

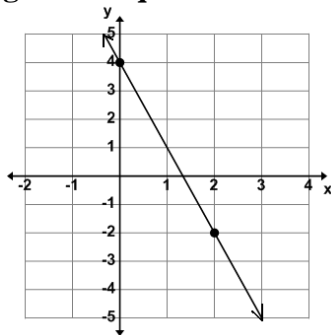
4. Simplify: $(a^{-3} \times b^3)^{-1} \div (a^2 \times b^{-2})^2 \times (a \div b)^3$

- (A) a^3b^{-3} (B) $a^{-2}b^{-4}$ (C) a^2b^{-2} (D) ab (E) a^6b^6

5. Bill Spender, Len Meekash, and Penni Les spent the day at the mall. Bill spent \$3.00 more than Len spent. Len spent twice as much as Penni spent. When they left the mall Bill still had \$5.00, Len had \$2.00 and Penni had \$.50. Together they spent \$23.00. How much money did Len have when they went into the mall?

- (A) \$11.00 (B) \$10.00 (C) \$8.50 (D) \$8.00 (E) \$4.50

6. Which of the following linear equations is best represented by this graph?



- (A) $3x + y = 4$ (B) $3x - 2y = 4$ (C) $2x + 3y = 4$ (D) $x - 3y = 4$ (E) $2x + 3y = 4$

7. Simplify: $\left(\frac{6x^2 + x - 2}{4x^3 - 16x^2 - x + 4}\right) \div \left(\frac{9x^2 + 12x + 4}{6x^2 + 7x + 2}\right)$

- (A) $x^2 - 8x + 16$ (B) $\frac{x-4}{x+4}$ (C) $x^2 - 16$ (D) $\frac{1}{x-4}$ (E) $x + 4$

8. Which of the following properties, is used to go from step 3 to step 4?

Step

1	$5(k - 2)$	$= 5$
2	$5k - 10$	$= 5$
3	$5k - 10 + 10$	$= 5 + 10$
4	$5k + 0$	$= 5 + 10$
5	$5k$	$= 15$
6	$5k \times \frac{1}{5}$	$= 15 \times \frac{1}{5}$
7	k	$= 3$

- (A) distributive (B) additive inverse (C) additive identity
 (D) multiplicative identity (E) transitive

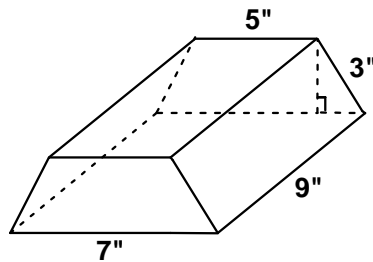
9. Willis A. Nutt mixed some almonds worth \$5.00 a pound with some cashews worth \$6.50 a pound. How many pounds of cashews did he mix with the almonds to make 10 pounds of mixed nuts that sells for \$6.00 a pound?

- (A) $4\frac{1}{3}$ lbs (B) $1\frac{1}{5}$ lbs (C) $6\frac{2}{3}$ lbs (D) 5 lbs (E) $1\frac{4}{5}$ lbs

10. The set $\{-1, 0, 1\}$ is closed under which of the following operations:
 + addition - subtraction \times multiplication \div division

- (A) + & \times (B) +, -, & \times (C) \times only (D) none of the four (E) all four

11. Find the lateral surface area of the isosceles trapezoid prism shown. (nearest sq. in). Drawing is not to scale.



- (A) 216 sq. in (B) 48 sq. in (C) 162 sq. in (D) 54 sq. in (E) 210 sq. in

12. The ratio of the length to the width of a rectangle is 10:6. If 5 units are added to both the length and the width, then the ratio of the length to the width is now 3:2. What is the difference in the areas of the two rectangles? (square units)

- (A) 225 (B) 250 (C) 275 (D) 300 (E) 325

13. The point of intersection of the 3 medians of a triangle is called a(n) _____.

- (A) center (B) centroid (C) circumcenter (D) incenter (E) orthocenter

14. If $\frac{A}{3x-2} + \frac{B}{2x+1} = \frac{x-10}{6x^2-x-2}$, where A and B are constants, then A + B equals:

- (A) -4 (B) -1 (C) 1 (D) 3 (E) 7

15. Let $A = \begin{bmatrix} -2 & -3 \\ 5 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 10 & -6 \\ 3 & -1 \end{bmatrix}$. Find $|A^T + B^T|$.

