



UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

District 1 • 2013



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YOU ARE INSTRUCTED TO DO SO!

1. Evaluate the following: $4! + (8)^{\frac{2}{3}} \div (12^{(-1)} \times \sqrt{16}) - 20$

- (A) $-29\frac{1}{3}$ (B) $-3\frac{11}{12}$ (C) 16 (D) 115 (E) 196

2. Clara Nett owns a music store. She has guitars on a "tax free" sale this weekend. If Vi Ollinn buys two guitars she gets 25% off the price for both guitars. And if Vi buys three more guitars she gets 40% off of the price for the three guitars. If the price of a guitar is \$150.00, how much will it cost Vi if she buys all five guitars?

- (A) \$510.00 (B) \$495.00 (C) \$487.50 (D) \$450.00 (E) \$255.00

3. 440 inches per second = _____ miles per hour.

- (A) 88 (B) 70 (C) 55 (D) 40 (E) 25

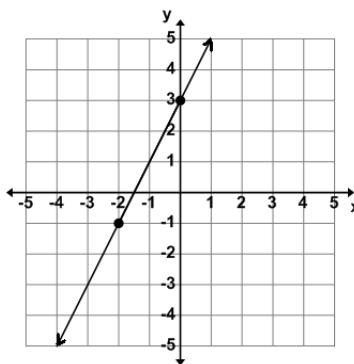
4. $3 \times 5 \times 7 = 15 \times 7 = 105$ and $3 \times 5 \times 7 = 3 \times 35 = 105$ are examples of the _____ properties of equality.

- (A) associative (B) commutative (C) distributive (D) identity (E) inverse

5. Simplify: $\left(\frac{x^2 + 6x + 9}{x - 3}\right) \left(\frac{x^2 - 6x + 9}{x^2 - 9}\right)$

- (A) $x - 3$ (B) $x + 3$ (C) $x^2 + 9$ (D) $x^2 - 9$ (E) $x^2 + 3x - 6$

6. A line perpendicular to the line shown intersects the line at point $(-1, 1)$ and has a y-intercept at point (x, y) . Find y.



- (A) 0 (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

7. Truth tables are mathematical tables used in an area of math best developed by which of the following mathematicians?

- (A) Goldbach (B) Boole (C) Venn (D) Cantor (E) Agnesi

8. One year ago, L. Paso was two and a half times as old as his daughter, K. Paso. In five years her father will be twice as old as her. What is the sum of their ages now?

- (A) 31 years (B) 36 years (C) 42 years (D) 44 years (E) 54 years

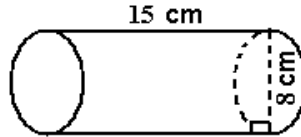
9. If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is always a(n) _____ triangle.

- (A) acute (B) isosceles (C) equilateral (D) obtuse (E) scalene

10. A regular icosahedron has F faces, E edges, and V vertices. Find $F + E + V$.

- (A) 70 (B) 62 (C) 52 (D) 26 (E) 14

11. Find the total surface area of the right cylinder shown. (nearest cm^2)



- (A) 377 cm^2 (B) 478 cm^2 (C) 528 cm^2 (D) 578 cm^2 (E) 754 cm^2

12. Which of the following points of concurrency are always on the interior of any triangle?

- (1) circumcenter (2) centroid (3) orthocenter (4) incenter

- (A) 2 only (B) 2, 3, & 4 (C) 1 & 3 (D) 2 & 4 (E) all of them

13. Harry Face uses an after-shave lotion that contains 50% ethyl alcohol. How much water does Harry need to add to 6 fluid ounces of his lotion to reduce it to 25% ethyl alcohol?

- (A) 3 fl. oz. (B) 4.5 fl. oz. (C) 6 fl. oz. (D) 6.75 fl. oz. (E) 18 fl. oz.

14. If $\frac{3x+4}{2x-3} + \frac{x+2}{4x-5} = \frac{Ax^2+Bx+C}{Px^2+Qx+R}$, then $\frac{A+B+C}{P+Q+R}$ equals:

- (A) $-\frac{1}{3}$ (B) $-\frac{10}{29}$ (C) -4 (D) -10 (E) -14

15. The length of the latus rectum of $16x^2 - 25y^2 = 400$ is:

- (A) 6.4 (B) 12.5 (C) 16 (D) 20 (E) 25

16. Find the area of the circle, $x^2 + y^2 - 10x + 6y - 18 = 0$. (nearest tenth)

- (A) 45.3 (B) 56.6 (C) 106.8 (D) 163.4 (E) 171.1

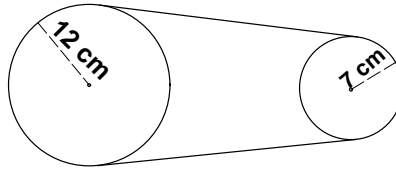
17. Smokie D. Bare spots two camp fires from his ranger station. One camp fire is 3 miles on a bearing of 75° and the other is 5 miles on a bearing of 225° . How far apart are the two camp fires? (nearest tenth)

- (A) 8.0 mi (B) 7.7 mi (C) 7.0 mi (D) 5.5 mi (E) 4.4 mi

18. Let $(2 + 3i)^4 = a + bi$. Find $a - b$.

- (A) -239 (B) -193 (C) 625 (D) 31 (E) 1

19. A belt joins the two pulleys shown. If the larger pulley is rotating at 10 rpm, then the smaller pulley is rotating at _____ rpm . (nearest tenth)

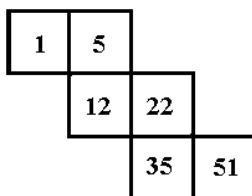


- (A) 17.1 (B) 18.4 (C) 19.0 (D) 5.0 (E) 5.8
20. Let $f(x) = 4x - 3$ and $g(x) = 2x + 5$. Find $f(f(x)) - g(g(x))$.
- (A) $12x - 22$ (B) $2x - 8$ (C) $20x - 2$ (D) $12x - 30$ (E) $20x$
21. Perri Mitter drew this figure on dot paper. The dots are 1 cm apart from each other both vertically and horizontally. Find the perimeter of the shaded region. (nearest tenth)
-
- (A) 6 cm (B) 15 cm (C) 17.7 cm (D) 19.3 cm (E) 37.8 cm
22. Seymore Sand can fill Pebble's sand box in 3 hours. If Saul T. Water helps Seymore they can fill the sand box in 2.25 hours working together. How long would it take Saul to fill it by himself?
- (A) 2 hrs 37.5 min (B) 3 hrs 22.5 min (C) 5 hrs 15 min (D) 6 hrs 45 min (E) 9 hrs 0 min
23. If $f'(x) = 6x - 4$ and $f(-1) = 12$, find $f(2)$.
- (A) 6 (B) 8 (C) 9 (D) 11 (E) 24
24. The series $\frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \frac{1}{21} + \dots$ converges to _____.
- (A) ∞ (B) 2 (C) $\frac{\sqrt{5}+1}{2}$ (D) e (E) $\frac{\pi^2}{6}$
25. The polar graph of $r = 3 - 3\sin \theta$ is symmetric to the: (1) polar axis (2) pole (3) line $\theta = \frac{\pi}{2}$
- (A) 1 only (B) 2 only (C) 3 only (D) 2 & 3 (E) 1, 2, & 3
26. The Texannas and the Calgirls play two games during the season. The Texannas are three times as likely to win any game as is the Calgirls. What is the probability that the Texannas will win both games?
- (A) 75% (B) $66\frac{2}{3}\%$ (C) $62\frac{1}{2}\%$ (D) $56\frac{1}{4}\%$ (E) 50 %

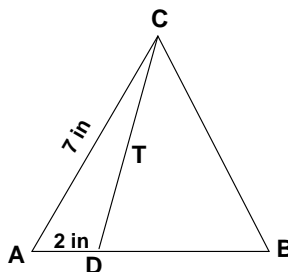
27. April Showers charted the Spring rainfall data as follows. What was the average rainfall for the 4 Spring months? (nearest hundredth)

| <u>Month</u> | <u>Inches of rain</u> |
|--------------|-----------------------|
| March | 4.5 |
| April | 6 |
| May | 4.6 |
| June | 3.1 |

- (A) 4.27" (B) 4.31" (C) 4.43" (D) 4.55" (E) 4.66"
28. Using Blaise Pascal's triangle, determine the sum of all of the numbers in the first 12 rows.
- (A) 1,023 (B) 2,048 (C) 4,095 (D) 6,145 (E) 8,192
29. Plato folds the net shown into a cube. He adds the numbers on each pair of opposite sides. What is the largest sum possible?