- (A) 35 (B) -24 (C) 120 (D) 118 (E) -1

16. Find the value of $(4 + i^1) + (3 + i^2) + (2 + i^3) + (1 + i^4)$.

- (A) -10 (B) 0 (C) 2 (D) 9 (E) 10

17. Which of the following mathematicians is considered to be the "Father of Symbolic Logic"?

- (A) Alicia Stott (B) John Venn (C) George Boole (D) John Napier (E) Georg Cantor

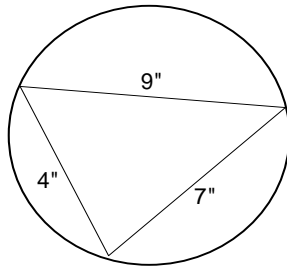
18. The *Wildflower* seed company's research data shows that the probability that a seed will germinate and grow into a plant is 70%. What are the odds that it won't germinate and grow into a plant?

- (A) $\frac{3}{10}$ (B) $\frac{1}{3}$ (C) $\frac{3}{4}$ (D) $\frac{3}{7}$ (E) $\frac{7}{10}$

19. How many 8-letter code words can be formed using the letters in the word COMMERCE?

- (A) 5,040 (B) 6,720 (C) 336 (D) 248 (E) 20,160

20. Find the diameter of the circle. Drawing is not to scale. (nearest tenth)



- (A) 9.4 " (B) 10.0 " (C) 11.9 " (D) 12.6 " (E) 13.3 "

21. The graph of the parametric equations $x = t^2 + t$ and $y = 2t - 1$ is a(n) _____.

- (A) semicircle (B) ellipse (C) line (D) cycloid (E) parabola

22. The frequency of the graph of $y = 1 + 2\sin^2(\frac{\pi}{6}x - 3)$ is:

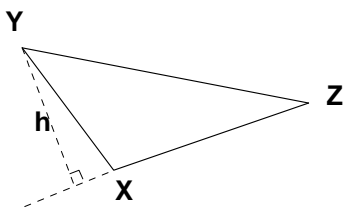
- (A) .08333... (B) 0.1666... (C) 0.333... (D) 0.314... (E) 0.261666...

32. Will E. Pikett randomly selects an odd integer less than 100 that is a multiple of 3. Betty Wont randomly selects an odd integer less than 100 that is a multiple of 5. What is the probability that they selected the same number? (nearest tenth)
- (A) 2.8% (B) 3.0% (C) 5.3% (D) 3.7% (E) 1.8%
33. The sum of the first three *lucky prime numbers* is:
- (A) 10 (B) 11 (C) 12 (D) 19 (E) 23
34. $(323_5 + 201_5) \times 4_5 = \underline{\hspace{2cm}}_5$
- (A) 4011 (B) 4101 (C) 4121 (D) 4201 (E) 4211
35. The square root of 1161 in base 8 is:
- (A) 31_8 (B) 34_8 (C) 27_8 (D) 41_8 (E) 37_8
36. Cookie Baykur made a batch of cookies. She gave 60% of the cookies to her daughter for her class party. She kept $\frac{2}{3}$ of the cookies she had left for her evening tea party. She gave 4 of the remaining cookies to her son and his friend for an afternoon snack. There were 2 cookies left for her husband. How many cookies did she bake originally?
- (A) 48 (B) 45 (C) 40 (D) 36 (E) 35
37. Line m contains points $(4, 1)$ and $(-2, -3)$. Which of the following equations of line n exists such that line n contains the point $(3, -2)$ and $m \perp n$.
- (A) $2x + 3y = 5$ (B) $2x - 3y = -5$ (C) $5x + 3y = -2$ (D) $3x - 2y = -5$ (E) $3x + 2y = 5$
38. If the roots of $2x^3 + bx^2 + cx + d = 0$ are $-4, 2,$ and $6,$ then $b + c + d$ equals:
- (A) 16 (B) -24 (C) 48 (D) -12 (E) 4
39. The point $(3, -3)$ lies on a circle whose center is $(-3, -3)$. Where does the point $(1, 2)$ lie in reference to the circle?
- (A) on the circle (B) outside the circle (C) inside the circle
(D) in quadrant II (E) cannot be determined
40. The range of the relation $(x + 3)^2 + (y + 3)^2 < 36$ is:
- (A) $[-6, 6]$ (B) $(-8, 2)$ (C) $(-2, 5)$ (D) $(-4, 4)$ (E) $(-9, 3)$
41. How many integral values of n exist such that $n \geq 0$ and $\frac{(n+2)!}{n!} \leq 20$
- (A) none (B) 3 (C) 4 (D) 6 (E) 9