- (A) 52 (B) 56 (C) 63 (D) 73 (E) 86
30. $111_4 + 1111_2 = \text{_____}_2$
- (A) 10110 (B) 1010 (C) 11001 (D) 100100 (E) 1001110
31. Juan Prop flies his small plane 120 miles with the wind in 1 hour 20 minutes. The return trip against the wind takes 1 hours 40 minutes. What was the speed of the wind?
- (A) 4.5 mph (B) 6 mph (C) 9 mph (D) 10 mph (E) 13.5 mph
32. If $\frac{x-15}{x+16} + \frac{x+16}{x-15}$ is written as the mixed number $A \frac{B}{C}$, then B is?
- (A) 1 (B) 31 (C) 32 (D) 62 (E) 961
33. Given the equilateral triangle shown, find T^2 .



- (A) 21 (B) 22.5 (C) 39 (D) 45 (E) 63.5

34. If $a_1 = -3$, $a_2 = 3$ and $a_n = (a_{n-2}) \div (a_{n-1})$ for $n \geq 3$, then a_7 equals:

- (A) $\frac{1}{3}$ (B) $\frac{1}{9}$ (C) $-\frac{1}{27}$ (D) -3 (E) -9

35. If $A + B = 20$ and $A \times B = 16$ then $|B - A| = \underline{\hspace{2cm}}$.

- (A) $4 + 2\sqrt{5}$ (B) $5\sqrt{42}$ (C) $2\sqrt{5} - 4$ (D) $4\sqrt{11}$ (E) $4\sqrt{21}$

36. Let $f(x) = \begin{cases} kx - 1 & \text{if } x < 2 \\ kx^2 & \text{if } 2 \leq x. \end{cases}$ Find the value of k that makes $f(x)$ continuous on $(-\infty, +\infty)$.

- (A) $-\frac{1}{2}$ (B) $-\frac{1}{4}$ (C) 4 (D) 2 (E) $\frac{1}{6}$

37. The Texas Senate consists of 31 senators. Twelve of them are selected to serve on three special committees. In how many ways can the 12 senators be divided, so that 5 senators are on the budget committee, 3 on the governor's committee, and 4 on the highways committee?

- (A) 50,388 (B) 27,720 (C) 13,860 (D) 7,440 (E) 828

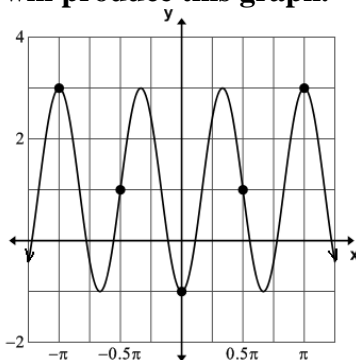
38. $FACF_{16} \div D_{16} = \underline{\hspace{2cm}} 16$

- (A) 1012 (B) 1234 (C) 1347 (D) 13A5 (E) 134B

39. Which of the following is not in the solution set of $2|3 + 5x| - 7 < 11$?

- (A) $-2\frac{2}{3}$ (B) $-2\frac{1}{7}$ (C) $-1\frac{7}{11}$ (D) $\frac{1}{2}$ (E) $1\frac{1}{6}$

40. The equation $y = \underline{\hspace{2cm}}$ will produce this graph.



- (A) $2\cos(3x) - 1$ (B) $2\sin(x) - 3$ (C) $-2\cos(3x) + 1$ (D) $2\cos(3x) + 1$ (E) $2\sin(3x) - 1$

41. Use the Fibonacci characteristic sequence ... p , q , -3 , r , s , 3 , ... to find $(p + q) - (r + s)$.

- (A) 12 (B) 9 (C) 0 (D) -3 (E) -6

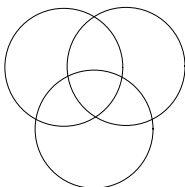
42. Let $A = \begin{bmatrix} -1 & 1 \\ 2 & -3 \end{bmatrix}$. Find $\det(A + A^T)$.