42. If $a_1 = -2$, $a_2 = 2$, $a_3 = 4$, $a_n = (a_{n-1}) \div (a_{n-3}) - (a_{n-2})$, where $n \geq 4$, then a_6 equals:

- (A) 6 (B) 4.5 (C) 3 (D) 2.5 (E) 1.5

43. A triangle is drawn as shown. Find h if $m\angle XZY = 25^\circ$, $XY = 33''$, and $YZ = 75''$. (nearest tenth)



- (A) 36.9'' (B) 33.2'' (C) 31.4'' (D) 31.7'' (E) 30.8''

44. $\triangle DEF$ exists such that $m\angle DEF = 90^\circ$ and point M is the midpoint of segment DF . If $EM = 12.5$ cm, and $DE = 24$ cm, find $m\angle FME$. (nearest hundredth)

- (A) 31.39° (B) 32.52° (C) 35.50° (D) 36.87° (E) 37.15°

45. The harmonic mean of the real roots of $2x^3 + 9x^2 + 3x - 4 = 0$ is ? (nearest tenth)

- (A) 1.3 (B) 4.0 (C) 5.5 (D) -2.3 (E) -1.5

46. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. Find $\text{GCD}(f_{16}, f_{12})$.

- (A) 3 (B) 4 (C) 5 (D) 8 (E) 14

47. The probability that statement P is false is $\frac{3}{10}$, and the probability that statement Q is true is $\frac{7}{8}$. Determine the probability that $P \rightarrow Q$ is false.

- (A) $\frac{21}{80}$ (B) $\frac{23}{40}$ (C) $\frac{3}{40}$ (D) $\frac{7}{80}$ (E) $\frac{3}{80}$

48. Find the area bounded by $y = 1 - x^2$, $y = x - 6$, $x = -1$, and $x = 1$. (square units).

- (A) 12 (B) $12\frac{5}{6}$ (C) $13\frac{1}{3}$ (D) $13\frac{2}{3}$ (E) $14\frac{1}{6}$

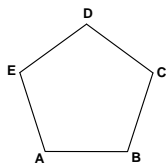
49. The function $f(x) = x^4 - x^3 + 1$ has inflection points at $x = a$ and $x = b$. Find $a + b$.

- (A) $\frac{1}{2}$ (B) 0 (C) 1 (D) -1 (E) $-\frac{1}{2}$

50. Which of the following equations in polar form can be written as $3x - 2y = 2$ in rectangular form?

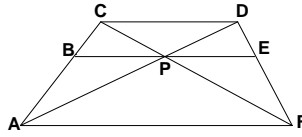
- (A) $r(3\sin \theta - 2\cos \theta) = 2$ (B) $r(3\cos \theta + 2\sin \theta) = \sqrt{2}$ (C) $r(2\cos \theta - 3\sin \theta) = 2\sqrt{2}$
 (D) $r(3\cos \theta - 2\sin \theta) = 2$ (E) $r(3\sin \theta + 2\cos \theta) = 4$