- (A) 3 (B) 2 (C) 1 (D) -2 (E) -17

43. How many distinguishable arrangements could Plato make from the letters DODECAHEDRA?

- (A) 1,663,200 (B) 1,200,000 (C) 483,840 (D) 40,320 (E) 7,920

44. M. T. Pockets surveys 26 men and finds out that 5 have dimes, 14 have nickels, and 10 have pennies. One has only dimes and pennies, 3 have only nickels and dimes, and 4 have only pennies and nickels. If no one has all three coins, how many men do not have any of these coins? (This diagram might help.)



- (A) 0 (B) 2 (C) 5 (D) 6 (E) 8

45. Which of the following is considered to be an *equidigital* number?

- (A) 21 (B) 22 (C) 33 (D) 36 (E) 44

46. If the roots of $x^3 + bx^2 + cx + d = 0$ are $-5, 3,$ and $8,$ then $b + c + d$ equals:

- (A) -145 (B) -95 (C) 0 (D) 6 (E) 83

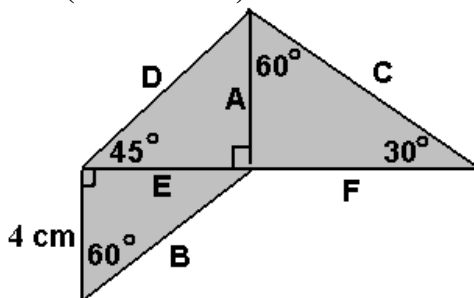
47. Simplify: $\frac{\cos^2(\theta)}{(1 - \sin^2(\theta))^2}$

- (A) $\sec^2(\theta)$ (B) $\csc^2(\theta)$ (C) 1 (D) $\cos(\theta)$ (E) $\sec(\theta)$

48. Dee Kart connected the points $A(1, 3), B(4, -4),$ and $D(-3, -2)$ forming an acute, scalene triangle. What is the area of this triangle?

- (A) 20 sq. units (B) 21.5 sq. units (C) 22 sq. units (D) 22.5 sq. units (E) 23 sq. units

49. Find the area of the shaded region. (nearest cm^2).



- (A) 79.4 cm^2 (B) 69.3 cm^2 (C) 94 cm^2 (D) 83.1 cm^2 (E) 112 cm^2

50. Find the slope of the tangent line to $25x^2 - 16y^2 = 400$ at point $P(-5, 3.75)$.

- (A) $-2\frac{1}{12}$ (B) $-1\frac{2}{3}$ (C) $-1\frac{5}{12}$ (D) $-1\frac{1}{3}$ (E) $-1\frac{11}{64}$

51. Let $\|V_1\| = 18$, $\|V_2\| = 10$, where the direction angles of V_1 and V_2 are 20° and 80° , respectively. Find the direction angle of $V_1 + V_2$. (nearest degree)

- (A) 21° (B) 41° (C) 50° (D) 60° (E) 100°

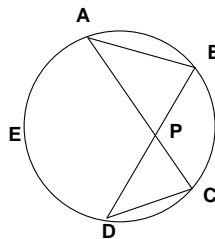
52. Each of the first five "happy" numbers, 1, 7, 10, 13, and 19 are written on blank sheets of paper. Three pieces of paper are randomly selected. What are the odds that the sum of the "happy" numbers selected is an even number?