51. Let $f(x) = x^2 - bx + c$. If $f(x)$ is divided by $x - 2$ the remainder is 6 and if $f(x)$ is divided by $x + 3$ the remainder is 1. Find $b + c$.
- (A) -4 (B) -2 (C) 0 (D) 1 (E) 5
52. The expansion of $(3x - 1)(3x - 2)(3x - 3)$ is $ax^3 + bx^2 + cx + d$. Find $(ab) \div (cd)$.
- (A) $6.888\dots$ (B) $7.363636\dots$ (C) $1.222\dots$ (D) $8.181818\dots$ (E) $3.444\dots$
53. Len Meech borrowed \$500.00 to help pay for college books. Part of the loan was at the simple interest rate of 4% per year and the rest of the loan was at the simple interest rate of 6% per year. At the end of the year he paid off the loan and the interest totaling \$526.50. How much of the loan was at 4%?
- (A) \$375.00 (B) \$325.00 (C) \$250.00 (D) \$175.00 (E) \$125.00
54. Ranger Chris P. Kritter looks down from the Woodworth Tower and sees a forest fire coming toward the ranger station tower. The tower is 175 feet tall and the angle of depression from his eyes to the base of the fire is 10° . The speed of the fire is estimated to be moving at 3 feet per minute. How long does Chris have before the fire reaches the tower? (nearest minute).
- (A) 5 hrs 0 min (B) 5 hrs 31 min (C) 5 hrs 15 min (D) 5 hrs 7 min (E) 6 hrs 5 min
55. Seymore Wirk and Doug Upp can dig a trench for a water line in 8 hours when working at the same time. Seymore notices that Doug works twice as fast as he does. How long would it take Doug to dig the trench by himself?
- (A) 24 hrs (B) 16 hrs (C) 12 hrs (D) 4 hrs (E) $2.666\dots$ hrs
56. A right triangle has a hypotenuse of length 20". If one of the acute angles is decreasing at the rate of 6° per second, how fast is the area of the triangle decreasing when this acute angle is 21° ? (nearest tenth)
- (A) $3.4 \text{ in}^2/\text{sec}$ (B) $15.8 \text{ in}^2/\text{sec}$ (C) $13.5 \text{ in}^2/\text{sec}$ (D) $7.8 \text{ in}^2/\text{sec}$ (E) $15.6 \text{ in}^2/\text{sec}$
57. Willie Drawrite puts 5 country DVDs, 7 rock DVDs, and 3 blank DVDs in a bag. He randomly chooses 3 DVDs, without replacement. What is the probability that Willie chose one country, one rock, and one blank DVD? (nearest per cent)
- (A) 8% (B) 12% (C) 34% (D) 58% (E) 23%
58. Given the regular pentagon shown, find BD if $AB = 8''$. (nearest tenth)



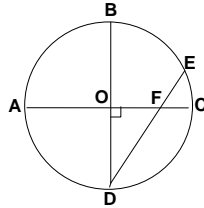
- (A) 11.3" (B) 12.9" (C) 12.0" (D) 13.9" (E) 6.5"

59. Given the trapezoid shown where segments AF , BE , and CD are parallel to each other and the three interior segments are concurrent at point P , find the length of segment BP if $CD = 18$ cm and $AF = 26$ cm. (nearest hundredth)



- (A) 10.82 cm (B) 9.91 cm (C) 11.00 cm (D) 10.35 cm (E) 10.64 cm

60. Given the circle O with perpendicular diameters and a chord, find BE if $DE = 11''$ and $DF = 7''$. (nearest tenth)



- (A) 5.2'' (B) 6.3'' (C) 7.1'' (D) 6.1'' (E) 5.7''

**University Interscholastic League
MATHEMATICS CONTEST
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Answer Key**

- | | | |
|-------|-------|-------|
| 1. A | 21. E | 41. C |
| 2. D | 22. B | 42. D |
| 3. D | 23. E | 43. D |
| 4. C | 24. C | 44. B |
| 5. B | 25. B | 45. B |
| 6. A | 26. A | 46. A |
| 7. D | 27. C | 47. D |
| 8. B | 28. D | 48. C |
| 9. C | 29. C | 49. A |
| 10. C | 30. B | 50. D |
| 11. C | 31. C | 51. A |
| 12. A | 32. E | 52. B |
| 13. B | 33. E | 53. D |
| 14. B | 34. E | 54. B |
| 15. C | 35. A | 55. C |
| 16. E | 36. B | 56. E |
| 17. C | 37. E | 57. E |
| 18. D | 38. C | 58. B |
| 19. A | 39. B | 59. E |
| 20. A | 40. E | 60. E |