- (A) 1 to 4 (B) 3 to 2 (C) 1 to 2 (D) 2 to 3 (E) 2 to 1

53. The volume, nearest cubic inch, of a regular octahedron with edge length 4" is:

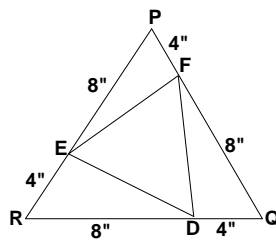
- (A) 6 cu. in. (B) 8 cu. in. (C) 23 cu. in. (D) 30 cu. in. (E) 32 cu. in.

54. \overline{AB} , \overline{AC} , \overline{BD} , and \overline{CD} are chords of circle O. If $m\widehat{CD} = 42^\circ$ and $m\widehat{AB} = 68^\circ$ then $m\angle BPC = ?$



- (A) 55° (B) 62.5° (C) 70° (D) 110° (E) 125°

55. Find the area of $\triangle DEF$ to the nearest inch.



- (A) 62" (B) 35" (C) 27" (D) 21" (E) 12"

56. Let $f(x) = \frac{x^3 + 7x + 1}{x^2 - x + 1}$ and $s(x)$ be the slant asymptote of f . Find the value of $s(-1)$.

- (A) $-2\frac{1}{3}$ (B) -2 (C) 0 (D) $2\frac{2}{3}$ (E) 3

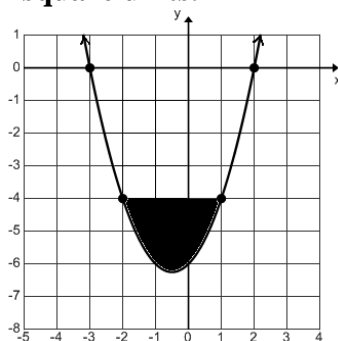
57. An infinite geometric sequence has a common ratio of $\frac{3}{4}$ and a sum of $26\frac{2}{3}$. What is the first term of the sequence?

- (A) $35\frac{5}{9}$ (B) $6\frac{2}{3}$ (C) 20 (D) $3\frac{2}{9}$ (E) $6\frac{1}{12}$

58. Determine the concavity of the graph of $f(x) = 2\sin(x) + 3\cos(x)$ at $x = \frac{\pi}{4}$.

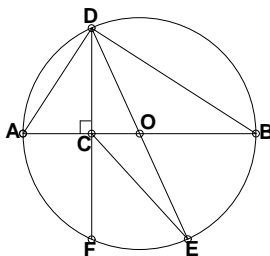
- (A) $-\frac{\sqrt{2}}{2}$ (B) $-\frac{5\sqrt{2}}{2}$ (C) -1 (D) $\sqrt{2}$ (E) $\frac{3\sqrt{2}}{2}$

59. Find the area of the shaded region in square units.



- (A) 3 (B) 3.25 (C) 4 (D) 4.5 (E) 4.75

60. G. I. Amaree drew a circle with center O with $BC = 2AC$, $CD = 3\sqrt{2}$ ", and $AC = 3$ ". Based on the information and the drawing what is the area of $\triangle CDE$? (nearest tenth)



- (A) 2.1 sq. in. (B) 3.2 sq. in. (C) 6.4 sq. in. (D) 12.7 sq. in. (E) 19.1 sq. in.

**University Interscholastic League
MATHEMATICS CONTEST
HS • District 1 • 2013
Answer Key**

- | | | |
|-------|-------|-------|
| 1. C | 21. C | 41. E |
| 2. B | 22. E | 42. A |
| 3. E | 23. C | 43. A |
| 4. A | 24. B | 44. C |
| 5. B | 25. C | 45. A |
| 6. E | 26. D | 46. E |
| 7. B | 27. D | 47. A |
| 8. D | 28. C | 48. B |
| 9. A | 29. C | 49. A |
| 10. B | 30. D | 50. A |
| 11. B | 31. C | 51. B |
| 12. D | 32. E | 52. B |
| 13. C | 33. C | 53. D |
| 14. D | 34. C | 54. E |
| 15. A | 35. E | 55. D |
| 16. D | 36. A | 56. C |
| 17. B | 37. B | 57. B |
| 18. E | 38. E | 58. B |
| 19. A | 39. A | 59. D |
| 20. D | 40. C | 60. C |